TEST OF THE CLUSTER-KNOCKOUT MODEL USING THE REACTION $Li^6(p, p\alpha)d$†

P. G. Roos, Hogil Him, Mahavir Jain, and H. D. Holmgren
University of Maryland, College Park, Maryland
(Received 10 January 1969)

The reaction $Li^6(p,p\alpha)d$ was studied as a function of angle at 61.5 MeV. The cross section was measured for ten proton angles. Using the quasifree knockout approximation, the $Li^6(p,p\alpha)d$ angular distribution was compared with the free p-α elastic-scattering angular distribution. The agreement between the two is extremely good.

In recent years a number of cluster-knockout reactions, such as $(p,p\alpha)$, have been studied, and have been found to be consistent with the quasifree cluster-knockout model obtained using the impulse approximation. In order to test this model more thoroughly, we have studied the reaction $Li^6(p,p\alpha)d$ as a function of angle at 61.5 MeV. In the impulse approximation the cross section for the reaction $Li^6(p,p\alpha)d$ can be written as

$$\frac{d^3\sigma}{d\Omega_p d\Omega_\alpha dE_p} = (\text{kinematic factor}) \frac{d\sigma}{d\Omega}\bigg|_{p-\alpha} |\varphi(q)|^2,$$

where $(d\sigma/d\Omega)|_{p-\alpha}$ is the free p-α elastic-scattering cross section resulting from the replacement of the off-the-energy-shell t matrix for the p-α reaction by the on-the-energy-shell t matrix. The exact value of $(d\sigma/d\Omega)|_{p-\alpha}$ to be used in the above expression, however, is not uniquely determined, as some ambiguity exists in the choice of energy and scattering angle at which the free cross section is to be evaluated. The kinematic factor is a smoothly varying function; but its specific form depends on the choice of free cross section. For the choice of $(d\sigma/d\Omega)|_{p-\alpha}$ we have followed the prescription used by Epstein[1] in which the energy and observed scattering angle are transformed into the system in which the α particle is initially at rest.

The quantity $\varphi(q)$ is the momentum wave function of relative motion for an α particle and a deuteron in Li^6, where q is the momentum measured in the rest frame of the Li^6 nucleus. In general, this is only one component of a cluster expansion for the Li^6 ground-state wave function. One hopes that by studying the cluster-knockout reaction, $\varphi(q)$ can be determined. This determination would not only result in the momentum distribution of the cluster, but also the cluster parentage of the target-nucleus ground state. However, one must first show that the impulse approximation is valid in the treatment of cluster-knockout reactions.

To this end, we have measured the $Li^6(p,p\alpha)d$ cross section as a function of angle in order to compare the knockout cross section with the free p-α elastic-scattering cross section. The Li^6 nucleus was chosen as a target for two reasons. Firstly, the small separation energy of Li^6 into an α particle and a deuteron, and the success of cluster-model calculations for Li^6 based on an $(\alpha+d)$ structure,[2] indicate that the Li^6 ground state has a large overlap with $(\alpha+d)$. Secondly, previous measurements[3] on the reaction $Li^6(p, p\alpha)d$ at 57 MeV show large contributions from the quasifree knockout process and are consistent with the impulse approximation.

The reaction $Li^6(p,p\alpha)d$ has been studied previously by Ruhla et al.[4] at 155 MeV. In order to test the impulse approximation at 155 MeV, they also measured the $Li^6(p,pd)\alpha$ cross section. Since $\varphi(q)$ is identical in both measurements, the ratio of the two knockout cross sections should be equal to the ratio of the free p-d and the free p-α elastic-scattering cross sections. They find reasonable agreement between the two ratios.

Two other experiments, similar to our measurements, have recently been performed to test the impulse approximation for cluster knockout. Pugh et al.[5] have studied the reaction $Li^6(\alpha,2\alpha)d$ as a function of angle and energy and have compared the angular and energy dependence of the $(\alpha,2\alpha)$ cross section with that of the free α-α cross section. They find remarkably good agreement. The other experiment was a study of the cross section of $Be^9(p,p\alpha)He^5$ as a function of angle at 160 MeV.[6] Again reasonable agreement was found between the cluster-knockout cross section and the free p-α cross section.

The 61.5-MeV proton beam from the Oak Ridge isochronous cyclotron (ORIC) was used to bombard a 0.8-mg/cm^2 self-supporting Li^6 foil. Coincident proton and α-particle events from the reaction $Li^6(p,p\alpha)d$ were detected in two counter telescopes which were coplanar and on opposite sides of the beam. The cross section was measured for ten proton angles ranging from 45° to 140°. The corresponding α-particle angles were chosen so that it was kinematically possible for the deuteron to have zero recoil momentum for some point on the three-body kinematic curve in the two-dimensional energy spectrum of E_p-E_α.

In the impulse approximation this zero-recoil-momentum point also corresponds to the zero-momentum point of the alpha cluster before the collision; i.e., $\vec{q}_{d,\mathrm{recoil}} = -\vec{q}_\alpha$. Therefore, the $(p,p\alpha)$ cross section at the zero-recoil point should be proportional to $|\varphi(q=0)|^2$. As $|\varphi(q=0)|^2$ should be independent of the kinematic conditions, the knockout cross section at the zero-recoil-momentum point should be proportional to the product of the phase-space factor (a known kinematic expression) and the free $(p$-$\alpha)$ cross section, if the impulse approximation is valid.

In previous $(p,p\alpha)$ experiments[7] near 60 MeV we have found that the maximum in $|\varphi(q)|^2$ is often shifted from the point $q_{\mathrm{recoil}} = 0$. We, therefore, first measured the $(p,p\alpha)$ angular correlation for $\theta_p = 105°$ by varying the α-particle angle from 29° to 32°. We found that the peak in $|\varphi(q)|^2$ is within ±0.5° of the zero-recoil-momentum point at $\theta_\alpha = 30°$. For all other proton angles the α-particle angle was chosen to include the zero-recoil-momentum point.

The data for the ten proton angles and the appropriate α-particle angles are shown in Fig. 1. The zero-recoil-momentum point is indicated

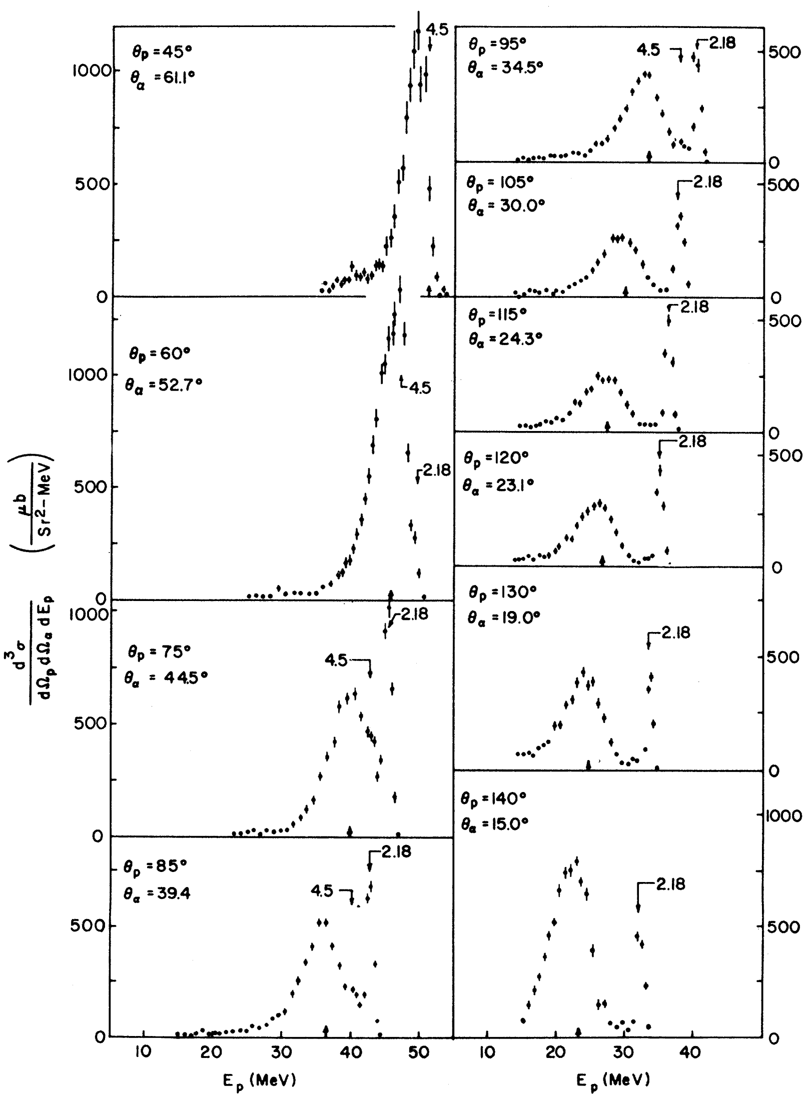

FIG. 1. $Li^6(p,p\alpha)d$ spectra as a function of proton energy. The arrows on the energy axis indicate the zero-recoil-momentum point. The arrows labeled 2.18 and 4.5 indicate the positions of excited states of Li^6 which undergo α decay.

with an arrow on the energy scale. At all angles there are strong contributions from quasifree knockout. It is interesting to note that at all angles the quasifree peak is shifted slightly away (~5 MeV/c) from the zero-recoil-momentum point. In addition to the "quasifree" knockout contributions in the data, there are also contributions from the sequential process

$$p + \mathrm{Li}^6 \to p + \mathrm{Li}^{6*} \to \alpha + d$$

proceeding through the 2.18- and 4.5-MeV states in Li^6. The positions of these states are indicated by arrows in Fig. 1. For $\theta_p = 60°$ the 2.18-MeV state has been eliminated by a low-energy threshold (~8.5 MeV) on the α-particle counter which was used to eliminate deuterons in this counter. Because of the kinematic relationship such a threshold leads to an upper proton-energy limit of approximately 50 MeV. As a result of this limit the $q = 0$ point in the 45° data was cut off. Hence, the data at this angle are somewhat unreliable.

The quasifree knockout data for all angles $\theta_p \geqslant 75°$ have reasonably similar shapes when plotted as a function of recoil momentum, and have a full width at half-maximum of 75 ± 7 MeV/c. This width is approximately 20% larger than the width measured by Ruhla et al.[4] This effect will be investigated more carefully when a more detailed comparison of theory and experiment is performed. For $\theta_p = 45°$ and $\theta_p = 60°$ we have fitted an average shape to the data on the low-energy side of the curve in order to extract the maximum value of the cross section.

The maximum quasifree cross section for each θ_p was then divided by the kinematic factor, and plotted as a function of center-of-mass angle in the proton-alpha system. In the impulse approximation this plot should be proportional to the center-of-mass cross section for free p-α scattering. Figure 2 shows a plot of the data

$$\frac{1}{\text{kinematic factor}} \times \frac{d^3\sigma}{d\sigma_p d\Omega_\alpha dE_p}$$

compared with measured values of free p-α scattering cross section from 31.0 to 93.0 MeV.[8] The error bars on the $\mathrm{Li}^6(p,p\alpha)d$ data include both statistical uncertainties and an estimate of the uncertainty in the extraction of the maximum cross section. In addition, we estimate that the uncertainty in the absolute value of the cross section is about 30%.

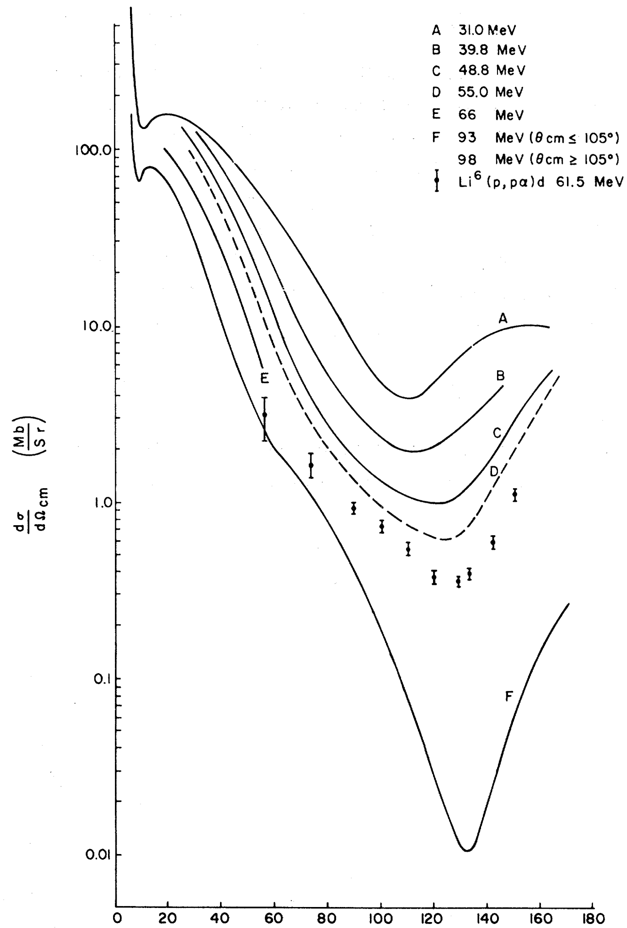

FIG. 2. $(d^3\sigma/d\Omega_p d\Omega_\alpha dE_p) \times (\text{kinematic factor})^{-1}$ as a function of the center-of-mass scattering angle for the p-α system compared with free p-α scattering. The free p-α cross sections were taken from the summary of Ref. 7. The normalization of the $(p,p\alpha)$ cross section is indicated in Ref. 8.

In Fig. 2 we see that with the exception of the $\theta_p = 45°$ point, the data follow the expected shape of the free p-α cross section at 61.5 MeV over a change in free cross section of more than a factor of 4. At the most forward angle, $\theta_p = 45°$, the results fall below the free cross section, but since the $q = 0$ point was eliminated from the data by the energy threshold, this measurement can only be considered a lower limit.

A 2S oscillator wave function has been fitted to the experimentally determined $|\varphi(q)|^2$. The results give an $(\alpha + d)$ cluster parentage of (30 ± 10)% for Li^6. This value seems significantly lower than expected, but is in reasonable agreement with other measurements. The small cluster probability generally obtained from cluster-knockout analyses may indicate the importance of multiple-scattering effects. However, more careful calculations must be performed before one can draw any such conclusions.

At the present one can say that the cross section for the reaction $Li^6(p, p\alpha)d$ follows the free p-α cross section extremely well as predicted by the impulse approximation and lends support to the use of the impulse approximation in the analysis of cluster-knockout reactions even at these relatively low energies. The small value obtained for cluster-parentage coefficient is somewhat puzzling and indicates that multiple-scattering effects may have to be included. If these effects are important they could lead to an overall reduction in cross section, and possibly a distortion of the $|\varphi(q)|^2$ as a function of q. Further experiments should be performed in which one attempts to study the distribution $|\varphi(q)|^2$ with rather high accuracy under various conditions.

The authors would like to thank Dr. H. G. Pugh for his valuable discussions concerning both the experiment and the results of the experiment. We are indebted to the ORIC staff for their assistance and hospitality during the experiment. We would particularly like to thank Dr. C. A. Ludemann for his help on the ORIC computer which enabled us to perform much of the analysis during the run. We would also like to thank the University of Maryland Computer Science Center for the computer time used in the latter phase of our data analysis. Finally, two of us (H.D.H. and P.G.R.) wish to thank Oak Ridge Associated Universities for a grant to cover travel expenses.

†Work supported by the U. S. Atomic Energy Commission.

[1]M. Epstein, thesis, University of Maryland (unpublished). See also P. G. Roos, H. G. Pugh, Mahavir Jain, H. D. Holmgren, and Martin Epstein, to be published.

[2]K. Wildermuth and W. McClure, Cluster Representations of Nuclei (Springer-Verlag, Berlin, Germany, 1966).

[3]M. Jain, M. Epstein, H. D. Holmgren, H. G. Pugh, and P. G. Roos, Bull. Am. Phys. Soc. 12, 466 (1967).

[4]C. Ruhla, M. Riou, M. Gusakow, J. C. Jacmart, M. Liu, and L. Valentin, Phys. Letters 6, 292 (1963).

[5]H. G. Pugh, J. W. Watson, D. A. Goldberg, P. G. Roos, D. I. Bonbright, and R. A. J. Riddle, to be published.

[6]S. Kannenberg, thesis, Northeastern University (unpublished).

[7]Roos, Pugh, Jain, Holmgren, and Epstein, Ref. 1.

[8]The $Li^6(p, p\alpha)d$ data were normalized at the minimum by plotting the minimum p-α cross sections as a function of energy and interpolating for 61.5 MeV.

CATASTROPHIC EVENTS AND GAUGE INVARIANCE

Kurt Haller
University of Connecticut, Storrs, Connecticut 06268

and

Leon F. Landovitz
Belfer Graduate School of Science, Yeshiva University, New York, New York
(Received 20 November 1968)

Electromagnetic effects of catastrophic processes are examples of physical phenomena in which the usual treatment of the Lorentz gauge gives incorrect results. The recently proposed new formulation of the Lorentz gauge leads to results identical to those of the Coulomb gauge.

In earlier work we have called attention to the fact that the computational procedures currently followed in quantum electrodynamics in the Lorentz gauge use state vectors that are not consistent with the proper form of the subsidiary condition.[1,2] In Ref. 1 we developed a new formulation of quantum electrodynamics in the Lorentz gauge in which this difficulty is overcome. We showed that because of the adiabatic nature of scattering processes, there is no physical discrepancy between the two formulations in the case of collisions, though the off-shell transition amplitudes of the old formulation are inconsistent with the consequences of the proper subsidiary condition. In this paper we will demonstrate that for catastrophic processes, the conventional form of the Lorentz gauge and the Coulomb gauge lead to different physical predictions, and that the new formulation of the Lorentz gauge disagrees with the old formulation, but agrees with the Coulomb gauge.[3] Thus if one were to apply the Lorentz gauge in its conventional formulation to such catastrophic processes, one would be making a