EFFECT OF QUADRUPOLE DEFORMATION ON HIGH-ENERGY SCATTERING BY DEUTERONS*

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The high-energy elastic scattering cross sections of the deuteron are shown, because of the d-state admixture, to be strongly dependent on the polarization of the deuteron spin. The polarization dependence is shown to explain the absence of a pronounced minimum in the differential cross section for unpolarized deuterons. For polarized deuteron targets, on the other hand, the minimum should assume a clearly observable form.

It has recently been shown[1] that a number of features of the scattering of high-energy particles by light nuclei can be explained as the effects of multiple diffractive collision processes.[2,3] In particular the differential cross section for elastic proton-deuteron scattering[4,5] shows two varieties of angular dependence: an intense and narrow peak in the forward direction and a surrounding region in which the intensity falls off much more slowly with increasing angle. Recent measurements of the pion-deuteron differential cross section[6] reveal a similar structure. The intensities observed at small angles are approximately those characteristic of single diffractive collision processes while the weaker intensities observed at larger angles are describable as the result of double-collision processes.

Since the single- and double-collision processes are mutually coherent their amplitudes interfere. If the interaction of an arbitrary incident particle with the neutron and proton is largely absorptive, as is usually the case, then it is easily shown[3] that the amplitudes for single- and double-collision processes are roughly 180° out of phase; these amplitudes may be expected, in other words, to interfere destructively. The region of intermediate scattering angles for which the single and double scattering amplitudes are of comparable magnitude should therefore show a pronounced minimum in the scattered intensity. The measurements of p-d and π-d cross sections made to date[4-6] have shown, on the other hand, only shoulders or possibly the shallowest of minima at the appropriate angles.

Interference minima of essentially the same nature are observed[5] as predicted however for the cases of proton scattering by the spin-0 nuclei ^{4}He, ^{12}C, and ^{16}O. It is not unreasonable to assume therefore that the absence of an observed minimum for the deuteron is connected with the unit value of its spin. One possible source of the effect is the intrinsic spin dependence of the amplitudes for scattering by the neutron and the proton. Calculations using various spin-dependent amplitudes[7] have shown that spin dependence is indeed of quantitative importance but that it is not likely to wipe away completely the interference minimum for deuterium while leaving it intact, for example, for ^{4}He.

Another possible source of the effect suggested recently by Harrington[8] and by Coleman and Rhoades[9] is the quadrupole deformation of the deuteron. We wish to report here detailed calculations of deuteron cross sections which take the d-state admixture fully into account. These results explain the importance of including the d state and show that in addition to rendering the interference minimum almost invisible for experiments performed on unpolarized deuterons, it leads to a number of other effects which should be easily observable with polarized deuteron targets. In particular differential cross sections measured with polarized targets should clearly reveal the missing interference minima.

We write the internal wave function for a deuteron in the magnetic substate m ($m=\pm1,0$) relative to an axis of quantization $\hat{p}$ as

$$\Psi_{\hat{p},m}(\vec{r}) = (4\pi)^{-1/2}r^{-1}[u(r)+8^{-1/2}S_{12}w(r)]|\hat{p},m\rangle, \quad (1)$$

where $|\hat{p},m\rangle$ is a spin-1 spinor, u and w are the s- and d-state radial wave functions, and S_{12} is the tensor operator

$$S_{12}=[3\vec{\sigma}_1\cdot\vec{r}\,\vec{\sigma}_2\cdot\vec{r}-\vec{\sigma}_1\cdot\vec{\sigma}_2r^2]r^{-2}. \quad (2)$$

The structure of the deuteron only enters the scattering calculation through the form factor for the deuteron density distribution,[3] which takes on a second-order spherical harmonic dependence on the direction of the momentum transfer $\hbar\vec{q} = \hbar(\vec{k}-\vec{k}')$ relative to that of the spin vector $\vec{J}$ for the deuteron. This dependence is most conveniently expressed by defining a form-factor operator

$$S(\vec{q},\vec{J}) = S_0(\vec{q}) - S_2(\vec{q})[3(\vec{J}\cdot\hat{q})^2-2] \quad (3)$$

in which $\hat{q}$ is a unit vector in the direction of the momentum transfer, S_0 is the isotropic component of the form factor

$$S_0(q) = \int_0^\infty j_0(qr)[u^2(r)+w^2(r)]dr, \quad (4)$$

and S_2 is the quadrupole form factor

$$S_2(q) = 2^{1/2}\int_0^\infty j_2(qr)w(r)[u(r)-8^{-1/2}w(r)]dr. \quad (5)$$

To discuss scattering processes in which the deuteron has arbitrary initial and final angular momentum states it is convenient also to define a scattering amplitude operator $F[\vec{q},S]$. We write the amplitude for a collision in which the deuteron goes from the m state to the m' state (relative to the quantization axis $\hat{p}$) as

$$F_{m',m}(\vec{q}) = \langle \hat{p}, m' | F[\vec{q},S] | \hat{p}, m\rangle. \quad (6)$$

The appropriate operator $F[\vec{q},S]$ is a linear functional of the form factor operator S and may be written as

$$F[\vec{q},S] = F_1[\vec{q},S] + F_2[\vec{q},S] \quad (7)$$

in which F_1 and F_2 are the operators representing the single and double scattering amplitudes, respectively. These are

$$F_1[\vec{q},S] = f_1(\vec{q})S(\tfrac{1}{2}\vec{q}) + f_2(\vec{q})S(-\tfrac{1}{2}\vec{q}), \quad (8)$$

and

$$F_2[\vec{q},S] = \frac{i}{4\pi k}\int S(\vec{q}')\{f_1(\tfrac{1}{2}\vec{q}+\vec{q}')f_2(\tfrac{1}{2}\vec{q}-\vec{q}') + f_2(\tfrac{1}{2}\vec{q}+\vec{q}')f_1(\tfrac{1}{2}\vec{q}-\vec{q}')$$
$$-c_I[f_1(\tfrac{1}{2}\vec{q}+\vec{q}')-f_2(\tfrac{1}{2}\vec{q}+\vec{q}')][f_1(\tfrac{1}{2}\vec{q}-\vec{q}')-f_2(\tfrac{1}{2}\vec{q}-\vec{q}')]\}d^2q', \quad (9)$$

where f_1 and f_2 are the elastic scattering amplitudes for collisions between the incident particle and particles 1 and 2 of the deuteron. The coefficient c_I is 1 for incident particles of isospin $\frac{1}{2}$ and is $\frac{1}{2}$ for incident particles of unit isospin. The evaluation of the double scattering amplitude F_2 may be simplified considerably[8] by neglecting any dependence of the products $f_i(\frac{1}{2}\vec{q}+\vec{q}')f_j(\frac{1}{2}\vec{q}-\vec{q}')$ on the angle between the vectors $\vec{q}$ and $\vec{q}'$. This approximation becomes exact whenever f_i and f_j have similar Gaussian shapes as they do in the example we consider presently.

The initial momentum of the incident particle $\hbar\vec{k}$ and the final momentum $\hbar\vec{k}'$ define an orthogonal triad of unit vectors which we may write as $\hat{\kappa} = (\vec{k}+\vec{k}')/|\vec{k}+\vec{k}'|$, $\hat{q} = (\vec{k}-\vec{k}')/|\vec{k}-\vec{k}'|$, and $\hat{n} = \hat{q}\times\hat{\kappa}$. For the three axes of spin quantization $\hat{p} = \hat{\kappa}$, $\hat{q}$, and $\hat{n}$ we find that the deuteron spin transitions obey the selection rule $\Delta m = 0, \pm 2$. The transition amplitudes which do not vanish according to this rule are given as follows: For the three principal axes of quantization and the initial state $m=1$ we have

$$\langle\hat{\kappa},1|F[\vec{q},S]|\hat{\kappa},1\rangle = F_1[\vec{q},S_0+\tfrac{1}{2}S_2] + F_2[\vec{q},S_0+\tfrac{1}{2}S_2], \quad (10a)$$

$$\langle\hat{\kappa},-1|F[\vec{q},S]|\hat{\kappa},1\rangle = F_1[\vec{q},-\tfrac{3}{2}S_2], \quad (10b)$$

$$\langle\hat{q},1|F[\vec{q},S]|\hat{q},1\rangle = F_1[\vec{q},S_0-S_2] + F_2[\vec{q},S_0-\tfrac{1}{4}S_2], \quad (10c)$$

$$\langle\hat{q},-1|F[\vec{q},S]|\hat{q},1\rangle = F_2[\vec{q},-\tfrac{3}{4}S_2], \quad (10d)$$

$$\langle\hat{n},1|F[\vec{q},S]|\hat{n},1\rangle = F_1[\vec{q},S_0+\tfrac{1}{2}S_2] + F_2[\vec{q},S_0-\tfrac{1}{4}S_2], \quad (10e)$$

$$\langle\hat{n},-1|F[\vec{q},S]|\hat{n},1\rangle = F_1[\vec{q},-\tfrac{3}{2}S_2] + F_2[\vec{q},-\tfrac{3}{4}S_2], \quad (10f)$$

in which c-number form factors have been substituted for the operators S occuring in the definitions (8) and (9). For the initial states $m=-1$ the amplitudes can be found from the rule $\langle -m'|F\times|-m\rangle = \langle m'|F|m\rangle$, and for the initial states $m=0$ we have

$$\langle\hat{\kappa},0|F[\vec{q},S]|\hat{\kappa},0\rangle = F_1[\vec{q},S_0-S_2] + F_2[\vec{q},S_0-S_2], \quad (11a)$$

$$\langle\hat{q},0|F[\vec{q},S]|\hat{q},0\rangle = F_1[\vec{q},S_0+2S_2] + F_2[\vec{q},S_0+\tfrac{1}{2}S_2], \quad (11b)$$

$$\langle\hat{n},0|F[\vec{q},S]|\hat{n},0\rangle = F_1[\vec{q},S_0-S_2] + F_2[\vec{q},S_0+\tfrac{1}{2}S_2]. \quad (11c)$$

We have made use, for numerical purposes, of

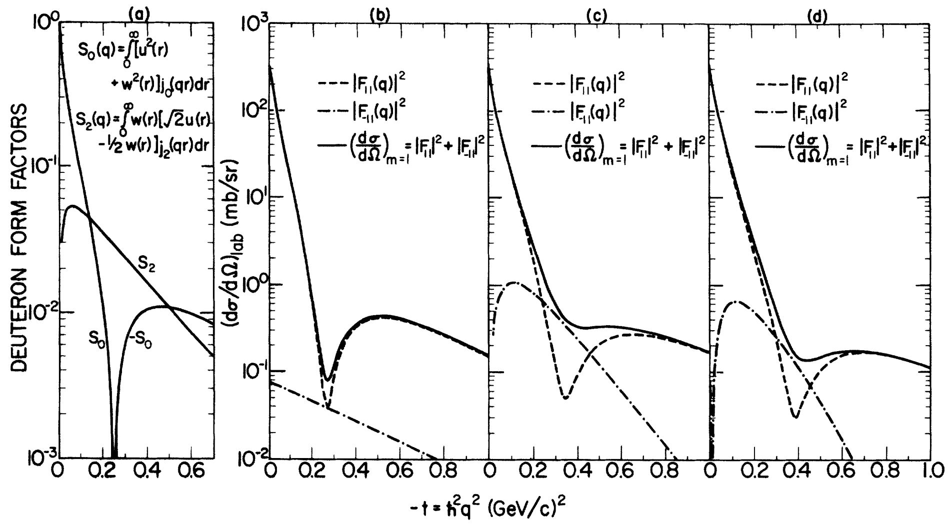

FIG. 1. (a) Deuteron form factors S_0 and S_2. (b)-(d) Cross sections for elastic scattering of 1-GeV protons by fully polarized deuterons ($m=1$ states) for the different polarization directions (b) $\hat{p}=\hat{q}$, (c) $\hat{p}=\hat{\kappa}$, and (d) $\hat{p}=\hat{n}$. The dashed curves represent the intensities for scattering with no spin reorientation, $\Delta m=0$. The dash-dotted curves represent the intensities for the spin-flip transitions. The solid curves represent the total intensities.

the most accurate fit of Moravcsik to the Gartenhaus wave function.[10] Although the amount of d-state admixture in the deuteron is constrained to a rather small value, 6.7 % for the wave function used, the fact that the form factor $S_2(q)$ increases with the amplitude rather than the squared amplitude of the d-state means that away from the forward direction, $\vec{q}=0$, it tends not at all to be small in comparison with $S_0(q)$. We see in fact from Fig. 1(a) that S_2 becomes equal to S_0 near $\hbar q=0.36$ GeV/c and exceeds S_0 for larger momentum transfers.

To illustrate the behavior of the various differential cross sections for the deuteron we have taken the neutron and proton scattering amplitudes to be of the form

$$f_j(\vec{q}) = k\sigma_j(i+\rho_j)e^{-\frac{1}{2}aq^2}/4\pi, \quad j=1,2, \tag{12}$$

and have given the parameters σ_j, ρ_j, and a the values which correspond to 1-GeV proton-nucleon collisions.[11] The cross sections for proton scattering by fully polarized deuterons ($m=1$ states) are shown in Figs. 1(b)-1(d) and 2(a) for the different polarization directions. The curves of Figs. 1(b)-1(d), which represent the intensities for scattering with no spin reorientation, $\Delta m=0$, all show pronounced minima brought about by the destructive interference of the single and double scattering amplitudes. The minima for the different polarization directions occur at rather different values of q since in that range of momentum transfers $S_2(\frac{1}{2}q)$ happens to be comparable in magnitude with $S_0(\frac{1}{2}q)$. For spins polarized along the $\hat{q}$ direction, for example, the form factor $\langle\hat{q},1|S|\hat{q}1\rangle = S_0 - S_2$ decreases much more rapidly with increasing momentum transfer than the two form factors $\langle\hat{\kappa},1|S|\hat{\kappa},1\rangle = \langle\hat{n},1|S|\hat{n},1\rangle = S_0+\frac{1}{2}S_2$ and thus leads to a minimum at an appreciably smaller momentum

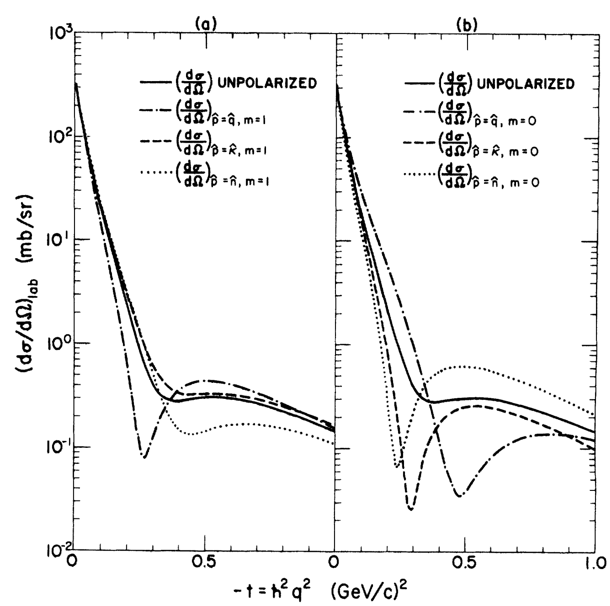

FIG. 2. Scattering of 1-GeV protons by unpolarized deuterons (solid curves) together with those found for the three principal polarization directions for (a) $m=1$ and (b) $m=0$.

transfer.

For the three axes of quantization considered the spin-flip transition $\Delta m \neq 0$ only takes place for initial states $m=\pm1$. The differential cross sections for these transitions are extremely sensitive to the polarization directions as may be seen from the curves shown in Figs. 1(b)-1(d). The origin of this dependence may be seen in Eqs. (10b), (10d), and (10f). The amplitude (10d) shows that for polarization in the $\hat{q}$ direction spin flip cannot be induced by the single-collision mechanism; it is weak because it stems entirely from double collisions. For polarization in the $\hat{n}$ direction both single and double scattering contribute (approximately 180° out of phase) to the spin-flip amplitude. For polarization in the $\hat{\kappa}$ direction, on the other hand, the suppression of the double scattering mechanism leaves the strong single-collision contribution fully in evidence and causes the most intense of the spin-flip transitions.

The spin-flip transitions are not distinguished experimentally from the $\Delta m=0$ transitions. Because the observed cross sections are the sums

$$(d\sigma/d\Omega)_{\hat{p},m} = \sum_{m'} |\langle \hat{p}, m'|F|\hat{p}m\rangle|^2 \tag{13}$$

the spin-flip intensities tend to fill in the minima in the $\Delta m=0$ differential cross sections. We see, for example, from Fig. 1(c) that for the polarization $\hat{p}=\hat{\kappa}$ the spin-flip intensity completely fills the interference minimum. For polarization $\hat{p}=\hat{n}$, [Fig. 1(d)] the spin-flip transitions are not quite strong enough to fill the minimum completely. For $\hat{p}=\hat{q}$ [Fig. 1(b)], on the other hand, the weakness of the spin-flip transitions leaves the minimum nearly intact; this minimum should be easily observable with an appropriately polarized target.

For unpolarized deuteron targets the observed cross section is the average of those found for the three principal polarizations

$$(d\sigma/d\Omega)_{\rm av} = \tfrac{1}{3} \sum_{\hat{p}=\hat{\kappa},\hat{q},\hat{n}} (d\sigma/d\Omega)_{\hat{p},m} \tag{14}$$

for $m=\pm1$ or 0. This cross section is shown together with the others for $m=1$ in Fig. 2(a) and for $m=0$ in Fig. 2(b). The fact that the minima (or stationary points) in the curves for the individual polarizations lie at such widely different momentum transfers leads to the disappearance of any minimum in the averaged cross section. We show in Fig. 3(a) the comparison of the calculated cross section for unpolarized deuterons with p-d scattering measurements[5] that have been made at 1.0 GeV. The measurements[4] which have been made at 2.0 GeV are shown in Fig. 3(b) together with the cross section calculated for that energy.[12] The agreement of the theoretical and experimental results is at least as satisfactory as the assumed fit of Eq. (12) to the proton-nucleon data. The development of polarized deuteron targets should allow a substantially more detailed check of the theory. Scattering measurements made in the interference region of the elastic scattering cross section evidently offer a sensitive probe of the quadrupole deformation of the deuteron.

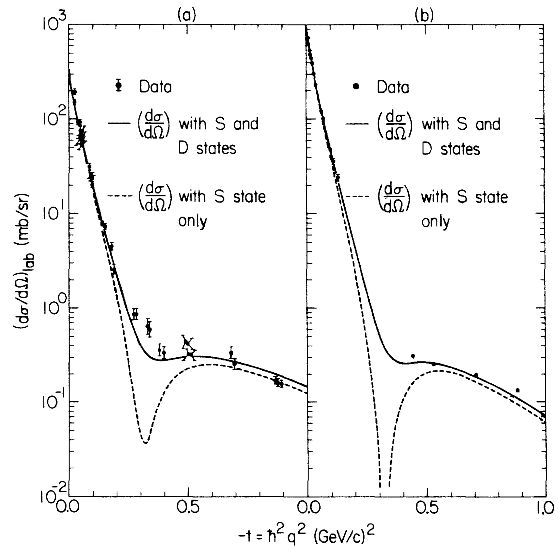

FIG. 3. Cross sections calculated for scattering of (a) 1-GeV and (b) 2-GeV protons by unpolarized deuterons and the measured values [Bennett et al., Ref. 5; Coleman et al., Ref. 4; Zolin et al., Ref. 4; Kirillova et al., Ref. 4]. Also shown for comparison are the differential cross sections for a spherically symmetric model of the deuteron obtained by ignoring the d state and appropriately renormalizing the s-state component of the wave function.

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[1]V. Franco and E. Coleman, Phys. Rev. Letters 17, 827 (1966).

[2]R. J. Glauber, in Lectures in Theoretical Physics, edited by W. E. Brittin et al. (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315.

[3]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966); R. J. Glauber and V. Franco, Phys. Rev. 156, 1685 (1967).

[4]E. Coleman et al., Phys. Rev. 164, 1665 (1967); L. S. Zolin, et al., Zh. Eksperim. i Teor. Fiz.–Pis'ma Redakt. 3, 15 (1966) [translation: JETP Letters 3, 8

(1966)]; L. Kirillova et al. (unpublished).

[5]G. W. Bennett et al., Phys. Rev. Letters 19, 387 (1967); E. T. Boschitz, private communication; H. Palevsky et al., Phys. Rev. Letters 18, 1200 (1967); E. T. Boschitz et al., Phys. Rev. Letters 20, 1116 (1968).

[6]H. C. Hsiung et al., Phys. Rev. Letters 21, 187 (1968); R. C. Chase et al., in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (to be published); F. Bradamante et al., to be published.

[7]E. Kujawski, D. Sachs, and J. Trefil, Phys. Rev. Letters 21, 581 (1968); V. Franco, Phys. Rev. Letters 21, 1360 (1968).

[8]D. R. Harrington, Phys. Rev. Letters 21, 1496 (1968).

[9]E. Coleman and T. G. Rhoades, private communication.

[10]M. J. Moravcsik, Nucl. Phys. 7, 113 (1958).

[11]We have used $\sigma_p = 47.55$ mb, $\sigma_n = 39.97$ mb, $\rho_p = 0.10$, $\rho_n = -0.50$, and $a = 0.205$ F^2. These values have been obtained from measurements by D. V. Bugg et al., Phys. Rev. 146, 980 (1966); L. M. C. Dutton et al., Phys. Letters 25B, 245 (1967); J. L. Friedes (unpublished).

[12]For these calculations we have used $\sigma_p = 45.10$ mb, $\sigma_n = 43.00$ mb, $\rho_p = -0.12$, $\rho_n = 0.14$, and $a = 0.297$ F^2. These values have been obtained from measurements by Bugg et al. (Ref. 11) and from measurements by L. F. Kirillova et al., Yadern. Fiz. 1, 533 (1965) [translation: Soviet J. Nucl. Phys. 1, 379 (1965)]; L. F. Kirillova et al., Zh. Eksperim. i Teor. Fiz. 50, 76 (1966) [translation: Soviet Phys.–JETP 23, 52 (1966)]; L. Kirillova et al. (unpublished); L. F. Kirillova, private communication.

HIGH-ENERGY PHOTOPRODUCTION OF ρ^0 MESONS FROM HYDROGEN AND DEUTERIUM*

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We have measured the photoproduction of ρ^0 mesons from hydrogen and deuterium as a function of the photon energy, k_γ, and the square of the momentum transfer, t. The assumption that the ρ is produced by a purely diffractive mechanism is midly contradicted by the data.

The photoproduction of ρ^0 mesons on hydrogen and deuterium has been measured at the Cornell University 10-GeV electron synchrotron at average photon energies of 4 to 9 GeV. The apparatus is shown in Fig. 1. The bremsstrahlung beam, which is monitored by a thin ion chamber upstream, passes through a 5-in. liquid target and into a uranium beam stop. Photoproduced ρ's decay into $\pi^+\pi^-$ pairs; the π^+ and π^- go through separate "arms" of the magnet system. A sixfold coincidence in the scintillation counters triggers the optical spark chambers. The entire magnet system is mounted on a platform which rotates vertically (to ~7°) about the target, thus varying the production angle. The spark-chamber pictures are used for high-resolution analysis of the data taken on heavy nuclei,[1] and serve only for qualitative checks on the hydrogen-deuterium data.

We study the dependence on incident photon energy k_γ, and on $m_{\pi\pi}$, the rest mass of the $\pi\pi$ system. Mass spectra are obtained by varying the magnetic fields with their ratio fixed. These spectra are dominated by the ρ resonance. Some details of the shape of this resonance are discussed in the following Letter[1] which describes results from various nuclei. Similar spectra are observed from all nuclei. k_γ is varied by moving the target along the beam and changing the magnetic field.

The data presented in Figs. 2 and 3 all repre-

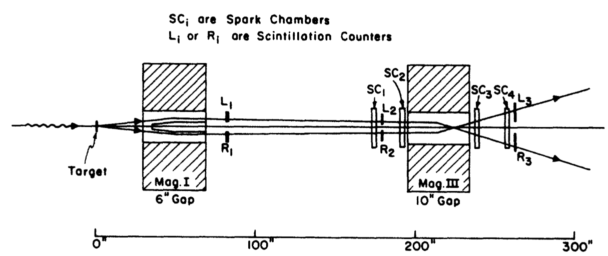

FIG. 1. Top view of apparatus.