This formula satisfies all the properties required by construction. The Regge behavior is already built in as can be shown easily in the following way: Let us keep s_3 and s_5 fixed and take $s_1 s_2$ and s_4 to $-\infty$. Then the region of integration that contributes corresponds to $f_1 f_2 f_4$ near 1. Expanding the logarithms of those functions around 1 it is easy to show that formula (10) tends to

$$[-\alpha(s_2)]^{\alpha(s_3)}[-\alpha(s_1)]^{\alpha(s_5)}\Gamma(-\alpha(s_3)) \times \int_0^\infty dx\, e^{-x} x^{-\alpha(s_5)-1} \times [1-x\alpha(s_4)/\alpha(s_1)\alpha(s_2)]^{\alpha(s_3)}.$$

For $s_4 \approx s_1 s_2$ this expression reduces to

$$\sum[-\alpha(s_2)]^{\alpha(s_3)-k}[-\alpha(s_1)]^{\alpha(s_5)-k} s_4^{\,k} \times \Gamma(-\alpha(s_3)+k)\Gamma(-\alpha(s_5)+k)/\Gamma(k+1),$$

which shows the characteristic double Regge behavior.

The author acknowledges useful discussions with C. Goebel and B. Sakita.

---

*Work supported in part by the University of Wisconsin Research Committee, with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-88 and No. COO-210.

[1] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1772 (1968); C. Schmid, Phys. Rev. Letters 20, 689 (1968).

[2] G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

[3] G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[4] J. Shapiro and J. Yellin, to be published; C. Lovelace, CERN Report No. TH/950 (unpublished); M. A. Virasoro, to be published.

[5] M. Ademollo et al., Phys. Rev. (to be published).

[6] M. A. Virasoro, Phys. Rev. (to be published).

STUDY OF THE 1450-MeV $p\pi^+\pi^-$ ENHANCEMENT IN THE REACTION $K^+p \to K^+\pi^+\pi^-p$ AT 5.5 BeV/c*

P. Antich, A. Callahan, R. Carson, B. Cox, D. Denegri, L. Ettlinger, D. Feiock,
D. Gillespie,† G. Goodman, G. Luste,‡ R. Mercer, A. Pevsner, and R. Zdanis
The Johns Hopkins University, Baltimore, Maryland

(Received 12 November 1968)

A highly peripheral $p\pi^+\pi^-$ enhancement centered near 1450 MeV, observed in $K^+p \to K^+\pi^+\pi^-p$ at 5.5 BeV/c, is shown to be consistent with a $\Delta^{++}\pi^-$ system with $I=\frac{1}{2}$. We observe a large asymmetry in the $\Delta^{++}\pi^-$ angular distribution which is not consistent with a pure resonant effect. Dissociation-type models fit the data reasonably well when cuts are made on the $K^+\pi^-$ mass, and this is interpreted as evidence for diffractive $K^+\pi^-$ virtual scattering in this $K^+\pi^-$ mass region.

Considerable interest has recently been directed toward nucleon-pion spectra observed in missing-mass spectrometer experiments[1,2] and various bubble-chamber experiments with a proton beam incident at various momenta from 6 to 30 BeV/c.[3,4] Of special interest is the wide (200-MeV) enhancement near 1400 MeV, whose interpretation as a resonance, possibly the Roper resonance predicted by πp phase-shift analysis,[5] is uncertain due to its position at the low end of the mass spectrum.[6]

We report here a study of the reaction

$$K^+p \to K^+\pi^-\pi^+p \qquad (1)$$

occurring in the 80-in. Brookhaven National Laboratory hydrogen bubble chamber exposed to 5.5-BeV/c incident momentum kaons. The film analyzed to date corresponds to 5.3 events/μb in Reaction (1).

The $p\pi^+\pi^-$ spectrum in this reaction exhibits an enhancement centered near 1470 MeV with width in excess of 200 MeV, as shown in Fig. 1. In this figure, we omit those events in which either the $K^*(890)$ or $K^*(1400)$ is formed; that is, events for which 840 MeV $<M(K^+\pi^-)<$ 940 MeV or 1320 MeV $<M(K^+\pi^-)<$ 1480 MeV. Figure 1(a) shows events for which the momentum-transfer squared from target proton to the final $p\pi\pi$ system is less than 0.15 (BeV/c)^2. The absence of an enhancement in the 1450 region when the cut $0.15<\Delta^2<0.5$ is made, Fig. 1(b), indicates the highly peripheral nature of the enhancement.

As we later discuss the effect on the $p\pi\pi$ mass of making various $K\pi$ cuts, we show in Fig. 1(c) the $K^+\pi^-$ invariant-mass spectrum for those events in which Δ^2(incident K^+ to final K^+) <0.5

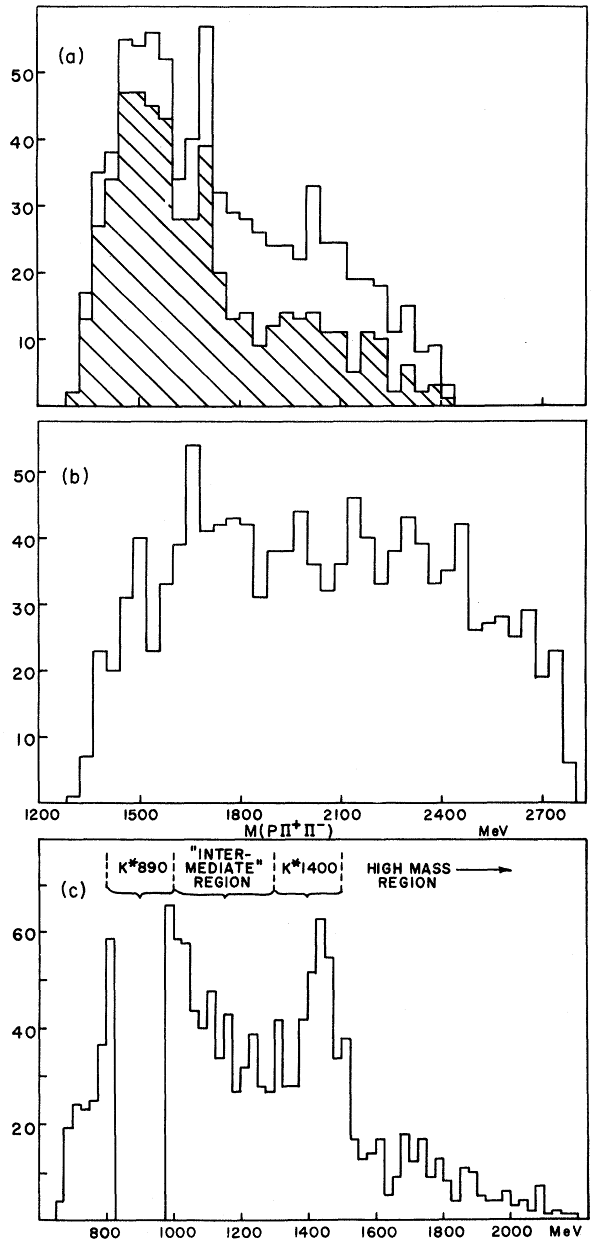

FIG. 1. (a) $p\pi^+\pi^-$ invariant-mass distribution for events with the $K^+\pi^-$ mass outside the K^* regions and $\Delta^2(K_{\text{beam}}\to K_{\text{out}}) < 0.15$. The shaded events are those for which the $(p\pi^+)$ mass lies in the $\Delta^{++}(1236)$ band and (b) $0.15 < \Delta^2 < 0.5$. (c) $K^+\pi^-$ invariant-mass distribution for $\Delta^2(K_{\text{beam}}\to K_{\text{out}}) < 0.5$, $\Delta^{++}(1236)$ in, and $K^*(890)$ out. [We define $K(890)$ by 840 MeV $< M(K\pi) <$ 940 MeV and $K^*(1400)$ by 1320 MeV $< M(K^+\pi^-) <$ 1480 MeV.]

and the Δ^{++} is formed [1150 MeV $< M(p\pi^+) <$ 1340 MeV].

The analysis of the $p\pi\pi$ enhancement, hereafter referred to as the $N(1450)$, has produced the following conclusions:

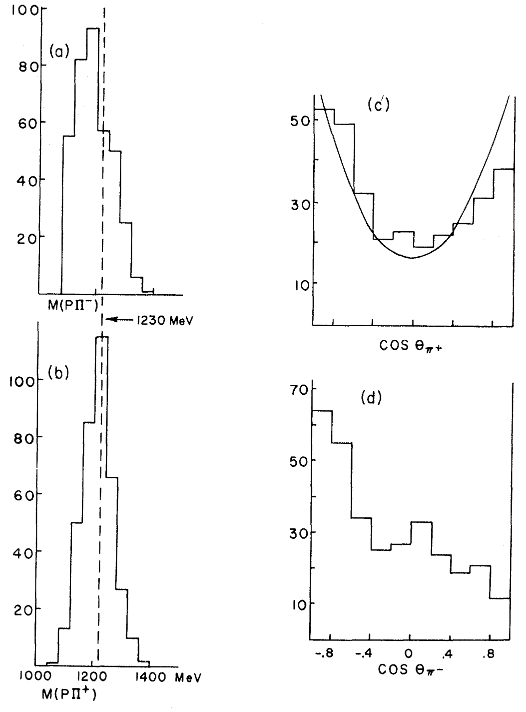

FIG. 2. (a) and (b) $p\pi^-$ and $p\pi^+$ mass distributions for events in the $(p\pi^+\pi^-)$ enhancement, i.e., 1350 MeV $< M(P\pi^+\pi^-) <$ 1600 MeV, both K^*'s out, and $\Delta^2(K_{\text{beam}}\to K_{\text{out}}) < 0.5$. (c) Cosine distribution of the angle between target and final protons in the $(p\pi^+)$ center-of-mass system for events in the $p\pi^+\pi^-$ enhancement, both K^*'s out, $\Delta^2 < 0.5$, and $\Delta^{++}(1236)$ in. (d) Cosine distribution of the angle between target proton and final π^- with the same cuts as (c).

(a) *The enhancement decays principally via the $\Delta^{++}\pi^-$ mode.*—The low mass of the enhancement forces both the $p\pi^+$ and $p\pi^-$ distributions to lie near the $\Delta^{++}(1238)$ region, causing difficulty in separating the $\Delta^{++}\pi^-$ mode from other modes such as $\Delta^0\pi^+$ or $p\pi\pi$ phase space. Nevertheless, three features of the enhancement favor this mode: (i) As shown in Figs. 2(a) and 2(b), there is a marked difference in the $p\pi^+$ and $p\pi^-$ distributions. The former is centered near the Δ^{++} mass of 1236 MeV with width 120 MeV and hence is well fitted by a Δ^{++} Breit-Wigner formula. The $p\pi^-$ mass distribution peaks below 1200 MeV and has width in excess of 160 MeV. (ii) Figure 2(c) shows the distribution of $\cos\theta_{\pi^+}$, the cosine of the angle between the π^+ and the target-proton directions in the $p\pi^+$ center-of-mass system for events in the $N(1450)$ region. The distribution is reasonably symmetric and gives a good fit to $1 + 3\cos^2\theta_{\pi^+}$. This $\cos^2\theta$ dependence is a characteristic decay distribution for the Δ^{++} when its spin component in the target-proton direction is restricted to $\pm\frac{1}{2}$. (iii) In Fig. 2(d) we show the distribution of $\cos\theta_{\pi^-}$, the cosine of the angle between the final π^- and target-proton directions as viewed in the $p\pi^+\pi^-$ center-of-mass system. The π^- strongly favors the direction opposite the target proton. The corresponding π^+ angular distribution is similar to that of Fig. 2(c). The noticeable difference in the two angular distributions cannot be explained if the enhancement is principally a $p\sigma$ system or $p\pi\pi$ phase space. The isospin-zero, spin-zero σ would lead to identical π^+ and π^- distributions. The asymmetry in Fig. 2(c) has been observed also in a $p+\mathrm{Ne}$ exposure at 25 GeV/c.[7]

(b) *The enhancement is not a pure resonant state.*—The highly asymmetric decay into $\Delta^{++}\pi^-$ is indicative of a high degree of parity mixing and rules against a resonance interpretation.

(c) *The decay rates of the enhancement are consistent with its being a pure $T=\frac{1}{2}$ state.*—The isospin channels available are $T=\frac{1}{2}$ and $T=\frac{3}{2}$ ($T=\frac{5}{2}$ is excluded by the production process). In the four-prong–topology kinematics, overlap of the Δ^{++} and Δ^0 prevents a determination of the $\Delta^{++}\pi^-/\Delta^0\pi^+$ ratio. However, we can compare Reaction (1) with the (two-prong+"vee") reactions:

$$K^+p \to K^0(p\pi^+), \tag{2}$$

$$K^+p \to K^0(p\pi^+\pi^0), \tag{3}$$

$$K^+p \to K^0(n\pi^+\pi^+). \tag{4}$$

The parentheses indicate the system where the enhancement is looked for.

From isospin, if $T=\frac{3}{2}$, then we expect three times as many doubly charged $N(1450)$ events as singly charged ones. If we assume $T=\frac{3}{2}$, the cross section for $N(1450)$ production in Reaction (1) predicts at least 47 events in the channels (2)-(4) after corrections are made for the size of the film sample used in the K^0 events to data. Our data show less that 10 events in all of these channels together which demonstrates that the $N(1450)$ is not a $T=\frac{3}{2}$ enhancement and that it is consistent with pure $T=\frac{1}{2}$.

The low-mass peaking and asymmetric decay are suggestive of a diffraction-scattering mechanism. This is further suggested by Figs. 3(a)-3(c) in which we have plotted the $\Delta^{++}\pi^-$ distribution for three cuts on the $K^+\pi^-$ mass: (a) 800 MeV $< M(K^+\pi^-) <$ 1000 MeV ("K^*890" region), (b) 1000 MeV $< M(K\pi) <$ 1300 MeV ("intermediate" region), and (c) $M(K\pi) >$ 1500 MeV. The distributions are notably different in appearance, especially in the sharp peaking that results in (b) and the very broad peak in (a) when the K^+ and π^- resonate.

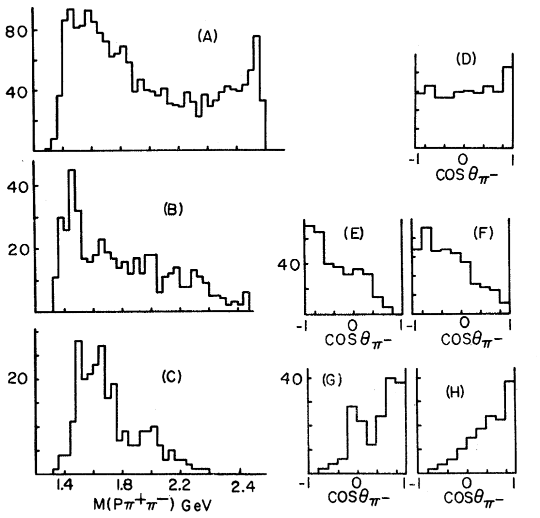

FIG. 3. (a)-(c) $p\pi^+\pi^-$ mass distributions for $\Delta^{++}(1236)$ in, $\Delta^2<0.5$, and (a) $K^*(890)$ in, (b) 1000 MeV $<M(K^+\pi^-)<$ 1300 MeV, and (c) $M(K^+\pi^-)<$ 1500 MeV. (d)-(h). Cosine distribution of the angle between target proton and final π^- with $\Delta^{++}(1236)$ in, and $\Delta^2<0.5$ for events in the $p\pi^+\pi^-$ enhancement. (d) Ross-Yam prediction for both K^*'s out. (e) Real data for 1000 MeV $<M(K^+\pi^-)<$ 1300 MeV. (f) Ross-Yam predictions for the same cut. (g) Real data for $M(K^+\pi^-)<$ 1500 MeV. (h) Ross-Yam prediction for the same cuts.

In studying the mechanism involved, we cannot use Deck-type pion exchange[8] as the K^+ distribution in the $K^+\pi^-$ center-of-mass system is not isotropic about the direction of the exchange pion. Consequently we have applied the Ross-Yam diffraction-dissociation model to this reaction, using the coherent sum of three terms representing $K^+\pi^-$ scattering, $K^+\Delta^{++}$ scattering, and K^+p scattering, with the proton dissociating into $\Delta^{++}\pi^-$.[9] The resulting $\Delta^{++}\pi^-$ distribution peaks at the right mass, but, as is characteristic of the model, its width is several hundred MeV too large. Furthermore, the model predicts a flat distribution in $\cos\theta_{\pi^-}$ as shown in Fig. 3(d). If, however, we make the same $K\pi$ cuts in the Ross-Yam calculation as we make in the real data, we find the following: (i) The predicted width of the peak, though remaining larger than that in the data, does get narrower when the intermediate $K\pi$ cut is made, just as the data do. (ii) In both the intermediate and high $K^+\pi^-$ mass regions, the $\cos\theta_{\pi^-}$ distributions predicted by the model agree qualitatively with the real data, as shown in Figs. 3(d) to 3(h). (iii) The Treiman-Yang distribution[10] predicted by the model also shows qualitative agreement with the data for both $K\pi$ cuts.

We conclude that the model, as we have applied it, errs in that it favors the high $K\pi$ mass region over the intermediate region (by a 5:3 ratio), whereas the data favor the intermediate region by a 2:1 ratio. This suggests that the events in the $\Delta^{++}\pi^-$ enhancement whose $K^+\pi^-$ mass lies in the intermediate region are principally produced by virtual $K^+\pi^-$ elastic scattering, as that diagram in the model favors lower $K^+\pi^-$ masses. Restricting ourselves to that $K\pi$ mass region, we are able to deduce the following about the nature of this scattering.

Pure $I=1$ exchange (ρ exchange) predicts the ratio $(K^+p \to K^0\pi^0\Delta^{++})/(K^+p \to K^+\pi^-\Delta^{++})=2/1$, which is not consistent with the absence of a $\pi^-\Delta^{++}$ enhancement in Reaction (3). This still leaves the possibility of $I=0$ exchange. Formation of an $I=\frac{3}{2}$ $K\pi$ state with subsequent decay into $K^+\pi^-$ or $K^0\pi^0$ would give the same ratio as above and is ruled out. $I=\frac{1}{2}$, however, predicts a ratio of $\frac{1}{2}$ and cannot be ruled out with our limited (two-prong + "vee") data.

In summary, we observe a $p\pi^+\pi^-$ enhancement near threshold which decays principally via $\Delta^{++}\pi^-$ and can be interpreted to be an $I=\frac{1}{2}$ state. The asymmetric decay rules against a pure resonant interpretation. The Ross-Yam model is moderately successful in predicting angular distributions, but does not stress adequately $K\pi$ scattering at low $K\pi$ masses. The low mass of the enhancement and its peripherality, along with the observed dependence of the shape of the spectrum on the $K\pi$ mass cut made, corroborates $K\pi$ diffractive scattering. Restricting ourselves to consideration of $K\pi$ scattering in the intermediate $K\pi$ mass region, we are able to rule out $I=1$ exchange as the mechanism involved. However, our data are consistent with both $I=0$ exchange and the formation of an $I=\frac{1}{2}$ $K\pi$ state with subsequent decay.

We acknowledge the many discussions we have had with Dr. G. Feldman, Dr. D. Griffiths, and Dr. L. Madansky. We thank Dr. T. Fulton for discussions on the subject of this paper and on the manuscript. We thank Dr. G. Kane for many discussions and his aid in setting up the diffraction-dissociation model. We are grateful to Dr. R. Shutt, the Brookhaven National Laboratory 80-in.-bubble-chamber crew and the beam No. 3 crew. Finally, we thank Mrs. D. Ellis for her aid in using the Berkeley TVGP-SQUAW routines.

---

*Work supported in part by the National Science Foundation, the U. S. Air Force Office of Scientific Research, and the U. S. Atomic Energy Commission Computation Center.

†Present address: Physics Department, University of Maryland, College Park, Md.

‡Present address: Stanford Linear Accelerator Center, Stanford, Calif.

[1] E. W. Anderson et al., Phys. Rev. Letters 16, 855 (1966); I. M. Blair, A. E. Taylor, W. S Chapman, P. I. P. Kalmus, J. Litt, M. C. Miller, D. B. Scott, H. J. Sherman, A. Astbury, and T. G. Walker, Phys. Rev. Letters 17, 789 (1966).

[2] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys Rev. Letters 19, 397 (1967).

[3] E. Gellert, G. A. Smith, S. Wojcicki, E. Colton, P. E. Schlein, and H. K. Ticho, Phys. Rev. Letters 17, 884 (1966); G. Alexander, O. Benary, G. Czapek, B. Haber, N. Kidron, B. Reuter, A. Shapira, E. Simopoulou, and G. Yekutieli, Phys. Rev. 154, 1284 (1967).

[4] W. E. Ellis, D. J. Miller, T. W. Morris, R. S. Panvini, and A. M. Thorndike, Phys. Rev. Letters 21, 697 (1968).

[5] P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. 165, 1730 (1968).

[6] E. L. Berger, E. Gellert, G. A. Smith, E. Colton, and P. E. Schlein [Phys. Rev. Letters 20, 964 (1968)] do a Regge-pole–exchange calculation regarding such a low-mass peaking.

[7] F. R. Huson, D. J. Miller, and J. S. O'Neall, in the Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (to be published).

[8] R. T. Deck, Phys. Rev. Letters 13, 169 (1964).

[9] M. Ross and Y. Y. Yam, Phys. Rev. Letters 19, 546 (1967). This paper refers to earlier papers on the subject. We are indebted to Dr. G. Kane of the University of Michigan for working out the equations used in the calculation. In the factors σe^{-Bt} that appear, we used $\sigma_{\pi-k}=20$ mb, $\sigma_{\Delta^{++}k^+}=20$ mb, $\sigma_{pk^+}=17$ mb, $B_{\pi-k}=6$ $(\mathrm{BeV}/c^2)^{-2}$, $B_{\Delta^{++}k^+}=8$ $(\mathrm{BeV}/c^2)^{-2}$, and $B_{pk^+}=6$ $(\mathrm{BeV}/c^2)^{-2}$.

[10] In this case we refer to the azimuthal angle in the $\Delta^{++}\pi^-$ center-of-mass system with respect to that coordinate system whose z axis is in the direction of the target proton and whose y axis lies along the cross product of the outgoing K^+ direction and the z axis. This is actually the Gottfried-Jackson coordinate system, but the azinuthal angle is equivalent to the Treiman-Yang angle.