es exchange degeneracy implies the vanishing of Regge-pole contributions to amplitudes when the trajectory function passes through wrong-signature values. Two of these cases are πN charge-exchange scattering at $\alpha_\rho = 0$ and π^+p backward elastic scattering at $\alpha_N = -\frac{1}{2}$. These predictions are not affected by the existence of fixed poles at wrong-signature values.

---

*Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under Contract No. AFOSR F44620-68-C-0075.

[1] F. Arbab and C. B. Chiu, Phys. Rev. 147, 1045 (1966); C. B. Chiu and J. D. Stack, Phys. Rev. 153, 1575 (1967).

[2] C. E. Jones and V. L. Teplitz, Phys. Rev. 159, 1271 (1967); S. Mandelstam and L. L. Wang, Phys. Rev. 160, 1490 (1967).

[3] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters 2, 239 (1962).

[4] R. Oehme, Phys. Letters 28B, 122 (1968).

[5] C. B. Chiu and J. Finkelstein, Phys. Letters 27B, 510 (1968).

[6] P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, Phys. Rev. Letters 20, 1395 (1968).

[7] L. Van Hove, in Particle Interactions at High Energy, edited by T. W. Preist and L. L. J. Vick (Oliver and Boyd, Edinburgh, Scotland, 1966), p. 63; R. Henzi, Nuovo Cimento 52A, 772 (1967).

[8] This requirement leads to a generalization of the idea of exchange degeneracy originally proposed by R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

[9] G. Veneziano, Nuovo Cimento 57A, 190 (1968); C. Lovelace, CERN Report No. CERN TH 950, 1968 (to be published); J. Shapiro and J. Yellin, University of California Lawrence Radiation Laboratory Report No. UCRL-18500, 1968 (unpublished); K. Igi, CERN Report No. CERN TH 959, 1968 (to be published).

[10] V. Barger and D. Cline, University of Wisconsin Report No. COO-203, 1968 (to be published); C. Schmid, CERN Report No. CERN TH 960, 1968 (to be published).

[11] C. B. Chiu and J. Finkelstein, "The Hybrid Model: Further Results," to be published.

SMALL HADRONIC PARAMETERS AND A PRINCIPLE OF COMPLEMENTARITY FOR BOOTSTRAP MODELS*

Geoffrey F. Chew
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 16 January 1969)

It is conjectured that, although no model is capable of explaining its motivating small parameters, an assortment of complementary models collectively may span the entire hadronic S matrix, with all parameters determined.

According to the S-matrix bootstrap hypothesis, the combined requirements of Lorentz invariance, unitarity, analyticity, and Regge asymptotic behavior are supposed to define a unique S matrix that approximates actually observed hadronic phenomena. If the strong-interaction picture were devoid of small dimensionless parameters, verification of this bootstrap idea might be a practical impossibility. To understand anything might require a simultaneous understanding of everything. In fact, small parameters occur in the hadron S matrix, and although these parameters are supposed ultimately to be determined by the bootstrap, their existence has allowed the construction of various tractable and meaningful models. In this note we identify a working principle for bootstrap investigations which recognizes the inevitability of a variety of complementary models, each model depending for its validity on certain small parameters which the model itself cannot explain. Given the special parameters that motivated its construction, each model is capable of approximately "explaining" a portion of the hadronic S matrix which may include the small parameters underlying a different model. It is conjectured that no single model will encompass all hadronic phenomena but that an assortment of models collectively may be able to do the job, with no parameters left undetermined.

Let us recall some significant existing hadronic models, in each case identifying the motivating small parameters and the limitation in scope.

(1) The "potential" or "Yukawa" model (also sometimes called the "strip" model) is characterized by its neglect in unitarity sums of all but a few low-lying channels. It also keeps only "nearby" singularities in crossed channels. Construction of the model is motivated by the nearness of these crossed singularities, which fol-

lows from the smallness of certain meson masses. These small masses constitute the underlying parameters. Classical nuclear physics as a whole is a potential model; so is the static model of baryon resonances. In a potential model the energy region near the included low-lying channel thresholds is adequately described, but the model is unable to handle poles lying far below these thresholds, such as those corresponding to the least massive mesons. The model is thus powerless to explain its motivating parameters. It also fails to describe phenomena at energies high above the included thresholds.

(2) A second category of small quantities comprises resonance widths, which are typically a good deal smaller than the spacing between resonances. The major hadron-theoretical discovery of the past two years has been that widths are small enough for meaningful models to ignore the nonlinear unitarity condition, basing their dynamical content entirely on Lorentz invariance, analyticity, and Regge asymptotic behavior. "Zero-width" models, which began in a crude form suggested through finite-energy sum rules by Dolen, Horn, and Schmidt[1] and independently by Mandelstam,[2] are currently receiving intense study in an elegant form discovered by Veneziano.[3] Optimistically viewed, the zero-width model seems to explain the approximate linearity and universal slope of the leading trajectories as well as the general form and ratios of residue functions. Threshold phemonena are ignored. Also not explained are the trajectory height, the number of different trajectories, and, most important, the magnitudes of residues. Since these latter correspond to resonance widths, we see that the model does not explain its motivating small parameters.

(3) The "multiperipheral" model is based on the small mean transverse momentum of particles produced in high-energy collisions. This smallness allows a decomposition of the phase space into regions that overlap only slightly, in each region a factoring of the amplitude being possible that leads to a tractable integral equation.[4] The model depends explicitly on unitarity and gives relations between trajectory height, spacing, and magnitudes of residues. It cannot describe threshold effects, nor can it be continued from one physical region to another (crossing). The model is valid only for small momentum transfer and thus cannot be used to explain why amplitudes decrease exponentially at large values of final transverse momentum. Again we see that the motivating parameter is inaccessible.

Let us now consider how the above models might be used to complement one another. Beginning at the arbitrary point in a possible bootstrap cycle, we note that the zero-width (e.g., Veneziano) model explains the sharp decrease with increasing transverse momentum and thus provides motivation for the multiperipheral model. The zero-width model furthermore gives an explicit form for the trajectories and residues which are to be fed into the inhomogeneous term and kernel of the multiperipheral integral equation. The number of trajectories and their heights and residues near zero momentum transfer ($t=0$) should then be determinable by a bootstrap matching of integral-equation output with input. One hopes that the residues will turn out small enough to justify the zero-width model. Once normalized near $t=0$, the zero-width model tells how to extrapolate to large positive and negative values of t. Finally, given the physical particles' masses and couplings, which one hopes will correctly include the low-mass mesons, the potential model can be used to fill in the details of what happens near thresholds.

The above scheme is vague and no doubt will be much modified in practice, but it illustrates the idea of complementarity and mutual interdependence of different models. Although esthetically our picture may seem ugly, perhaps this is only because ideas of beauty in physics are based on theories of a nonbootstrap character. An essential feature of a bootstrapped regime is the absence of any central parameter or set of parameters on which exhaustive theoretical analysis can be based. All elements of a bootstrapped S matrix are equivalently nonfundamental, and it is only the limited human capacity for understanding that focuses attention on certain special parameters.

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[2] S. Mandelstam, Phys. Rev. 166, 1539 (1968).

[3] G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[4] For a review of recent work on multiperipheral dynamics, see W. Frazer, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 415.