± 2.2 W.u.) and Doppler-shift attenuation[7] ($\Gamma = 24.5 \pm 2.2$ W.u.). In the rotational model, the present value of 20.3 ± 1.4 W.u. corresponds to a static quadrupole moment of $|Q_{\rm rot}| = 0.185 \pm 0.013$ b.

The measured value of Q is larger by about 30%. It would not be reasonable to ascribe this difference, which may not be significant in view of the errors, to a breakdown of the semiclassical approximation made in the calculation of the Coulomb excitation process, because the distance of closest approach is about 50 times the deBroglie wavelength of the projectile for the present experimental conditions. It is interesting to note that the static quadrupole moments derived from the $4^+ \to 2^+$, $6^+ \to 4^+$, and $8^+ \to 6^+$ transitions seem to decrease as one goes up the $K=0$ band.[11] This in turn would increase the discrepancy with the measured Q of the 2^+ state and thus indicate a significant deviation from the predictions of the rotational model.

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EXCHANGE DEGENERACY AND REGGE DIPS*

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It is shown that exchange degeneracy leads to dips in differential cross sections when trajectories pass through nonsense, wrong-signature values. The proof does not assume the absence of Gribov-Pomeranchuk fixed poles.

One of the more important successes of Regge-pole theory has been the prediction of dips in differential cross sections at values of momentum transfer at which trajectories pass through nonsense, wrong-signature values.[1] This mechanism has been used to explain the dips in the πN charge-exchange differential cross section at the value of t (≈ -0.5 GeV^2) at which $\alpha_\rho = 0$, and in the $\pi^+ p$ elastic differential cross section at the value of u (≈ -0.2 GeV^2) at which $\alpha_N = -\frac{1}{2}$. However, considerable doubt has been cast on these explanations by the realization[2] that the existence of Gribov-Pomeranchuk fixed poles[3] invalidates the theoretical proof of the necessity of dips. As has been recently emphasized by Oehme,[4] dips might exist in spite of fixed poles; whether they do or not is a question of dynamics. It is the purpose of this note to suggest that the dynamics associated with exchange degeneracy in certain cases leads to dips in differential cross sections, independently of any question involving Gribov-Pomeranchuk poles.

As was reviewed in a recent paper by Chiu and Finkelstein,[5] work on finite-energy sum rules[6] and on the structure of the overlap function[7] indicates that, in reactions with quantum numbers such as to forbid coupling to any known resonance, the imaginary parts of the exchanged Regge trajectories (excluding the Pomeranchukon) must cancel.[8] To see how this requirement can lead to dips, consider first the amplitude for $\pi^+\pi^+ \to \pi^+\pi^+$, at the value of t at which $\alpha_\rho = \alpha_{f^0} = -2$. The f^0 contribution to the amplitude is purely real (since the f^0 trajectory is of right signature), and the ρ contribution is purely imaginary. Since the sum of these must be real, the ρ contribution to the amplitude must vanish at this value of t. There will not be any dip, since exchange degeneracy connects the ρ and the f^0 contributions in such a way that the sum is a smoothly varying function of t. However, if we now consider the reaction $\pi^+\pi^- \to \pi^0\pi^0$, the f^0 does not contribute; isospin symmetry and our previous argument tell us that the ρ contribution to this reaction vanishes when $\alpha_\rho(t) = -2$, and so this differential cross section would have a dip.

To restate the argument, the reason that the vanishing of the signature factor at $\alpha_\rho = -2$ does not in itself guarantee a dip is that the ρ residue could blow up at that point, so that the product was finite. However, the f^0 residue cannot blow up at $\alpha = -2$, and exchange degeneracy then tells us that the ρ residue cannot either. To obtain this result, we have assumed a dynamics in which trajectories are "built up" locally from resonances, and which has no resonances of exotic quantum numbers; we have not neglected, but rather have shown to be irrelevant, the existence of Gribov-Pomeranchuk poles. Since our assumptions are verified in models utilizing the Veneziano representation,[9] we can understand why these models necessarily exhibit nonsense, wrong-signature dips.

This analysis does not, however, predict dips in all cases in which trajectories assume nonsense wrong-signature values. For example, since the Pomeranchukon does not satisfy exchange degeneracy, we would not expect a dip in forward Compton scattering, even though $\alpha_P = 1$ is nonsense for this reaction. In fact, since we have made the assumption that there are no fixed poles at right-signature values, our results should be applied only with great caution to any amplitude involving photons.

Exchange degeneracy does predict a dip in the πN charge-exchange differential cross section at the value of t at which $\alpha_\rho = 0$. The ρ couplings to $N\bar{N}$ are related by exchange degeneracy to those of the A_2, and its coupling to $\pi\pi$ related to that of the f^0. Either of these two relations, together with factorization, is sufficient to show that the ρ contributes nothing to the πN amplitude when $\alpha_\rho = 0$. This is true even for the sense amplitude, since the tensor trajectories, and hence by exchange degeneracy the ρ trajectory, decouple from sense in order to avoid ghosts. We thus have to rely on other contributions to the amplitude, such as cuts, to prevent the predicted dip from being a zero.

We can also establish the vanishing of the nucleon-trajectory contribution to π^+p backward scattering at the value of u at which $\alpha_N = -\frac{1}{2}$ by relying on SU(3) symmetry. Because of the absence of resonances which couple to KN, we expect all baryon trajectories which contribute to KN backward scattering to be exchange degenerate.[10] [If the Σ approximately decouples from KN, it would not need any exchange-degenerate partner; degeneracy between the trajectories of the Λ and of the $Y_0^*(1520)$ is discussed in Ref. 10.] Just as the ρ does not contribute to $\pi\pi$ scattering when $\alpha_\rho = -2$, neither the Σ nor the Λ contributes anything to KN scattering at nonsense, wrong-signature values, such as the value $\alpha = -\frac{1}{2}$. Because of exchange degeneracy, we would not expect a zero in the KN backward amplitude when $\alpha_\Lambda = -\frac{1}{2}$, just as we did not expect a zero in the $\pi^+\pi^+$ elastic amplitude when $\alpha_\rho = -2$. However, if we now assume SU(3) symmetry for the baryon trajectory residue functions at fixed α, then from the fact that neither the Λ nor the Σ contribute to KN at $\alpha = -\frac{1}{2}$ we can see that the nucleon contributes nothing to π^+p backward scattering at $\alpha = -\frac{1}{2}$.

One can derive additional constraints on the baryon trajectories and residues by demanding cancellations in processes in which there are no resonances in the t channel. It is an interesting question whether these constraints are valid, because of the presence of baryon-antibaryon annihilation in the t channel; in any case, these additional constraints are not necessary to establish the existence of the dip in the π^+p amplitude.

The presence of any additional terms in the amplitude, such as those arising from cuts, does not affect our discussion of the Regge-pole term. If a sum of pole and cut contributions must be real, then the pole contributions by themselves must be real, and this was the condition that we used to establish the existence of zeros in the pole terms. The cuts could certainly fill in the dips in the differential cross sections. On the other hand, it has been argued[11] that when the pole terms have zeros for very small values of $|t|$ or $|u|$, the cuts tend to be smaller than they otherwise might be.

In summary, we have seen that in certain cases exchange degeneracy implies the vanishing of Regge-pole contributions to amplitudes when the trajectory function passes through wrong-signature values. Two of these cases are πN charge-exchange scattering at $\alpha_\rho = 0$ and $\pi^+ p$ backward elastic scattering at $\alpha_N = -\frac{1}{2}$. These predictions are not affected by the existence of fixed poles at wrong-signature values.

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SMALL HADRONIC PARAMETERS AND A PRINCIPLE OF COMPLEMENTARITY FOR BOOTSTRAP MODELS*

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It is conjectured that, although no model is capable of explaining its motivating small parameters, an assortment of complementary models collectively may span the entire hadronic S matrix, with all parameters determined.

According to the S-matrix bootstrap hypothesis, the combined requirements of Lorentz invariance, unitarity, analyticity, and Regge asymptotic behavior are supposed to define a unique S matrix that approximates actually observed hadronic phenomena. If the strong-interaction picture were devoid of small dimensionless parameters, verification of this bootstrap idea might be a practical impossibility. To understand anything might require a simultaneous understanding of everything. In fact, small parameters occur in the hadron S matrix, and although these parameters are supposed ultimately to be determined by the bootstrap, their existence has allowed the construction of various tractable and meaningful models. In this note we identify a working principle for bootstrap investigations which recognizes the inevitability of a variety of complementary models, each model depending for its validity on certain small parameters which the model itself cannot explain. Given the special parameters that motivated its construction, each model is capable of approximately "explaining" a portion of the hadronic S matrix which may include the small parameters underlying a different model. It is conjectured that no single model will encompass all hadronic phenomena but that an assortment of models collectively may be able to do the job, with no parameters left undetermined.

Let us recall some significant existing hadronic models, in each case identifying the motivating small parameters and the limitation in scope.

(1) The "potential" or "Yukawa" model (also sometimes called the "strip" model) is characterized by its neglect in unitarity sums of all but a few low-lying channels. It also keeps only "nearby" singularities in crossed channels. Construction of the model is motivated by the nearness of these crossed singularities, which fol-