ical curves have been smoothed by eye from the Monte Carlo results.

[6]The word "event" refers to a single configuration of the protons in Fig. 1(a) satisfying the condition $\Delta^2 > 1$. In consequence the quoted cross section for "All events" of Table I is larger than the cross section $1.66^{+0.13}_{-0.06}$ mb found for the physical process $pp \rightarrow pp\pi^+\pi^-$ at 16 GeV/c.

[7]E. L. Berger, E. Gellert, G. A. Smith, E. Colton, and P. E. Schlein, Phys. Rev. Letters 20, 964 (1968).

[8]G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

EFFECT OF BACKGROUND ON THE $I=2$ $\pi\pi$ SCATTERING PHASE SHIFTS*

K. Abe, A. D. Johnson, V. J. Stenger, and P. G. Wohlmut
University of Hawaii, Honolulu, Hawaii
(Received 2 December 1968)

We show that in the reaction $\pi^-p \rightarrow \pi^-\pi^-\pi^+p$ at 3.9 GeV/c, the peaking in the $\pi^-\pi^-$ "scattering angular distribution" above 0.6-GeV c.m. energy inferred from the one-pion exchange model can be interpreted as resulting from a background of competing processes. Similar background effects may be significant also in the reaction $\pi^+p \rightarrow \pi^+\pi^+n$, and conclusions from that reaction as to evidence for d waves in the $I=2$ amplitude below 1 GeV should be reserved pending a similar analysis.

The one-pion exchange model is widely used in a variety of interactions to extract information about $\pi\pi$ and $K\pi$ scattering.[1-18] In this Letter, we consider the reaction

$$\pi^-p \rightarrow \pi^-\pi^-\pi^+p, \tag{1}$$

in which there is good evidence for Δ^{++} production by one-pion exchange at a wide range of energies. It has been found in several experiments that, after applying cuts on $M(\pi^+p)$ and $-t(\pi^+p)$ in order to purify the sample, the $\pi^-\pi^-$ scattering angular distribution is isotropic for $M(\pi^-\pi^-) \lesssim 0.6$ GeV and becomes increasingly forward peaked as $M(\pi^-\pi^-)$ increases.[8-15] This is generally taken as evidence for the rapid onset of d waves or higher partial waves, although no structure in the distribution of $M(\pi^-\pi^-)$ is evident. We have results which suggest strongly that the forward peaking observed is a direct result of the background of competing processes.

We have fitted 7975 events of Reaction (1) at 3.9 GeV/c with a model of resonance production by one-particle exchange, using a maximum-likelihood technique.[19] We find that an excellent fit is obtained for a model in which there are the 11 processes listed in Table I adding incoherently. The only adjustable parameters of the fit are the relative amounts of the various processes, and the best values for these parameters are also listed in Table I. The diagrams considered are shown in Fig. 1. The model assumes only s-wave scattering at the $\pi^-\pi^-$ vertex in Fig. 1(a). This process, with and without Δ^{++} production, constitutes the "signal" of interest. The processes in Figs. 1(b)-1(d) are the major "noise" or

Table I. The 11 processes of our fit to the data corresponding to the four diagrams of Fig. 1. The processes are assumed to add incoherently and the relative amounts determined by the fit are shown. The errors quoted are purely statistical. Comments indicate form of Breit-Wigner used (no comment means fixed width form). Diffraction at πp vertices is included in several processes. A slope parameter $b = 9$ (GeV/c)$^{-2}$ was used. Results appear to be insensitive to this parameter.

Process $\pi^-p \rightarrow$	Diagram Fig.1	Mass and Width of Resonance (GeV)	Comments	Relative Amount (%)
$\Delta^{++}\pi^-\pi^-$	(a)	1.232, 0.12	Variable width Breit-Wigner form	15.6±0.5
$\rho^0 p\pi^-$	(b)	0.77, 0.14	Variable width Breit-Wigner form. Diffraction at π^-p vertex above 1.67 GeV.	17.1±0.6
$\pi^-\pi^-p\pi^+$	(a)		Diffraction at π^+p vertex above 1.67 GeV	5.1±0.7
$\Delta^0\pi^+\pi^-$	(b)	1.232, 0.12	Variable width Breit-Wigner form	1.5±0.5
$f^0 p\pi^-$	(b)	1.26, 0.14	Diffraction at π^-p vertex above 1.67 GeV	0.3±0.4
$A_2^- p$ $\hookrightarrow \rho^0\pi^-$	(d)	1.306, 0.081 0.77, 0.14		8.6±0.5
$A_1^- p$ $\hookrightarrow \rho^0\pi^-$	(d)	1.079, 0.13 0.77, 0.14		3.7±1.0
$\pi^-\pi^+\pi^-p$	(b)		Diffraction at π^-p vertex above 1.67 GeV	6.4±1.1
$N^{*+}\pi^-$ $\hookrightarrow \Delta^{++}\pi^-$	(c)	1.45, 0.21 1.232, 0.12		12.4±0.7
$\rho^0\Delta^0$	(b)	0.77, 0.14 1.232, 0.12	Variable width Breit-Wigner form	8.0±0.6
$\pi^-(1650)p$ $\hookrightarrow f^0\pi^-$	(d)	1.654, 0.109 1.26, 0.14		1.3±0.4

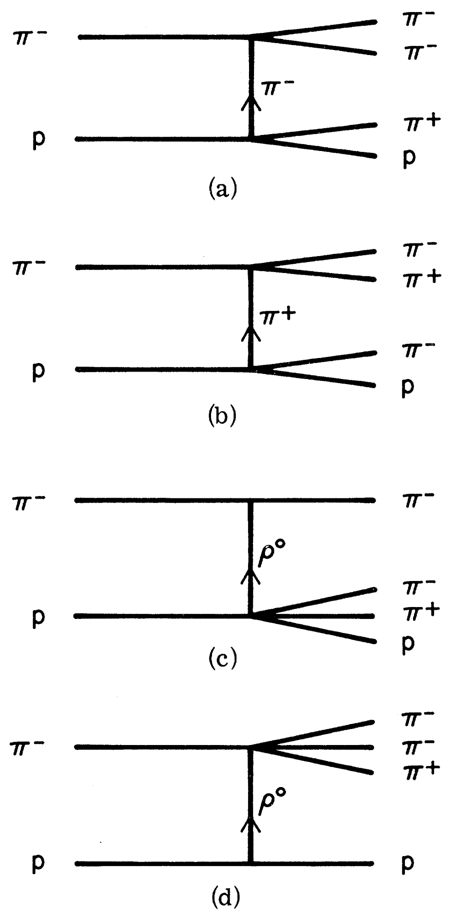

FIG. 1. Diagrams for the four basic processes used in the fit. Resonances produced at either vertex are considered separately in model, adding incoherently to the nonresonant background. For nonresonant vertices, s-wave scattering is assumed with diffraction scattering at the πp vertex above 1.67-GeV πp c.m. energy.

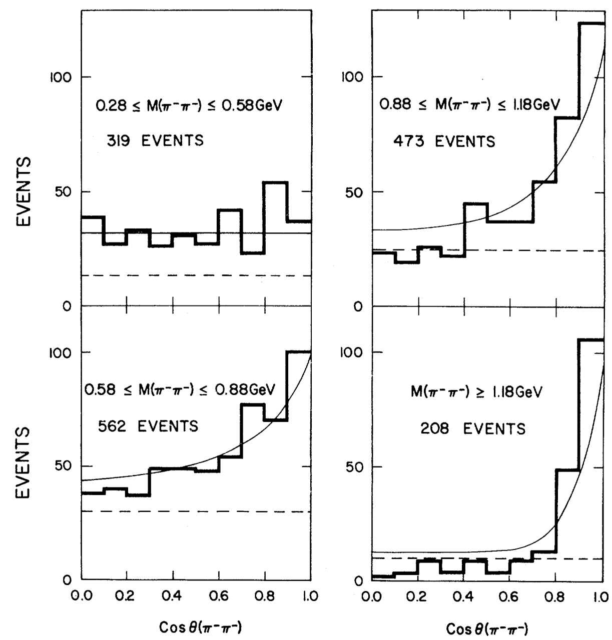

FIG. 2. Plots of $\cos\theta(\pi^-\pi^-)$, where $\theta(\pi^-\pi^-)$ is the scattering angle in the $\pi^-\pi^-$ c.m. system, for four ranges of $M(\pi^-\pi^-)$. The solid curve is the prediction of the Monte Carlo events generated according to the model. The dashed curve is the prediction of the Monte Carlo events generated according to the "signal" process Fig. 1(a), normalized with the signal-to-noise ratio given by the Monte Carlo. The data and Monte Carlo on the plots contain the cuts $1.13 \leq M(\pi^+) \leq 1.31$ GeV and $-t(\pi^+p) \leq 0.5$ GeV$/c)^2$.

background.

We have generated Monte Carlo events weighted according to this model and applied the same cuts to these and the actual measured data. In Fig. 2 we show the resulting $\cos\theta(\pi^-\pi^-)$ distributions in the $\pi^-\pi^-$ c.m. system for several regions of $M(\pi^-\pi^-)$. The Monte Carlo and the data show strikingly similar forward peaking behavior. Only for $M(\pi^-\pi^-) > 1.18$ GeV do the data appear significantly more peaked than the Monte Carlo.

In order to understand the effect, we have plotted separately in Fig. 2 the angular distributions for Monte Carlo events generated according to the "signal" diagram in Fig. 1(a), in which the two π^-'s are at the same vertex. The normalization is chosen to accurately reflect the fraction of "signal to noise" in each plot. We note that even for the lowest $M(\pi^-\pi^-)$ band the background is more than twice the signal. The angular distributions for the signal are isotropic as one would expect. Thus, it is the background which accounts for the increasing forward peaking with increasing $M(\pi^-\pi^-)$ observed in the data. This can be easily explained since there is a definite correlation between $M(\pi^-\pi^-)$ and $\cos\theta(\pi^-\pi^-)$ when the π^-'s are produced peripherally from different vertices: The more collinear the pions with respect to the incident pion, the higher their two-body c.m. energy.

While the cuts applied to the data purify the sample somewhat, the background processes in this particular reaction constitute a significant fraction of the data. From Table I we see that Δ^{++} production by π exchange contributes only 16 % to Reaction (1) at 3.9 GeV$/c$, with an additional 5 % of nonresonant π^+p production. On the other hand, the competing processes which produce the forward peaking constitute at least 37 %.

We conclude that it is possible to explain the observed forward peaking in the $\pi^-\pi^-$ scattering angular distribution as arising from a background of competing processes in which the two π^-'s are produced peripherally at opposite vertices.

Biswas et al.[13] have attempted to extract the low-energy π-π phase shifts from Reaction (1) by utilizing the interference with the Coulomb amplitude in the forward direction. Our results show that this procedure is questionable unless the background is carefully taken into account.

The $I=2$ $\pi\pi$ scattering angular distributions have also been studied in the reaction

$$\pi^+p \rightarrow \pi^+\pi^+n, \qquad (2)$$

and the increasing forward peaking phenomenon is observed here as well.[16-18] While we do not have data from Reaction (2) available to us for analysis, we suggest that a similar effect may occur. It is conceivable that there is a significant peripheral process in this reaction in which the two π^+'s emerge from opposite vertices. This cannot be π exchange, but may be diffraction dissociation[20] where $p \rightarrow \pi^+n$ or diffraction production of N^* states. The data show strong N^* production in the 1500- and 1680-MeV regions suggestive of a mechanism causing orbital exictation of the target nucleon.[16-18] Such a background to the π-exchange process would give an increasing forward peaking with rising $M(\pi^+\pi^+)$ analogous to the reaction studied here. This possibility is somewhat reinforced by the fact that the forward-peaking effect is not independent of incident π^+ momentum, as it should be if it resulted from higher partial waves in the $I=2$ $\pi\pi$ scattering amplitude. At $p_{\text{lab}}=1.6$ GeV/c the forward peaking disappears out beyond $M(\pi^+\pi^+)$ $=0.85$ GeV when a cut is made to include only those events with low momentum transfer to the neutron.[16] The same cut at higher momenta does not remove the forward peaking.[17,18] This observation is also consistent with the expectation that the π-exchange diagram will be stronger relative to the diffraction process at lower momenta since its cross section goes as p_{lab}^{-2} while the latter has a cross section which is presumably constant with incident momentum. It may be, however, that s-channel effects, which are expected to be important at 1.6 GeV/c, account for the observed discrepancy.

We feel that, in general, it is a dangerous procedure to utilize the one-pion exchange model to infer $\pi\pi$ or $K\pi$ scattering parameters from Reaction (1) or its K analog unless one can show that the pion-exchange process being studied dominates or that other processes produce no significant biases. The background to the π-exchange process is perverse, giving the same increasing forward peaking with increasing energy expected with the onset of higher partial waves. With respect to $\pi\pi$ scattering in the $I=2$ state, we have shown, at least for Reaction (1), that it is possible to find a reasonable model that fits the data and requires only s-wave $\pi^-\pi^-$ scattering up to 1 GeV. Until a similar analysis is carried out for Reaction (2), we suggest that conclusions as to higher partial waves in the $I=2$ $\pi\pi$ scattering amplitude below 1 GeV be held in abeyance.

We are grateful to Professor David Cline for helpful discussions and to the Goldhaber-Trilling and Powell-Birge groups at Lawrence Radiation Laboratory for loaning us the film used in this study.

---

*Work supported in part by the U. S. Atomic Energy Commission Contract No. AT(04-3)-511.

[1]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[2]E. Ferrari and F. Selleri, Nuovo Cimento 24, 453 (1962).

[3]F. Salzman and G. Salzman, Phys. Rev. 120, 599 (1960).

[4]G. Wolf, Phys. Letters 19, 328 (1965).

[5]L. J. Gutay, P. B. Johnson, F. J. Loeffler, R. L. McIlwain, D. H. Miller, R. B. Willman, and P. L. Csonka, Phys. Rev. Letters 18, 142 (1967).

[6]W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters 18, 630 (1967).

[7]E. Malamud and P. Schlein, Phys. Rev. Letters 19, 1056 (1967).

[8]P. H. Satterblom, W. D. Walker, and A. R. Erwin, Phys. Rev. 134, B207 (1964).

[9]S. U. Chung, O. I. Dahl, J. Kirz, and D. H. Miller, Phys. Rev. 165, 1491 (1968).

[10]A. W. Key, J. D. Prentice, N. R. Steenberg, E. West, T. S. Yoon, W. A. Cooper, W. Manner, L. Voyvodic, and W. D. Walker, Phys. Rev. 166, 1430 (1968).

[11]W. D. C. Moebs, III, University of Michigan Report No. COO-1112-3 (unpublished).

[12]C. Caso, F. Conte, G. Tomasini, P. Dittman, G. Drews, P. von Hamdel, P. Shilling, L. Mandelli, S. Ratti, G. Vegni, P. Daronian, A. Daudin, B. Gandois, C. Kochowski, C. Lewin, and L. Mosca, Nuovo Cimento 57A, 699 (1968).

[13]N. N. Biswas, N. M. Cason, P. B. Johnson, V. P. Kenney, J. A. Poirier, W. D. Shephard, and R. Torgerson, Phys. Letters 27B, 513 (1968).

[14]N. Schmitz, Nuovo Cimento 31, 255 (1966).

[15]Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 35, 1 (1965).

[16]P. Daronian, A. Daudin, M. A. Jabiol, C. Lewin, C. Kochowski, B. Ghindini, S. Mongelli, and V. Picciarelli, Nuovo Cimento 41A, 503 (1966).

[17]Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 37, 361 (1965).
[18]Aachen-Berlin-Birmingham-Bonn-Hamburg-London (Imperial College)-München Collaboration, Phys. Rev. 138, B897 (1965).
[19]Details of results to be published. For description of method see V. J. Stenger, University of Hawii Report No. UH-511-30-68 (unpublished). See also J. Friedman and R. Ross, Alvarez Group Memo No. P-102 (unpublished).
[20]W. D. Walker, M. A. Thompson, W. J. Robertson, B. Y. Oh, Y. Y. Lee, R. W. Hartung, A. F. Garfinkel, A. R. Erwin, and J. L. Davis, Phys. Rev. Letters 20, 133 (1968).

MODEL OF THE FINITE SELF-MASSES OF LEPTONS

Hidezumi Terazawa
Department of Physics, University of Tokyo, Tokyo, Japan
(Received 23 December 1968)

We consider a model of the weak interactions with scalar intermediate bosons $C^\pm$ with a negative metric in addition to the usual weak vector bosons $B^\pm$. With this modification, problems of divergence and high-energy behavior are greatly reduced. If the logarithmic weak and electromagnetic self-mass divergences are assumed to cancel each other, the coupling constant g and the mass m_B of the weak boson can be predicted.

Recently several authors[1,2] have proposed new approaches to determine the Cabibbo angle through the requirement that weak self-masses of hadrons be free of quadratic divergences. How about the self-masses of leptons? The purpose of this Letter is to consider a model of the weak interaction with less divergences and to show a few results from the model. Details and other effects will be discussed elsewhere.

We assume the existence of charged scalar $C^\pm$ bosons (mass m_C) with a negative metric[3] in addition to the usual charged, weak, intermediate vector bosons $B^\pm$ (mass m_B), and a fundamental Lagrangian for the weak interaction of the form

$$\mathcal{L}_{\mathrm{wk}} = gJ_\lambda^{\;+}[B_\lambda + (1/m_B)\partial_\lambda C] + \mathrm{H.c.}, \tag{1}$$

where J_λ is the weak current [e.g., $i\bar{\Psi}_e\gamma_\lambda(1+\gamma_5)\times\Psi_{\nu_e} + i\bar{\Psi}_\mu\gamma_\lambda(1+\gamma_5)\Psi_{\nu_\mu}$ for leptons] and g is a coupling constant. For simplicity we will take $m_B = m_C$ in the following although a variation of the $C^\pm$ mass keeps most of the following results unchanged.

It is easily seen that the effect of $C^\pm$ cancels the $k_\lambda k_\mu/m_B^2$ term in the $B^\pm$ propagator so that the effective propagator of these bosons becomes $\delta_{\lambda\mu}/(k^2+m_B^2)$. The effective Fermi interaction becomes better-behaved at high energies as follows[5]:

$$\mathcal{L}_{\mathrm{eff}} = g^2\int d^4x'\, J_\lambda^{\;+}(x)\Delta_{\lambda\mu}(x-x')J_\mu(x'), \tag{2}$$

where

$$\Delta_{\lambda\mu}(x-x') = \frac{1}{(2\pi)^4}\int d^4k\, e^{ik(x-x')}\frac{\delta_{\lambda\mu}}{k^2+m_B^{\;2}}. \tag{3}$$

The relation between the coupling constant g and the Fermi coupling constant G ($Gm_p^{\;2} \simeq 10^{-5}$) is

$$g^2/m_B^{\;2} = G/\sqrt{2}. \tag{4}$$

The weak self-masses of leptons come to have only logarithmic divergences and are calculated to be

$$\delta m_l^{\;\mathrm{wk}} = -\frac{g^2}{16\pi^2}m_l \ln\frac{\Lambda^2}{m_B^{\;2}} + \cdots \quad \text{for } l = e \text{ or } \mu \tag{5}$$

and

$$\delta m_\nu^{\;\mathrm{wk}} = 0 \quad \text{for the neutrinos,} \tag{6}$$

where Λ is a cutoff energy. Moreover, if the divergences are assumed to cancel those of electromagnetic self-masses of leptons[6] $\delta m_l^{\mathrm{e.m.}} = 3e^2[16\pi^2 m_l \ln(\Lambda^2/m_l^2)]^{-1}$, i.e.,

$$\delta m_l = \delta m_l^{\;\mathrm{e.m.}} + \delta m_l^{\;\mathrm{wk}} = \text{finite}, \tag{7}$$

the coupling constant g can be predicted to be

$$g^2 = 3e^2. \tag{8}$$

From this relation and Eq. (4), the mass of $B^\pm$ can also be determined:

$$m_B = (3\sqrt{2}\,e^2/G)^{1/2} \simeq 200 m_p. \tag{9}$$

This mass being so large, a direct proof of this model ($p+\bar{p} \to B^\pm + \pi^\mp, \mu+\mu \to B + \mu + \nu_\mu$, etc.) will not be completed until a p-$\bar{p}$ colliding-beam ma-