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MAGNETOSTRICTION AND ANOMALOUS THERMAL EXPANSION OF CHROMIUM

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Crystal distortions in Cr associated with antiferromagnetic ordering have been established using electrical-resistance strain gauges. Between 123 and 311°K the observations confirm the existence of an orthorhombic structure associated with the presence of a transversely polarized spin-density wave.

The antiferromagnetism of chromium is now well established.[1-11] The Néel temperature T_N is 311°K, below which the antiferromagnetic state is that of a spin-density wave (SDW). In the so called AF_1 phase, between 123 and 311°K, the polarization is perpendicular to the wave vector of the SDW; below 123°K the polarization and wave vectors are parallel. This longitudinally polarized phase (LSDW) is conventionally known as the AF_2 phase. The transition between the two phases is first order and is often described by the term "spin flip" (T_{SF} = 123°K). In the transversely polarized AF_1 phase (TSDW) the wave vector is a continuous function of temperature[3-6,11] and at T_N the phase transition to the paramagnetic state is also first order.[11]

The AF_1 phase exhibits marked elastic anomalies and magnetoelastic effects.[12-14] In particular the polycrystalline Young's modulus is anomalously low[12] and the distribution of SDW's among the six otherwise equivalent (100) directions is drastically affected by cooling a sample through T_N in the presence of either an external magnetic field[9,13-20] or a uniaxial stress.[9,20]

Many workers[6,7,11,14,16,18] have pointed out that the magnetic symmetry of the antiferromagnetic states is inconsistent with the retention of full cubic symmetry. The AF_2 state will, in general be tetragonal and the AF_1 state orthorhombic. It is generally agreed[13] that the loss of cubic symmetry would account, in principle, for both the stress-cooling and elastic-constant behavior. The former would have its origin in a macroscopic distortion parallel to the wave vector of the SDW. A lattice distortion associated with direction of the polarization vector of SDW would account for the latter since an elastic stress could remove the degeneracy associated with the otherwise equivalent two directions of polarization. The removal of the degeneracy is supposed to take place by the motion of antiferromagnetic domain walls separating two transversely polarized SDW's of the same wave vector. This mechanism could explain the observation that a large magnetic field reduces the elastic anomaly[14] since a magnetic field can induce wall movement, ultimately to a degree at which further stress-induced wall movement cannot occur.

Unfortunately, attempts to detect these lattice distortions have, so far, been unsuccessful.[11,21,22] The most ambitious of these to date[22] established that the lattice distortion is certainly less than 10^{-5}. Refined x-ray techniques capable of detecting distortions at least an order of magnitude smaller than this certainly exist but they are applicable only to materials more structurally perfect than polycrystalline or single-crystal metals and moreover are limited to investigations at or near ambient temperatures. It seems likely, therefore, that the distortions which are being sought are below the limits of sensitivity of presently applicable x-ray techniques.

We have succeeded in detecting the anticipated lattice distortions using standard magnetostriction techniques. Measurements were made on a single crystal of chromium in the form of a disk with a [001] plane in its plane. The crystal was annealed according to the recipe of Street.[12] Low-magnetoresistance strain gauges were fixed along the [010] direction and the strain was observed when a magnetic field of 25 kOe was rotated from the [100] direction to the [010] direc-

tion. The strain detection system was standard and the dummy gauge was mounted on polycrystalline tungsten, chosen because its thermal expansion matches that of chromium over the appropriate temperature interval.

In describing the results it is convenient to use the notation in which the wave vector of the SDW is denoted by $\vec{Q}$ and polarization vector by $\vec{\eta}$. In a perfect crystal and in the absence of external constraints, cubic symmetry requires that the AF_1 state should have six equivalent SDW's denoted $\vec{Q}_1\vec{\eta}_2$, $\vec{Q}_1\vec{\eta}_3$, $\vec{Q}_2\vec{\eta}_1$, $\vec{Q}_2\vec{\eta}_3$, $\vec{Q}_3\vec{\eta}_1$, and $\vec{Q}_3\vec{\eta}_2$ where the subscripts 1, 2, and 3 refer to the x, y, and z axes of the cubic crystal. At T_{SF} such a multi-$\vec{Q}$ crystal transforms to an AF_2 state with three equivalent LSDW's denoted $\vec{Q}_1\vec{\eta}_1$, $\vec{Q}_2\vec{\eta}_2$, and $\vec{Q}_3\vec{\eta}_3$. If, however, a crystal is cooled through T_N with either a large magnetic field or a tensile stress applied parallel to one of the cubic axes, the resulting AF_1 state has a single vector $\vec{Q}$ parallel to this direction. If, for example, a crystal is cooled through T_N with a magnetic field applied parallel to [100] the AF_1 state consists only of the two SDW's $\vec{Q}_1\vec{\eta}_2$ and $\vec{Q}_1\vec{\eta}_3$. This single-$\vec{Q}$ state transforms, on cooling through T_{SF}, to an AF_2 state with a single LSDW, $\vec{Q}_1\vec{\eta}_1$.

We have observed magnetostrictive strains in crystals in both multi-$\vec{Q}$ and single-$\vec{Q}$ states. However, in attempting to relate the observed strains to the required lattice distortions we are obliged to ignore, for the time being, the measurements on the multi-$\vec{Q}$ state crystals since we have, at present, no means of knowing what effect a magnetic field has on the SDW distribution under the conditions of our experiment. Measurements on the single-$\vec{Q}$ state crystal are much less ambiguous in this respect since we can make use of the observation by Werner, Arrott, and Atoji[15] that the single-$\vec{Q}$ state in chromium is very stable once it is produced and that the vector $\vec{Q}$ is neither rotated from its preferred direction parallel to a cube edge nor switched to another crystallographically equivalent direction by magnetic fields up to 27 kOe. We thus interpret the observed strains in the single-$\vec{Q}$ crystal in terms of a reorientation of the polarization vector $\vec{\eta}$.

We denote the strains associated with antiferromagnetic ordering by λ, μ, and ν such that λ is the strain parallel to $\vec{Q}$, μ is the strain parallel to $\vec{\eta}$, and ν is the strain perpendicular to both $\vec{Q}$ and $\vec{\eta}$. λ, μ, and ν are all functions of temperature and are measured from a fictitious nonmagnetic state obtained by extrapolation from the paramagnetic state. In order to establish which of these quantities our measurements determine we assume, for the time being, that the measuring field H_m suppresses completely the SDW with $\vec{\eta}$ parallel to H_m in favor of that for which $\vec{\eta}$ is perpendicular to H_m. A crystal which has been cooled through T_N in a field H_c parallel to [100] has equal volumes of SDW domains $\vec{Q}_1\vec{\eta}_2$ and $\vec{Q}_1\vec{\eta}_3$. With H_m parallel to [100] the SDW structure remains $\vec{Q}_1\vec{\eta}_2+\vec{Q}_1\vec{\eta}_3$ and the strain parallel to [010] is $\frac{1}{2}(\mu+\nu)$. When H_m is parallel to [010] the SDW structure becomes $\vec{Q}_1\vec{\eta}_3$ and the strain parallel to [010] becomes ν. The difference between the strains in the final and initial states is thus $\frac{1}{2}(\nu-\mu)$. This quantity denotes a lattice distortion perpendicular to $\vec{Q}$ and associated with the orientation of $\vec{\eta}$. It is the most likely source of the elastic anomalies in the AF_1 phase. The measured strain difference is shown in Fig. 1 as a function of field and in Fig. 2 (curve A) as a function of temperature at a constant field of 25 kOe. This field is evidently insufficient to suppress one SDW completely and the magnitude of $\frac{1}{2}(\nu-\mu)$ must be somewhat higher than the measured strains shown in Fig. 2.

If an otherwise identical measurement is carried out on a crystal field cooled with H_c parallel to [010], the strain parallel to [010] is λ for both directions of H_m and the strain difference is zero. Confirmation of this prediction is shown in Fig. 2 (curve B).

No effect could be observed in either the single-$\vec{Q}$ or multi-$\vec{Q}$ states in the AF_2 phase, confirming that $\vec{Q}$ switching does not take place to a significant extent in fields up to 25 kOe. This is in

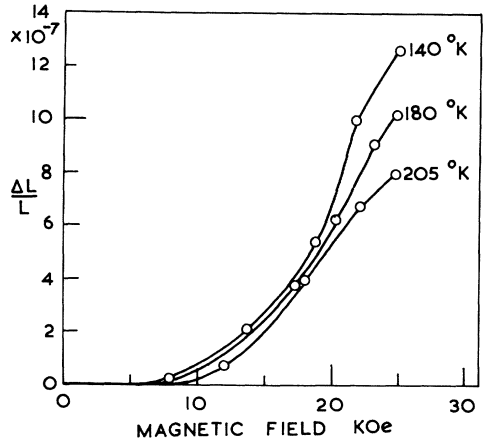

FIG. 1. Strain measured along [010] when magnetic fields of various strengths are rotated from [100] to [010]. Single-$\vec{Q}$ crystal with $\vec{Q} \parallel [100]$.

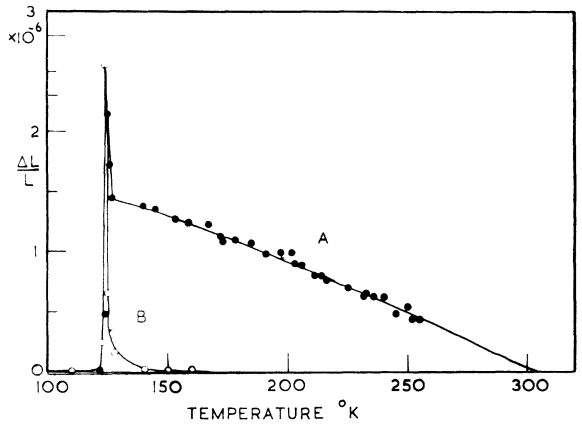

FIG. 2. Strain measured along [010] when a magnetic field of 25 kOe is rotated from [100] to [010]. Curve A: single-$\vec{Q}$ crystal with $\vec{Q} \parallel [100]$. Curve B: single-$\vec{Q}$ crystal with $\vec{Q} \parallel [010]$.

conformity with measurements of the elastic constants which show no anomalous behavior in the AF_2 state.[12]

The large strains observed close to T_{SF} are, we believe, associated with field-induced transitions $AF_1 \rightleftharpoons AF_2$. It is known from the neutron diffraction experiment of Werner, Arrott, and Atoji[15] that a magnetic field of 27 kOe applied parallel to the propagation vector of a single-$\vec{Q}$ state crystal depresses the spin-flip temperature by about 1°K. This is, roughly, the width of the peak in the magnetostriction shown in Fig. 2. We suggest that, over a narrow temperature interval close to T_{SF}, a magnetic field H favors a SDW for which $\vec{\eta}$ is perpendicular to H. Thus for example a multi-$\vec{Q}$ crystal close to T_{SF} in a magnetic field with H parallel to [010] splits up into two LSDW's $\vec{Q}_1\vec{\eta}_1$ and $\vec{Q}_3\vec{\eta}_3$ and two TSDW's $\vec{Q}_2\vec{\eta}_1$ and $\vec{Q}_2\vec{\eta}_3$. Rotation of the field to [100] yields a distribution $\vec{Q}_1\vec{\eta}_2$, $\vec{Q}_1\vec{\eta}_3$, $\vec{Q}_2\vec{\eta}_2$, and $\vec{Q}_3\vec{\eta}_3$ with consequent lattice distortion associated with the different mixture of AF_1 and AF_2 phases. At higher temperatures the field is too small to sustain the energy difference between the AF_1 and AF_2 phases, the crystal transforms to the AF_1 phase, and the only effect of the field is to remove the degeneracy of the two differently polarized TSDW states.

In view of the strong evidence that a magnetic field switches $\vec{\eta}$ but not $\vec{Q}$ it is clear that measurements of this kind cannot determine λ. Preliminary measurements of the anisotropic thermal expansion of a single-$\vec{Q}$ crystal are shown in Fig. 3. These indicate quite clearly that $\lambda > \frac{1}{2}(\mu + \nu)$, i.e., that there is an elongation parallel to $\vec{Q}$ greater than the strain perpendicular to $\vec{Q}$.

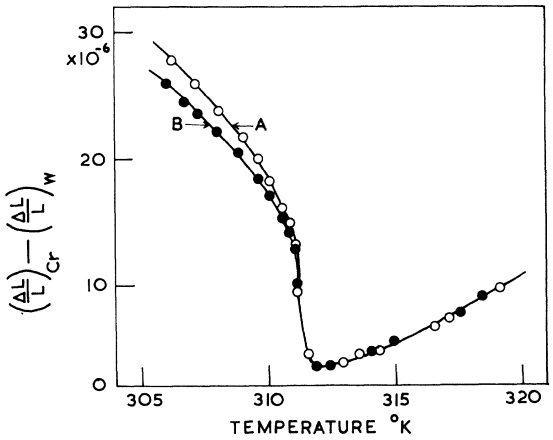

FIG. 3. Lattice expansivity of chromium relative to that of tungsten in the neighborhood of the Néel temperature measured in the [010] direction. Curve A with field cooling, H_c along [010]. Curve B with H_c along [100]. The difference $A-B=\lambda-\frac{1}{2}(\mu+\nu)$ indicating that the SDW axis is elongated with respect to the two perpendicular axes.

The sign of λ is consistent with the nature of the stress-cooling experiment.

Thermal expansion measurements over the entire AF_1 phase show quite clearly that λ, μ, and ν are very nearly equal. Thus the main distortion of the lattice associated with the antiferromagnetic ordering is a uniform volume dilation and the anisotropic strains are small superimposed perturbations. In this sense the lattice distortions in chromium are similar to those in normal ferromagnets but are complicated by the existence of the symmetry restrictions imposed by both $\vec{Q}$ and $\vec{\eta}$. The anisotropic part of the lattice distortion corresponds to a change of symmetry from cubic to orthorhombic in the AF_1 phase. The signs and magnitudes of the orthorhombic distortion are sufficient to account, in principle, for all the observed magnetoelastic effects.

A fuller account of this investigation, including evidence for a first-order transition at T_N and the lattice distortions associated with the spin-flip transition, will be presented elsewhere.

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SIMPLE METHOD FOR INVESTIGATING THE PARENTAGE OF STATES USING TWO-NUCLEON TRANSFER REACTIONS*

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We describe a promising method for investigating the parentage of nuclear states. It involves the simultaneous observation of (p,t) and $(p,{}^3\mathrm{He})$ reactions on 0^+ $(T>0)$ targets producing analog final states with the same isospin as the target. Experimental results for $20 \leq A \leq 36$ $(T=1)$ reveal a number of states with apparently very simple parentage for which j^2 "paired" pickup predominates.

The simultaneous observation of (p,t) and $(p,{}^3\mathrm{He})$ reactions on a target with isospin T_i has been used for some time[1-5] to locate and identify analog final states with isospin $T_f = T_i + 1$. The signature of a pair of such states is that the angular distributions of the corresponding tritons and ^{3}He particles have the same shape, and their magnitudes are related by a simple calculable factor. In this Letter we shall first examine what is now a fairly large body of relevant experimental data, and use it as a means of establishing the validity of the approximations made in calculating the cross-section ratio. We will then use the same approximations to calculate ratios for analog final states with $T_f = T_i$. In certain cases the same experimental techniques can be used not only to identify such states, but also to determine information regarding their structure. Experimental data presented on such states in sd-shell nuclei indicate a striking simplicity in their parentage. Evidently the experimental method described provides a new and useful spectroscopic tool for investigating the parentage of nuclear states.

An expression for the differential cross section for the two-nucleon pickup reaction $A(a,b)B$ can be derived using the distorted-wave Born approximation (DWBA). If spin-orbit forces are neglected, the result obtained is[6,7]

$$\frac{d\sigma}{d\Omega} \propto \frac{k_b}{k_a} \sum_{L\Lambda SJ} b_{ST}{}^2 (T_B{}^N{}_B{}^{TN} | T_A{}^N{}_A)^2 [D(S,T)]^2 \left| \sum_N G_{NLSJT} B_\Lambda{}^{LN}(\vec{k}_a, \vec{k}_b) \right|^2. \quad (1)$$

Here $b_{ST}{}^2$ is a spectroscopic factor for the light particles a and b; it equals $\frac{1}{2}(\delta_{S0}\delta_{T1} + \delta_{S1}\delta_{T0})$ for $(p,{}^3\mathrm{He})$ and $\delta_{S0}\delta_{T1}$ for (p,t). The function $D(S,T)$ depends[7,8] upon the strength of the spin- and isospin-exchange terms in the interaction potential. The term $B_\Lambda{}^{LN}$ contains distorted waves and the form factor of the transferred nucleons, while G_{NLSJT} contains all the information on nuclear struc-