# ANALYSIS OF $pp \to pp\pi^+\pi^-$ AT 16 GeV/$c$ BY THE MULTI-REGGE-POLE EXCHANGE MODEL

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Experimental distributions for the reaction $pp \to pp\pi^+\pi^-$ at 16 GeV/$c$ are consistent with predictions of a Regge-pole model incorporating exchange of a Reggeized pion. The Treiman-Yang angle distribution affords a distinctive test in favor of pion Reggeization for this reaction.

There has recently been considerable interest in the use of Regge-pole exchange models for the description of multiparticle production processes. Data are presented here on the reaction $pp \to pp\pi^+\pi^-$ at 16 GeV/$c$ from the CERN 2-m hydrogen bubble chamber, and all events are considered, not just those involving $\Delta^{++}(1236)$'s. Comparison of the data is made with a Reggeized-pion-exchange model (RPEM) and with the conventional one-pion-exchange model (OPEM) according to the basic diagram of Fig. 1(a). Our data would seem to show that in some respects agreement is improved by pion Reggeization, though the evidence is not conclusive.

We follow the Toller-variable form for a multi-Regge exchange amplitude as described by Bali, Chew, and Pignotti.[1] For the RPEM the absolute square of the invariant amplitude $M$, summed over final spins and averaged over initial spins, may be expressed as

$$\sum|M|^2 = \sum|M_{\pi^-p}|^2 \left[\frac{(\pi\alpha_\pi')^2 N(t^2, \Delta^2, t'^2)}{2[1-\cos\pi\alpha_\pi(\Delta^2)]} \times \left(\frac{S_{\pi\pi}}{S_0}\right)^{2\alpha_\pi(\Delta^2)}\right] \sum|M_{\pi^+p}|^2, \qquad (1)$$

where

$$S_{\pi\pi} = s_{\pi\pi} + t^2 + t'^2 - (m_\pi^2 + t^2 + \Delta^2)(m_\pi^2 + t'^2 + \Delta^2)/2\Delta^2,$$

$s_{\pi\pi}$ is the invariant mass squared of the $\pi\pi$ system, and where a linear pion trajectory, $\alpha_\pi = -\alpha_\pi'(\Delta^2 + m_\pi^2)$, is employed, with the commonly used value $\alpha_\pi' = 1$ (GeV)$^{-2}$. $N(t^2, \Delta^2, t'^2)$ embraces the factors belonging to the denominator of the $\cosh\xi$ variable of Bali, Chew, and Pignotti of which $S_{\pi\pi}$ is the numerator, and the unknown couplings at the two internal vertices which can also depend on two Toller angles. For simplicity we have taken $N(\Delta^2)$ only, an assumption which the data show to be adequate.

With the conventional OPEM the term in square brackets is replaced by $F(\Delta^2)/(\Delta^2 + m_\pi^2)$, where $F(\Delta^2)$ is a form factor.[2] When we further require identity of RPEM and OPEM at the pion pole $\Delta^2 = -m_\pi^2$, this gives $N(-m_\pi^2) = 1$. We have attempted to fit our data with the constant value $N(\Delta^2) = 1$, by adjusting the only other free parameter $S_0$.

The quantity $M_{\pi^-p}$ in Eq. (1) describes the elastic scattering of a Reggeized $\pi^-$ off a proton, and is approximated by the physical amplitude, evaluated at the scattering angle $\cos\theta(t^2, \Delta^2, s_{\pi^-p})$, namely

$$\sum|M_{\pi^-p}|^2 = 64\pi^2 s_{\pi^-p}\, d\sigma/d\Omega.$$

A corresponding expression is written for $M_{\pi^+p}$. This approximation is used in preference to a sum over the many diagrams containing all Regge trajectories believed to contribute to physical $\pi$-$p$ scattering. It was hoped that effects due to the scattering pion being off shell could be accommodated by adjustment of $S_0$,[3] in the same way as any $\Delta^2$ dependence of the two internal vertices.

To limit possible off-shell effects, and to comply with the diffractive nature of a Regge model, a cut of $\Delta^2 < 1$ (GeV/$c$)$^2$ has been applied to the data. This rejects some 25% of the event combinations, a significant proportion of which are three-body ($p\pi^+\pi^-$) resonances and some $\rho$ events. The presence of such resonances in the remaining events is discussed below.

An underlying assumption of this analysis has been the dominance of the pion trajectory in Fig. 1(a). Other trajectories having the required negative $G$ parity and isospin 1 are the $A_1$ and $A_2$ mesons. Our reasons for excluding them from consideration are similar to those stated by Berger.[4]

Calculations were done using the Monte Carlo program FOWL,[5] and contributions from each of the four diagrams obtained from Fig. 1 by permutation of the protons were included, provided $\Delta^2 < 1$. A value $S_0 = 0.3$ GeV$^2$ was found to provide a reasonable fit to the $\Delta^2$ distribution of all the

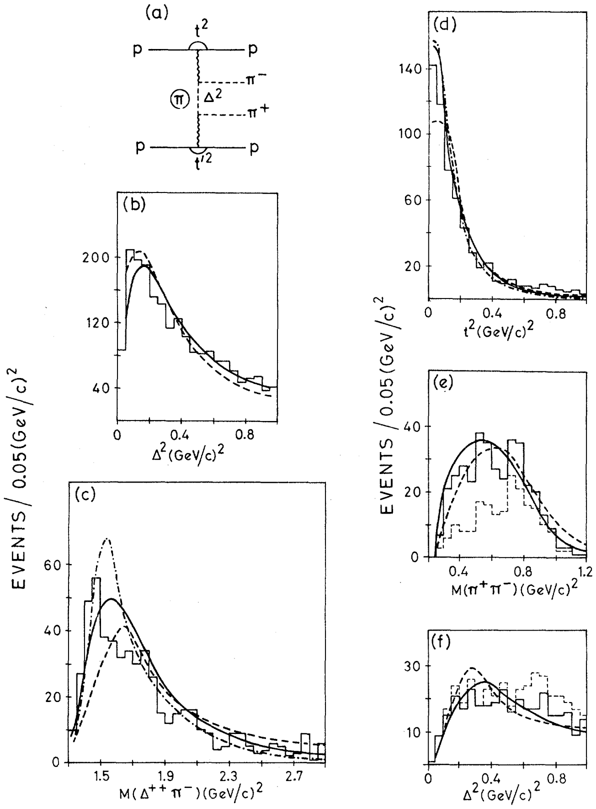


FIG. 1. (a) The basic diagram considered for both the RPEM and OPEM. The momentum transfers $t^2$, $\Delta^2$, and $t'^2$ are defined to be positive in the physical region. (b) The $\Delta^2$ distribution for all events. The plot contains 2011 combinations with $\Delta^2 < 1$. The solid curve is the prediction of RPEM, and the dashed curve that of OPEM. Both are normalized to the data. (c) The $\Delta^{++}\pi^-$ invariant mass distribution for the 656 events belonging to category 1 of Table I. The solid curve is the prediction of RPEM, the dashed curve that of OPEM, and the dash-dotted curve that of DRM. Each is normalized to the data. (d) The $t^2$ distribution for the 656 $\Delta^{++}$ events belonging to category 1 of Table I. Curves as in Fig. 1(c). (e) The $\pi^+\pi^-$ invariant mass distribution for the 396 events belonging to category 4 of Table I. Curves as in Fig. 1(b). The dashed histogram is of the subset of 209 events having $m(p\pi^+) > 1.45$ GeV/$c^2$. (f) The $\Delta^2$ distribution of those events belonging to category 4 of Table I which do not have a $\pi^+\pi^-$ system in the mass range of the $\rho$[0.7 GeV/$c^2$ $< m(\pi^+\pi^-) < 0.8$ GeV/$c^2$]. The plot contains 325 events, and the curves are as in Fig. 1(b). The dashed histogram is of all 396 events belonging to category 4 of Table I.

events,[6] as shown in Fig. 1(b). The experimental $t^2$ and $t'^2$ distributions were found to be closely matched by both models.

In Table I we present cross sections for various categories of events for comparison with the theoretical predictions of RPEM and OPEM. For the category in which a $p\pi^+$ combination lies within a $\Delta\Delta^{++}$ [1.15 GeV/$c^2 < m(p\pi^+) < 1.30$ GeV/$c^2$], an alternative formulation of the model (the double Regge-pole model, DRM) which regards the $p\pi^+$ system as a spin-$\frac{3}{2}$ particle is relevant, and has been applied with some success to $pp \to p\pi^-\Delta^{++}$ at 6.6 GeV/$c$ by Berger et al.[7] The difference from our model is functionally nontrivial, because instead of the factor

$$(S_{\pi\pi}/S_0)^{2\alpha_\pi(\Delta^2)}$$

there is an analogously defined

$$(S_{\Delta\pi}/S_0)^{2\alpha_\pi(\Delta^2)},$$

where

$$S_{\Delta\pi} = s_{\Delta\pi} + t^2 - m_p^2 - (M_\Delta^2 - m_p^2 + \Delta^2)(M_\pi^2 + t^2 + \Delta^2)/2\Delta^2.$$

Table I. Comparison of experimental and theoretical cross sections relating to $pp \to pp\pi^+\pi^-$ at 16 GeV/$c$.

| No. | Category Description of events[a] | Cross section (mb) Experiment | RPEM | OPEM | DRM |
|---|---|---|---|---|---|
| | All | $2.44^{+0.18}_{-0.08}$ | 2.28 | 2.04 | ... |
| 1 | $\Delta^{++}$ | $0.80^{+0.07}_{-0.04}$ | 0.74 | 0.78 | 0.79 |
| 2 | $\Delta^{++}$, $m(p\pi^-) > 1.75$ GeV/$c^2$ | $0.49^{+0.05}_{-0.03}$ | 0.51 | 0.47 | 0.54 |
| 3 | No $\Delta^{++}$ | $1.64^{+0.13}_{-0.06}$ | 1.54 | 1.26 | ... |
| 4 | No $\Delta^{++}$, $m(p\pi^-) > 1.75$ GeV/$c^2$ | $0.49^{+0.05}_{-0.03}$ | 0.46 | 0.41 | ... |

[a]Ref. 6.

A different scaling factor $S_0$ is required, and we have taken the value 0.8 GeV$^2$ used previously,[7] which now gives a satisfactory cross-section value at 16 GeV/$c$ and agreement with the experimental $\Delta^2$ distribution.

The Treiman-Yang angle offers an important test of pion Reggeization. It is defined in the $p\pi^-$ rest frame as

$$\varphi = \cos^{-1}\left[\left(\frac{\vec{p}_{\rm in}\times\vec{p}_{\rm out}}{|\vec{p}_{\rm in}\times\vec{p}_{\rm out}|}\cdot\frac{\vec{p}_{p\pi^+}\times\vec{p}_{\rm in}'}{|\vec{p}_{p\pi^+}\times\vec{p}_{\rm in}'|}\right)\right],$$

where the proton momenta $\vec{p}_{\rm in}$ and $\vec{p}_{\rm out}$ refer to the top vertex of Fig. 1(a), and the proton momentum $\vec{p}_{\rm in}'$ to the bottom vertex. Because more than one diagram contributes, OPEM predicts a little anisotropy in $\varphi$, though not enough to explain the data. Figure 2 gives the experimental distributions in $\varphi$ for each category of Table I, and they are seen to be reasonably well accounted for by the RPEM. The DRM is less satisfactory than at 6.6 GeV/$c$ in this respect.

The models predict Toller-angle distributions insufficiently distinctive from phase space to afford a meaningful test, the data being adequately accounted for.

In category 1, i.e., $\Delta^{++}p\pi^-$ events, the background of nonresonant $p\pi^+$ combinations is thought to be about 10%. Figure 1(c) shows the distribution in $m(\Delta^{++}\pi^-)$; the reaction $pp\to pN(1470)$ is probably contributing,[4] and would go some way towards explaining the discrepancy between the data and the theoretical curves. On the other hand, a recent conjecture of Chew and Pignotti[8] pictures such resonant processes as already included in the Regge amplitude in some average sense (though at fixed values of Toller angles and $t^2, \Delta^2$). At present it does not seem possible to resolve this question quantitatively. Figure 1(d) gives $t^2(p,p)$, and the OPEM is somewhat less successful than either Regge model.

Turning to non-$\Delta^{++}$ events, an interesting distribution (because of the $S_{\pi\pi}$ factor) is that relating to $m(\pi^+\pi^-)$, given in Fig. 1(e). To avoid $p\pi^-$ resonances we have made the further selection $m(p\pi^-)>1.75$ GeV/$c^2$. There is evidence for some $\rho$ production, which is emphasized in the subset of events having $m(p\pi^+)>1.45$ GeV/$c^2$ [see dashed histogram of Fig. 1(e)]. Once again any $\pi\pi$-resonance production might be thought of as already included in our four-body final-state Regge amplitude in the manner suggested by Chew and Pignotti. Exclusion of $\rho$ events gives only slightly better agreement of both the RPEM and OPEM with the $\Delta^2$ distribution [Fig. 1(f)].

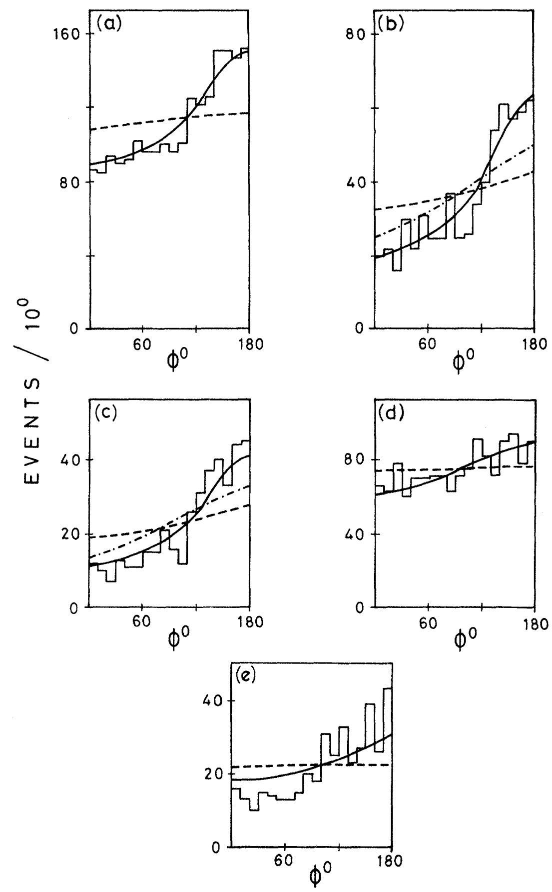


FIG. 2. Treiman-Yang angle distributions for: (a) All 2011 events. Curves as in Fig. 1(b). (b) The 656 events belonging to category 1 of Table I. Curves as in Fig. 1(c). (c) The 399 events belonging to category 2 of Table I. Curves as in Fig. 1(c). (d) The 1355 events belonging to category 3 of Table I. Curves as in Fig. 1(b). (e) The 396 events belonging to category 4 of Table I. Curves as in Fig. 1(b).

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[1]N. F. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. Letters 19, 614 (1967), and Phys. Rev. 163, 1572 (1967).

[2]The Ferrari-Selleri form factors were used: E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963). A value of $30m_\pi^2$ was taken for the cut-off parameter $\gamma$, to give reasonable agreement between the experimental and theoretical cross sections.

[3]$S_0$ is exactly equivalent to a factor $\exp[-d(\Delta^2+m_\pi^2]$ in Eq. (1).

[4]E. L. Berger, Phys. Rev. Letters 21, 701 (1968).

[5]CERN Computer Program Library W.505. A modified version of this program was used, and the theoret-

ical curves have been smoothed by eye from the Monte Carlo results.

[6]The word "event" refers to a single configuration of the protons in Fig. 1(a) satisfying the condition $\Delta^2 > 1$. In consequence the quoted cross section for "All events" of Table I is larger than the cross section $1.66^{+0.13}_{-0.06}$ mb found for the physical process $pp \rightarrow pp\pi^+\pi^-$ at 16 GeV/$c$.

[7]E. L. Berger, E. Gellert, G. A. Smith, E. Colton, and P. E. Schlein, Phys. Rev. Letters 20, 964 (1968).

[8]G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

# EFFECT OF BACKGROUND ON THE $I=2$ $\pi\pi$ SCATTERING PHASE SHIFTS*

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We show that in the reaction $\pi^- p \rightarrow \pi^-\pi^-\pi^+ p$ at 3.9 GeV/$c$, the peaking in the $\pi^-\pi^-$ "scattering angular distribution" above 0.6-GeV c.m. energy inferred from the one-pion exchange model can be interpreted as resulting from a background of competing processes. Similar background effects may be significant also in the reaction $\pi^+ p \rightarrow \pi^+\pi^+ n$, and conclusions from that reaction as to evidence for $d$ waves in the $I=2$ amplitude below 1 GeV should be reserved pending a similar analysis.

The one-pion exchange model is widely used in a variety of interactions to extract information about $\pi\pi$ and $K\pi$ scattering.[1-18] In this Letter, we consider the reaction

$$\pi^- p \rightarrow \pi^-\pi^-\pi^+ p, \qquad (1)$$

in which there is good evidence for $\Delta^{++}$ production by one-pion exchange at a wide range of energies. It has been found in several experiments that, after applying cuts on $M(\pi^+p)$ and $-t(\pi^+p)$ in order to purify the sample, the $\pi^-\pi^-$ scattering angular distribution is isotropic for $M(\pi^-\pi^-) \lesssim 0.6$ GeV and becomes increasingly forward peaked as $M(\pi^-\pi^-)$ increases.[8-15] This is generally taken as evidence for the rapid onset of $d$ waves or higher partial waves, although no structure in the distribution of $M(\pi^-\pi^-)$ is evident. We have results which suggest strongly that the forward peaking observed is a direct result of the background of competing processes.

We have fitted 7975 events of Reaction (1) at 3.9 GeV/$c$ with a model of resonance production by one-particle exchange, using a maximum-likelihood technique.[19] We find that an excellent fit is obtained for a model in which there are the 11 processes listed in Table I adding incoherently. The only adjustable parameters of the fit are the relative amounts of the various processes, and the best values for these parameters are also listed in Table I. The diagrams considered are shown in Fig. 1. The model assumes only $s$-wave scattering at the $\pi^-\pi^-$ vertex in Fig. 1(a). This process, with and without $\Delta^{++}$ production, constitutes the "signal" of interest. The processes in Figs. 1(b)-1(d) are the major "noise" or

Table I. The 11 processes of our fit to the data corresponding to the four diagrams of Fig. 1. The processes are assumed to add incoherently and the relative amounts determined by the fit are shown. The errors quoted are purely statistical. Comments indicate form of Breit-Wigner used (no comment means fixed width form). Diffraction at $\pi p$ vertices is included in several processes. A slope parameter $b=9$ (GeV/$c$)$^{-2}$ was used. Results appear to be insensitive to this parameter.

| Process $\pi^- p \rightarrow$ | Diagram Fig.1 | Mass and Width of Resonance (GeV) | Comments | Relative Amount (%) |
|---|---|---|---|---|
| $\Delta^{++}\pi^-\pi^-$ | (a) | 1.232, 0.12 | Variable width Breit-Wigner form | 15.6±0.5 |
| $\rho^0 p\pi^-$ | (b) | 0.77, 0.14 | Variable width Breit-Wigner form. Diffraction at $\pi^-p$ vertex above 1.67 GeV. | 17.1±0.6 |
| $\pi^-\pi^- p\pi^+$ | (a) | | Diffraction at $\pi^+p$ vertex above 1.67 GeV | 5.1±0.7 |
| $\Delta^0\pi^+\pi^-$ | (b) | 1.232, 0.12 | Variable width Breit-Wigner form | 1.5±0.5 |
| $f^0 p\pi^-$ | (b) | 1.26, 0.14 | Diffraction at $\pi^-p$ vertex above 1.67 GeV | 0.3±0.4 |
| $A_2^- p$ ↳ $\rho^0\pi^-$ | (d) | 1.306, 0.081<br>0.77, 0.14 | | 8.6±0.5 |
| $A_1^- p$ ↳ $\rho^0\pi^-$ | (d) | 1.079, 0.13<br>0.77, 0.14 | | 3.7±1.0 |
| $\pi^-\pi^+\pi^- p$ | (b) | | Diffraction at $\pi^-p$ vertex above 1.67 GeV | 6.4±1.1 |
| $N^{*+}\pi^-$ ↳ $\Delta^{++}\pi^-$ | (c) | 1.45, 0.21<br>1.232, 0.12 | | 12.4±0.7 |
| $\rho^0\Delta^0$ | (b) | 0.77, 0.14<br>1.232, 0.12 | Variable width Breit-Wigner form | 8.0±0.6 |
| $\pi^-(1650)p$ ↳ $f^0\pi^-$ | (d) | 1.654, 0.109<br>1.26, 0.14 | | 1.3±0.4 |