

H. Trilling, J. A. Kadyk, A. Rittenberg, D. M. Siegel, J. S. Linsey, and D. Merrill, in Proceedings of the Twelfth International Conference on High Energy Physics, Dubna, U. S. S. R., 1964 (Atomizdat., Moscow, U.S.S.R., 1966), Vol. 2, p. 541.

<sup>5</sup>S. M. Berman and R. J. Oakes, Phys. Rev. **135**, B1034 (1964).

<sup>6</sup>A. Kotanski and K. Zalewski, Nucl. Phys. **B4**, 559 (1968).

<sup>7</sup>There are 16  $R$ -decay parameters; however, normalization of the decay angular distribution requires  $R_{00}^{00} = 1$ .

<sup>8</sup>The Breit-Wigner functions are normalized so that

they have the value unity at the resonance mass.

<sup>9</sup>With this restriction on the  $p\pi^0$  and  $\bar{K}^0\pi^-$  invariant masses, background contamination to the  $\bar{K}^*\Delta$  sample is 10% in the most forward production-angle intervals increasing monotonically to 50% in the most backward.

<sup>10</sup>J. Friedman, Lawrence Radiation Laboratory Group A programming Note No. P-156, 1967 (unpublished).

<sup>11</sup>The values of the maximum-likelihood solution for the relative fractions of the resonance processes were found to be insensitive to the decay angular distribution assumed for the process.

<sup>12</sup>P. Minnaert, Phys. Rev. **151**, 1306 (1966).

<sup>13</sup>J. T. Donohue, Nuovo Cimento **52A**, 1153 (1967).

## HIGH-ENERGY ELECTROPRODUCTION AND THE CONSTITUTION OF THE ELECTRIC CURRENT\*

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The asymptotic behavior of electroproduction cross sections is shown to contain information about the constitution of the electric current.

One of the most interesting possibilities which has emerged from the study of local current algebra is the use of high-energy inelastic lepton-nucleon scattering as a probe of the fundamental constituents of the electromagnetic current. In particular, Bjorken<sup>1,2</sup> has derived sum rules for equal-time commutators (ETCR) of space components of currents in terms of backward electron-scattering cross sections. Such commutators depend crucially on the constitution of the current in terms of fields, but the resulting sum rules are very difficult to evaluate, involving either neutrino scattering<sup>1</sup> or electroproduction on polarized targets.<sup>2</sup>

We would like to show that a much simpler test of this kind follows from the ETCR of the electromagnetic current with its time derivative. Depending on the constitution of the current, one finds that either longitudinal or transverse virtual photons dominate electroproduction cross sections for large momentum transfer. Consequently, simple cross-section measurements at high energies can yield fundamental information about the underlying theory.

Consider, first of all, the amplitude for virtual Compton scattering on protons,

$$\begin{aligned} T_{\mu\nu}(p, q) &= i \int d^4x e^{iq \cdot x} \theta(x_0) \langle p | [J_\mu(x), J_\nu(0)] | p \rangle \\ &= T_1(q^2, \nu) \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + T_2(q^2, \nu) \left( p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right), \end{aligned} \quad (1)$$

where  $\nu = p \cdot q$ ,  $J_\mu$  is the electromagnetic current, and subtraction of the disconnected part as well as an average over proton spins are understood. Both  $T_1$  and  $T_2$  satisfy fixed- $q^2$  dispersion relations (for spacelike  $q^2$ ) which are conveniently written in terms of the variables  $\omega = -q^2/\nu$ :

$$T_1(q^2, \omega) = T_1(q^2, \infty) - \int_0^4 d\omega' \frac{W_1(\omega', q^2)}{\omega'^2 - \omega^2 + i\epsilon}, \quad (2a)$$

$$T_2(q^2, \omega) = -\omega^2 \int_0^4 \frac{d\omega'}{\omega'^2} \frac{W_2(\omega', q^2)}{\omega'^2 - \omega^2 + i\epsilon}. \quad (2b)$$

Following standard Regge lore, we have assumed that  $T_1$  is once subtracted, while  $T_2$  requires no subtraction. The quantity  $W_1$  ( $W_2$ ) is proportional to the differential cross section for backward (forward) inelastic lepton-proton scattering.

Setting  $q = (q_0, 0)$ , (1) becomes

$$T_{ij} = \delta_{ij} \left[ T_1(q^2, \infty) - \int_0^4 d\omega'^2 \frac{W_1(\omega', q^2)}{\omega'^2 - q_0^2/p_0^2} \right] - \frac{p_i p_j q_0^2}{m^2 p_0^2} \int_0^4 d\omega'^2 \frac{W_2(\omega', q^2)}{\omega'^2 - q_0^2/p_0^2}. \quad (3)$$

Bjorken<sup>3</sup> has shown that the weak-interaction assumption

$$\lim_{q^2 \rightarrow -\infty} (-q^2) \int_0^\infty \frac{d\nu}{\nu} W_2(q^2, \nu) < \infty \quad (4)$$

implies that both

$$F_1(\omega) = \lim_{q^2 \rightarrow -\infty} m W_1(\omega, q^2) \\ \omega \text{ fixed}$$

and

$$F_2(\omega) = \lim_{q^2 \rightarrow -\infty} \frac{\nu}{m} W_2(\omega, q^2) \\ \omega \text{ fixed} \quad (5)$$

exist. Therefore, if we multiply both sides of (3) by  $q_0^2$  and let  $q_0 \rightarrow i\infty$ , we obtain, with the help of Bjorken's asymptotic theorem,

$$C_{ij}(\vec{p}) = -\int d^4x \delta(x_0) \langle p | [\dot{J}_i(x), J_j(0)] | p \rangle \\ = \delta_{ij} \left[ \lim_{q^2 \rightarrow -\infty} (q^2) T_1(q^2, \infty) \right] + \frac{p_0^2}{m} \int_0^4 \frac{d\omega'^2}{\omega'} \left[ \delta_{ij} \omega' F_1(\omega') - \frac{p_i p_j}{p_0^2} F_2(\omega') \right]. \quad (6)$$

Finally, we multiply this equation left and right by  $m/p_0^2$  and take the limit  $p_0 \rightarrow \infty$ , keeping the direction of  $\vec{p}$  fixed:

$$\int_0^4 \frac{d\omega'^2}{\omega'} \{ \omega' F_1(\omega') (\delta_{ij} - \hat{p}_i \hat{p}_j) - [F_2(\omega') - \omega' F_1(\omega')] \hat{p}_i \hat{p}_j \} = \lim_{p_0 \rightarrow \infty} \frac{m}{p_0^2} C_{ij}(\vec{p}). \quad (7)$$

The two quantities in the integrand have a simple expression in terms of the total cross sections for photoabsorption of transverse and longitudinal virtual photons:

$$\omega F_1(\omega) = \frac{1}{4\pi^2\alpha} \lim_{q^2 \rightarrow -\infty} (q^2 \sigma_T(\omega, q^2)), \quad (8)$$

$$F_2(\omega) - \omega F_1(\omega) = \frac{1}{4\pi^2\alpha} \lim_{q^2 \rightarrow -\infty} (q^2 \sigma_L(\omega, q^2)). \quad (9)$$

Clearly, if we know something about  $C_{ij}$ , (7) has consequences for the asymptotic behavior of  $\sigma_{T,L}$ .

It is instructive to compare two interesting and popular models: the algebra of fields<sup>4</sup> and the quark model with forces carried by a neutral vector meson (the so-called "gluon model"). The electromagnetic current is constituted differently in the two models, in the one case being proportional to a spin-1 field and in the other being bilinear in spin- $\frac{1}{2}$  fields. The ETCR of the current with its time derivative, and therefore  $C_{ij}$ , can be computed in both theories if one uses the equations of motion:

$$\delta(x_0) [\dot{J}_i(x), J_j(0)] = \delta(x) C_{ab} J_i^a(0) J_j^b(0) + c\text{-numbers (algebra of fields)}, \quad (10)$$

$$\delta(x_0) [\dot{J}_i(x), J_j(0)] = -\delta(x) \bar{\psi} \{ i(\gamma_i \overleftrightarrow{\partial}_j + \gamma_j \overleftrightarrow{\partial}_i - 2\vec{\gamma} \cdot \overleftrightarrow{\partial} \delta_{ij}) - 2g(\gamma_i B_j + \gamma_j B_i - 2\vec{\gamma} \cdot \vec{B} \delta_{ij}) + 4M\delta_{ij} \} Q^2 \psi \\ \text{(quark model)}, \quad (11)$$

where  $J_{\mu}^a$  are the  $SU(3) \otimes SU(3)$  currents,  $C_{ab}$  is a numerical matrix,  $Q$  is the quark charge, and  $B_{\mu}$  is the neutral vector-meson field. Just from the Lorentz tensor character of the commutator, one sees that in the two cases,<sup>5</sup>

$$C_{ij}(\vec{p}) = A p_i p_j + A' \quad (\text{algebra of fields}),$$

$$C_{ij}(\vec{p}) = B(p_i p_j - \delta_{ij} \vec{p}^2) + B' \quad (\text{quark model}),$$

where  $A, A', B, B'$  are Lorentz scalars, about whose values we know nothing.

Upon applying this to (7) we get

$$\begin{aligned} \int_0^4 \frac{d\omega^2}{\omega^2} \{ \omega F_1(\omega) (\delta_{ij} - \hat{p}_i \hat{p}_j) - [F_2(\omega) - \omega F_1(\omega)] \hat{p}_i \hat{p}_j \} &= mA \hat{p}_i \hat{p}_j \quad (\text{algebra of fields}) \\ &= mB(\hat{p}_i \hat{p}_j - \delta_{ij}) \quad (\text{quark model}). \end{aligned} \quad (12)$$

As Eqs. (8) and (9) indicate,  $\omega F_1$  and  $F_2 - \omega F_1$  are positive quantities. Therefore, we must conclude that

$$4\pi^2 \alpha \omega F_1(\omega) = \lim_{q^2 \rightarrow -\infty} q^2 \sigma_T(\omega, q^2) = 0 \quad (\text{algebra of fields}), \quad (13)$$

$$4\pi^2 \alpha [F_2(\omega) - \omega F_1(\omega)] = \lim_{q^2 \rightarrow -\infty} q^2 \sigma_L(\omega, q^2) = 0 \quad (\text{quark model}). \quad (14)$$

It seems most likely, in fact, that the limit is approached as  $1/q^2$ , so that experimentally these two behaviors should be quite distinctive: Depending on which model of the current you choose  $\sigma_T/\sigma_L$  goes either as  $q^2$  or  $1/q^2$ . The remarkable thing is that this result follows even though we have no information about the numerical value of the commutators (10), (11).<sup>6</sup>

The theorem also appears to be more general than the above model. In the quark model no matter what nonderivative interaction you choose, so long as the current has the general form  $J_{\mu} = \bar{\psi} \gamma_{\mu} Q \psi$ , it remains true that  $q^2 \sigma_L(\omega, q^2)$  vanishes for large spacelike  $q^2$ . In that sense the asymptotic vanishing of  $q^2 \sigma_L$  indicates that the electromagnetic current is made only out of spin- $\frac{1}{2}$  fields. If the current is bilinear in spin-0 fields one in general obtains the same theorem as for the algebra of fields. Not surprisingly, then, if the current contains both spin-0 and spin- $\frac{1}{2}$  fields, neither  $q^2 \sigma_T$  nor  $q^2 \sigma_L$  will vanish in the limit.<sup>7</sup>

We feel that these results are interesting on several counts. First of all, the connection between the asymptotic behavior of photoabsorption cross sections and the constitution of the current is as surprising as it is elegant. Second of all, the experimental verification of these theorems should be, in contrast to the previously known consequences of local current algebra, quite clean, since no sum rules have to be evaluated and no data on neutrons are needed. In fact, the rapid accumulation of data at the Stanford Linear Accelerator Center makes it likely that some aspects of these theorems can be compared with experiment in the near future.

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<sup>1</sup>J. D. Bjorken, Phys. Rev. **163**, 1767 (1967). This sum rule can be rotated into an inequality for electroproduction on protons and neutrons. Unfortunately the integrals in these inequalities probably diverge logarithmically.

<sup>2</sup>J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

<sup>3</sup>J. D. Bjorken, Stanford Linear Accelerator Center Report, 1968 (to be published). We would like to thank Professor Bjorken for informing us of his results prior to their publication.

<sup>4</sup>T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

<sup>5</sup>We are assuming that the singular nature of a product of local operators does not prevent us from deducing its tensor nature. In our applications the vacuum expectation value of the product is, of course, explicitly removed.

<sup>6</sup>In the algebra of fields, such commutators are explicitly equal to bilinear products of currents, and one might hope to relate their matrix elements to nonleptonic decay amplitudes. Unfortunately, on account of the subtraction in  $T_1$  this does not work out.

<sup>7</sup>For completeness, we have also looked at the case of a canonical current constructed out of vector-meson fields. The arguments we have given must be modified since assumption (11) is violated. Nonetheless, one can conclude that  $\sigma_L/\sigma_T$  vanishes for large spacelike  $q^2$ .

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ERRATA

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**EFFECT OF LATTICE DISORDER ON THE SUPERCONDUCTING TRANSITION TEMPERATURE.** J. W. Garland, K. H. Bennemann, and F. M. Mueller [Phys. Rev. Letters 21, 1315 (1968)].

A factor  $(\omega_{\text{ph}}^{\text{max}})_0/\langle\omega_{\text{ph}}\rangle_0$  which should multiply the factor  $\langle\omega_{\text{ph}}\rangle/1.26T_c$  in the argument of the logarithm in Eq. (1) was inadvertently omitted. Also,  $g$  should be defined as  $g \equiv \ln(\omega_{\text{ph}}^{\text{max}}/1.26T_c)$ . As elsewhere in the Letter, the subscript 0 denotes the bulk crystalline state. Finally, in the second paragraph on p. 1317, one should read  $\delta A \approx -0.06$ , not  $\delta A \approx -0.6$ . The corrected formulas were used in all calculations; all arguments and conclusions remain unchanged.

**MODIFIED ORDINARY MODE IN MAGNETIZED PLASMAS WITH RELATIVE STREAMING.** Kai Fong Lee [Phys. Rev. Letters 21, 1439 (1968)].

$[v_T^2 + c^2\Omega_e^2/\omega_{pe}^2]^{1/2}$  in Eq. (6) and subsequent discussions should be replaced by  $(v_T + c\Omega_e/\omega_{pe})$ .

**DIFFUSION OF ELECTRON PLASMA OSCILLATIONS IN A LARGE-AMPLITUDE ION-WAVE BACKGROUND.** R. C. Davidson and R. Goldman

[Phys. Rev. Letters 21, 1671 (1968)].

All integrals except those defining  $D_0$ ,  $D(\theta e)$ , and  $N^l(k, t)$  [on the line preceding Eq. (15)] have three-dimensional vector variables of integration; as an example in Eq. (3),  $dk'$  should be replaced by  $d^3k'$ . On line 22 of p. 1672, the second "equals" sign should be changed to a "minus" sign. On the next to last line of p. 1673 and the twelfth line of p. 1675, the time scales are given by  $t \sim (2\gamma_{\Delta k}^S)^{-1}$  and the spectrum levels by  $-\frac{1}{2}s_{\Delta k}^S/\gamma_{\Delta k}^S$ . In Eq. (13) the summation over  $m'$  should extend up to  $m' = l'$ . The first inequality of p. 1675 should read  $k^5 < 25D_0k_0^2t$ .

**BEHAVIOR OF THE <sup>27</sup>Al NMR LINE SHAPE IN RUBY UNDER DYNAMIC NUCLEAR POLARIZATION.** H. H. Niebuhr, E. E. Hundt, and E. Brun [Phys. Rev. Letters 21, 1735 (1968)].

On page 1737, column 2, section 2, the sentence, "It might be possible that they are due to small field inhomogeneities and therefore are proportional to  $\Delta^2/\delta$  as proposed," should read, "It might be possible that they are due to small field inhomogeneities and therefore are proportional to  $\Delta/\delta$  rather than to  $\Delta^2/\delta$  as proposed."