

chine with energies more than 100 GeV/beam is available. From Eqs. (6), (8), and (9), the finite self-masses of the electron and muon becomes

$$\delta m_l \simeq \frac{3e^2}{16\pi^2} m_l \ln\left(\frac{m_B}{m_l}\right)^2$$

$$(\simeq 0.045m_e \text{ for } e), \quad (10)$$

which shows that the mass of the weak boson plays the role of an effective cutoff energy of quantum electrodynamics.

In conclusion we note that this model suggests the following possibility: The weak interaction is not weak but strong enough to compete with the electromagnetic interaction; namely, the weak interaction behaves like a weak interaction only because the masses of intermediate bosons are much heavier than those of known particles. We also note that masses of weak particles (leptons and intermediate bosons) and hadrons keep their balance, e.g.,  $m_B m_e \simeq m_\pi m_p$ .

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<sup>1</sup>R. Gatto, G. Sartori, and M. Tonin, Phys. Letters 28B, 128 (1968), and CERN Report No. TH 964, 1968 (unpublished).

<sup>2</sup>N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968).

<sup>3</sup>A possibility of the negative-metric states as unstable particles has been discussed by Lee and Wick. See T. D. Lee and G. C. Wick (to be published).

<sup>4</sup>This Lagrangian is similar to that in Stueckelberg formalism, but our model is not the same as it because  $C^\pm$  is independent of  $B^\pm$  in our case. See E. C. G. Stueckelberg, Helv. Phys. Acta 11, 225 (1938).

<sup>5</sup>For example, T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 381 (1965).

<sup>6</sup>A similar idea was proposed about twenty years ago by Sakata and Hara. See S. Sakata and O. Hara, Progr. Theoret. Phys. 2, 30 (1947).

### DISCREPANCY BETWEEN THE VECTOR-DOMINANCE MODEL AND PION PRODUCTION BY POLARIZED PHOTONS\*

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The helicity density matrix for the process  $\pi^- p \rightarrow \pi^+ \pi^- n$  has been studied as a function of  $M_{\pi\pi}$ . The ratio  $\rho_{1-1}^{\text{hel}}/\rho_{11}^{\text{hel}}$  shows no variation with  $M_{\pi\pi}$  and it thus seems unlikely that the large discrepancy between the vector-dominance prediction and the observed asymmetry of single pions produced by linearly polarized photons could be the result of density-matrix distortion caused by background interference.

The vector-dominance model has been successfully used<sup>1</sup> to relate single-pion photoproduction,

$$\gamma p \rightarrow \pi^+ n, \quad (1a)$$

$$\gamma n \rightarrow \pi^- p, \quad (1b)$$

to  $\rho^0$  production by pions,

$$\pi^- p \rightarrow \rho^0 n. \quad (2)$$

By taking the sum of the two photoproduction cross sections, the  $\omega\rho$  interference terms drop out; since the  $\omega$  term itself is expected to be only a few percent, it is neglected, giving the  $\rho$ -

dominance prediction

$$\frac{\sigma^+ + \sigma^-}{2} = \frac{\pi\alpha}{\gamma^2} \left[ \rho_{11} \frac{d\sigma}{dt} \right]_{\pi^- p \rightarrow \rho^0 n}, \quad (3)$$

where  $\sigma^+$  and  $\sigma^-$  are  $d\sigma/dt$  for Reactions (1a) and (1b), respectively.

Recently, experiments on Reactions (1a)<sup>2</sup> and (1b)<sup>3,4</sup> have been performed using linearly polarized  $\gamma$  rays produced coherently from crystals. These experiments yield the asymmetries

$$A^\pm = (\sigma_\perp^\pm - \sigma_\parallel^\pm) / (\sigma_\perp^\pm + \sigma_\parallel^\pm), \quad (4)$$

where, for example,  $\sigma_{\perp}^{+}$  is the differential cross section for Reaction (1a) with the electric vector of the photon perpendicular to the production plane. To eliminate the  $\omega\rho$  interference terms the two cross sections are again summed together in an appropriate way giving the asymmetry

$$A(\pi^{+} + \pi^{-}) \equiv \frac{(\sigma_{\perp}^{+} + \sigma_{\perp}^{-}) - (\sigma_{\parallel}^{+} + \sigma_{\parallel}^{-})}{(\sigma_{\perp}^{+} + \sigma_{\perp}^{-}) + (\sigma_{\parallel}^{+} + \sigma_{\parallel}^{-})} = \frac{A^{+} + RA^{-}}{1 + R}, \quad (5)$$

where

$$R = \sigma^{-}/\sigma^{+}. \quad (6)$$

Values for  $A(\pi^{+} + \pi^{-})$  have been calculated from the experimental data<sup>2-5</sup> at  $k = 3.4$  GeV and are shown as crosses in Fig. 1. Note that  $R$ ,  $A^{+}$ , and  $A^{-}$  are themselves ratios and there should be no normalization problems in combining the results of several experiments.

The quantity  $A(\pi^{+} + \pi^{-})$  can be compared with data from Reaction (2) using the  $\rho$ -dominance model; as before, the few percent  $\omega$  term is neglected. While the probability for the  $\rho^0$  in Reaction (2) to have helicity 1 is just  $\rho_{11}^{\text{hel}1}$ , the probability for it to have linear polarization normal to its direction of motion and parallel or perpendicular to the reaction plane is given by<sup>6</sup>

$$P_{\perp} = \rho_{11}^{\text{hel}1} + \rho_{1-1}^{\text{hel}1}, \\ P_{\parallel} = \rho_{11}^{\text{hel}1} - \rho_{1-1}^{\text{hel}1}. \quad (7)$$

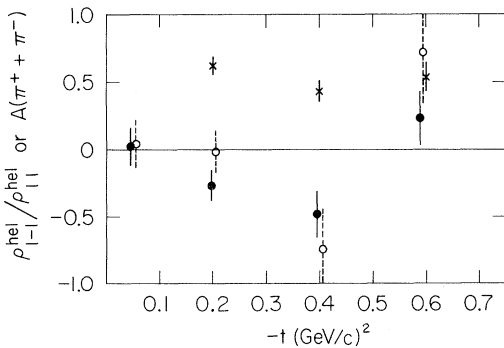


FIG. 1. Comparison of experimental values of  $A(\pi^{+} + \pi^{-})$ , the photoproduction asymmetry, at 3.4 GeV (crosses), with the ratio of helicity-frame density matrix elements  $\rho_{1-1}/\rho_{11}$  for the process  $\pi^{-}\rho \rightarrow \pi^{+}\pi^{-}n$  at 4 GeV/c. The closed circles are the ratio of density-matrix elements calculated from events in the  $\rho$  region ( $M_{\pi\pi}$  between 700 and 850 MeV) while the open circles were calculated with events in the background regions, 575 to 675 MeV and 875 to 975 MeV.

The  $\rho$ -dominance model then predicts

$$\sigma_{\perp}^{+} + \sigma_{\perp}^{-} = \frac{\pi\alpha}{\gamma^2} (\rho_{11}^{\text{hel}1} + \rho_{1-1}^{\text{hel}1}) \frac{d\sigma}{dt}, \\ \sigma_{\parallel}^{+} + \sigma_{\parallel}^{-} = \frac{\pi\alpha}{\gamma^2} (\rho_{11}^{\text{hel}1} - \rho_{1-1}^{\text{hel}1}) \frac{d\sigma}{dt}, \quad (8)$$

from which it follows that

$$A(\pi^{+} + \pi^{-}) = \rho_{1-1}^{\text{hel}1} / \rho_{11}^{\text{hel}1}. \quad (9)$$

The ratio of the density-matrix elements has been plotted with solid points in the figure. The density matrices were obtained by direct fits in the helicity frame to the angular distributions of the 4-GeV/c data of the Notre Dame-Purdue-Stanford Linear Accelerator Center Collaboration.<sup>7</sup> As can be seen from the figure, there is a large discrepancy,<sup>8</sup> especially at  $|t| = 0.2$  and  $0.4$  (GeV/c)<sup>2</sup> where  $\rho_{1-1}^{\text{hel}1}/\rho_{11}^{\text{hel}1}$  is negative while  $A(\pi^{+} + \pi^{-})$  is near 0.5. This discrepancy has been pointed out by the DESY group.<sup>3</sup>

Unfortunately, there is a non- $\rho$  background of perhaps 20% in the  $\rho$  region (taken as  $700 \leq M_{\pi\pi} \leq 850$  MeV in the fits). To test whether the discrepancy could be due to interference of the  $\rho$  with the background, we have studied the density matrix as a function of the effective  $\pi\pi$  mass from 500 to 1050 MeV. One might expect that any appreciable distortion of the density matrix by backgrounds would also lead to a rapid variation of the matrix elements as  $M_{\pi\pi}$  passes through  $M_{\rho}$ . In each  $t$  interval the ratio  $\rho_{1-1}^{\text{hel}1}/\rho_{11}^{\text{hel}1}$  shows no trend with  $M_{\pi\pi}$ , however, all mass intervals giving a ratio consistent with that obtained for the  $\rho$  region. Values of  $\rho_{1-1}^{\text{hel}1}/\rho_{11}^{\text{hel}1}$  obtained from fits to events with  $M_{\pi\pi}$  between 575 and 675 MeV plus those between 875 and 975 MeV are shown with open circles in the figure; in this background region there are roughly equal numbers of  $\rho$  and background events. The lack of variation with  $M_{\pi\pi}$  would seem to eliminate background interference effects as a possible source of the vector-dominance discrepancy.

At high energies  $\sigma_{\perp}$  and  $\sigma_{\parallel}$  correspond to  $P(-1)^J = +1$  and  $-1$  (natural and unnatural parity) exchange, respectively.<sup>9</sup> The data thus indicate that while photoproduction proceeds principally via natural-parity exchange (~75% of the time), the reaction  $\pi^{-}p \rightarrow \rho^0 n$  has roughly equal contributions from the two spin-parity sequences, depending somewhat on  $t$ .

Taking the discrepancy at face value, the relative strength of the  $\rho$ -dominance terms in  $\pi^{\pm}$  pho-

toproduction can be estimated by assuming the photoproduction cross section to be the sum of two contributions, the first being the  $\rho$ -dominance contribution, with the ratio of natural- to unnatural-parity exchanges determined by the model as

$$\frac{(\sigma_{\perp}^{+} + \sigma_{\perp}^{-})_{\rho}}{(\sigma_{\parallel}^{+} + \sigma_{\parallel}^{-})_{\rho}} = \frac{1 + \rho_{1-1}/\rho_{11}}{1 - \rho_{1-1}/\rho_{11}}, \quad (10)$$

and the second contribution being an *ad hoc* term contributing only to  $\sigma_{\perp}^{+} + \sigma_{\perp}^{-}$ . These assumptions give an upper limit to the fraction of pions photoproduced via the  $\rho$ -dominance terms:

$$F = [1 - A(\pi^{+} + \pi^{-})] / (1 - \rho_{1-1}/\rho_{11}). \quad (11)$$

Using the data shown in the figure, this upper limit is  $(30 \pm 6)\%$ ,  $(39 \pm 7)\%$ , and  $(60 \pm 21)\%$  at  $-t = 0.2, 0.4,$  and  $0.6 \text{ GeV}^2$ , respectively. Thus, the present data imply that the original success of the  $\rho$ -dominance model in fitting the photoproduction differential cross section<sup>1</sup> must be regarded as fortuitous and that the unknown second term must have a  $t$  dependence similar to the  $\rho$ -dominance term; furthermore, the value of  $\gamma_{\rho}^{-2}$  must be roughly  $\frac{1}{2}$  (or less) that used previously.<sup>10</sup>

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<sup>1</sup>C. Iso and H. Yoshii, *Ann. Phys. (N.Y.)* **47**, 424 (1968); A. Dar, V. F. Weisskopf, C. A. Levinson, and H. J. Lipkin, *Phys. Rev. Letters* **20**, 1261 (1968); M. Krammer, *Phys. Letters* **26B**, 633 (1968), and **27B**, 260(E) (1968); R. Diebold and J. A. Poirier, *Phys. Rev. Letters* **20**, 1532 (1968).

<sup>2</sup>Chr. Geweniger *et al.*, quoted in B. Richter, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968* (CERN Scientific Information Service, Geneva, Switzerland, 1968).

<sup>3</sup>Chr. Geweniger *et al.*, *Phys. Letters* **28B**, 155 (1968).

<sup>4</sup>Z. Bar-Yam *et al.*, quoted in Richter, Ref. 2.

<sup>5</sup>P. Heide *et al.*, *Phys. Rev. Letters* **21**, 248 (1968); Z. Bar-Yam *et al.*, *Phys. Rev. Letters* **19**, 40 (1967).

<sup>6</sup>See M. Krammer and Dieter Schildknecht, *Nucl. Phys.* **B7**, 583 (1968), for a derivation of similar formula.

<sup>7</sup>Notre Dame-Purdue-Stanford Linear Accelerator Center Collaboration, "Compilation of  $\pi^{-}p$  Data at 4 GeV/c" (to be published).

<sup>8</sup>In this momentum-transfer region  $R$  is considerably less than 1 and it is the  $\pi^{+}$  asymmetry which dominates  $A(\pi^{+} + \pi^{-})$ ; even if one were to ignore the experimental values and arbitrarily set  $A^{-}$  equal to the limit  $-1$ , there would still be a large discrepancy at  $|t| = 0.2$  and  $0.4 \text{ (GeV/c)}^2$  where  $A(\pi^{+} + \pi^{-})$  calculated in this way would be  $0.19 \pm 0.08$  and  $0.23 \pm 0.09$ , respectively.

<sup>9</sup>P. Stichel, *Z. Physik* **180**, 170 (1964); J. P. Ader *et al.*, *Nuovo Cimento* **56A**, 952 (1968).

<sup>10</sup>This would imply  $\gamma_{\rho}^2/4\pi \geq 1$ , in agreement with the preliminary values reported by the Cornell and Stanford Linear Accelerator Center  $\rho$ -photoproduction groups, Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (unpublished).

## EXTENSION OF THE VENEZIANO FORM TO $N$ -PARTICLE AMPLITUDES

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The extension of Veneziano's form  $V(s, t)$  to the  $N$ -particle amplitude is given.

We give here a form  $V^{(N)}(s_1, s_2, \dots)$  which is the extension to the  $N$ -particle amplitude of Veneziano's form  $V^{(4)}(s, t)$  for the four-particle amplitude.<sup>1,2</sup> [The arguments  $s_i$  are invariant masses squared of various groups (so called "channels"  $i$ ) of the  $N$  particles.] Like  $V^{(4)}(s, t)$ , the form  $V^{(N)}(s_i)$  has an infinite number of poles in each  $s_i$ , corresponding to linear Regge trajectories of resonances, and it has Regge asymptotic behavior for large  $s_i$ ; it has no cuts (hence does not satisfy unitarity), and the Regge behav-

ior is linked with duality.

The choice of the set of the arguments  $s_i$  of  $V^{(N)}(s_i)$  is guided by duality. Suppose that  $V^{(N)}(s_i)$  has a pole term  $\sim 1/\prod_i'(m_i - \alpha_i)$  where each  $m_i$  is an integer,  $\alpha_i(s_i)$  is the leading trajectory in channel  $i$ , and the product is over the  $N-3$  internal lines of a particular  $N$ -particle tree diagram, such as in the first column of Fig. 1. Then duality implies that  $V^{(N)}$  also has pole terms corresponding to other tree diagrams. A minimal set consists of all tree diagrams which are derivable