MgO and CaO, respectively. Although δE_{EPC} for CaO is slightly greater than ω_0, it is less than $(1+\gamma)\omega_0$ unless $\gamma \leq 0.2$. A band gap at Γ for MgO of 7.775 ± 0.010 eV is consistent with having four Lorentzian shaped peaks and a band edge rising as $E^{1/2}$. A Γ band-gap energy for CaO of 7.034 ± 0.022 eV was estimated as being between the high-energy end of the first and second EPC. From the Γ band gaps we computed the $j = \frac{3}{2}$ exciton binding energies ϵ_B of 85 ± 10 and 104 ± 22 meV for MgO and CaO, respectively. The value of ϵ_B/ω_0 is 0.95 ± 0.11 for MgO and 1.5 ± 0.3 for CaO.

A hydrogenic fit to the measured binding energy of MgO, based on the effective mass value of $\mu = 0.23m$ of Cohen and Fong,[10] can be obtained only by using an effective dielectric constant $\epsilon = 6.0$ intermediate between the optical $\epsilon_0 = 2.97$ and the static dielectric constant $\epsilon_\infty = 9.8$.

The intensity ratio of the oscillator strength of the EPC to that of the Γ exciton is 0.63 ± 0.07 for MgO and 0.40 ± 0.20 for CaO. Toyozawa and Hermanson[9] predict that the intensity ratio increases with ϵ_B/ω_0. Our two experimental data points for MgO and CaO do not support this feature of the theory.

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PHONON AVALANCHE IN Ce-DOPED LANTHANUM MAGNESIUM DOUBLE NITRATE. MEASUREMENT OF THE PHONON LIFETIME AND OF THE ACOUSTIC SPECTRUM

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We have investigated the phonon avalanche in Ce-doped lanthanum magnesium double nitrate using pulsed microwave and pulsed magnetic-field techniques. In these experiments we have been able to determine the lifetime of the avalanche phonons and to obtain some evidence for coherence in the spin-phonon interaction.

We have investigated the phonon avalanche in Ce-doped lanthanum magnesium double nitrate[1,2] (LMN-Ce) by means of electron spin-echo and pulsed magnetic-field techniques. The experiments were conducted at 9.1 Gc/sec and at 1.5°K on a nominally 0.3% doped sample of dimensions 0.2 cm × 0.3 cm × 0.3 cm. The resonance line had a full width at half-height of 10 G. Microwave pulses were generated by a Litton L5022 traveling wave tube (output $\gtrsim 1$ kW). Pulsed magnetic fields were obtained by winding a few turns of wire around the sample. In these experiments we have been able to determine the phonon lifetime in the sample. We have also been able to obtain some information regarding the form of the acoustic pulses which develop during the avalanche.

The phonon lifetime was investigated by using a three-pulse sequence as follows. Pulse I, a 180° pulse, inverts the Ce-spin system and operates effectively on the whole resonance line. Pulses II and III generate a two-pulse spin echo by means of which we sample the magnetization a time t after the inverting pulse. Since we wish to sample only the central portion of the line (i.e., the portion which is "burnt out" by the avalanche), pulses II and III are characterized by a lower microwave field H_1 than pulse I.[3] The echo signal is integrated by means of a boxcar unit, and its amplitude is traced out as a function of t as shown in Fig. 1.

The lowest curve in Fig. 1 (curve a) shows the normal evolution of the inverted magnetization as the avalanche proceeds, and is similar to the curve shown in Fig. 1 of Ref. 1. In obtaining the upper curves we interrupted the growth of the avalanche by switching the Zeeman field 15 G to one side of its initial value at a time $\simeq 8$ μsec after the inverting pulse. As a result of this sudden change in field the hot phonons were left in

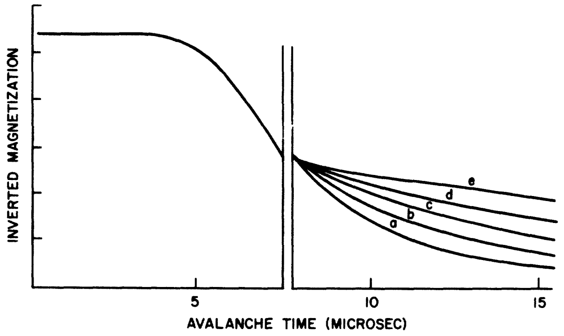

FIG. 1. Curves showing the decay of the inverted magnetization in the LMN-Ce sample. Curve a shows the normal evolution of the phonon avalanche. In the remaining cases the avalanche was temporarily interrupted 8 μsec after the inverting pulse by switching the Zeeman field 15 G to one side for a time t_H. Curves b, c, d, and e correspond to values $t_H = 5$, 10, 20, 30 μsec. The time interval t_H has been eliminated from the horizontal scale.

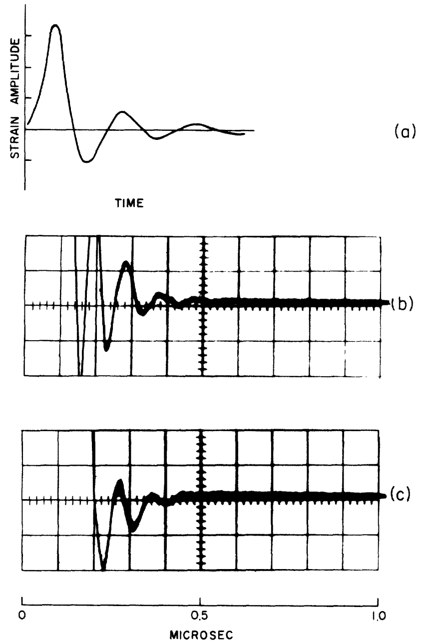

FIG. 2. (a) Acoustic π pulse computed according to the growing-wave models described in Ref. 5. (b) Induction signal corresponding to hole burnt out in center of line by avalanche 12 μsec after initial inverting pulse. (c) Induction signal corresponding to hole burnt out of the site of the line. The resonance line was suddenly displaced 12 μsec after initial inverting pulse. The Fourier transform of this induction signal gives an approximate measure of the power spectrum of the acoustic field 12 μsec after the start of the avalanche.

communication with spins in the tail of the resonance line (where the line intensity was down by a factor of <0.1), while the central portion of the line was placed in communication with a band of cold phonons. At a time t_H later the Zeeman field was switched back to its initial value, thus permitting the avalanche to continue. The diminished slope of M_z after this operation gave us a measure of the loss in intensity of the hot phonons during t_H.[4]

By combining a number of results such as those shown in Fig. 1 we were able to deduce a phonon lifetime $\approx$20 μsec in our sample. This is longer than the avalanche time itself and implies that little acoustic energy is lost during the evolution of the avalanche. It is also considerably longer than the transit time (~2 μsec) of sound waves across the sample, thus suggesting that there are no significant macroscopic variations in sound intensity throughout the sample.

The phase-memory time (i.e., the effective T_2) of the spins was measured in a normal two-pulsed echo experiment and found to be 3 μsec. As may be seen from Fig. 1, this time is approximately the same as the time during which the major change in M_z occurs. These magnitudes suggest that the transfer of energy from the spin system to the acoustic field is likely to be characterized by some degree of coherence, and cannot be reliably analyzed by means of rate equations. A number of authors have treated this type of problem in connection with the growth of light waves in a laser.[5] These treatments do not start out by assuming fixed transition probabilities. Instead, the growing wave is specified in terms of a slowly varying amplitude and phase, while the dynamics of the two-level systems are described by means of Bloch equations [see, for example, Eqs. (10)-(14) of Tang and Silverman].

Although our own problem differs in some respects from the laser problems considered in Ref. 5, we felt it would be worthwhile to make computations of the growth of traveling acoustic waves in LMN-Ce. Initial amplitudes were chosen to correspond with the acoustic energy present at 1.5°K in the band covered by the LMN-Ce resonance line. Input wave forms were adopted which rose smoothly from zero, and lasted for times $\gtrsim T_2$. Acoustic loss was ignored. Inhomogeneous broadening was assumed. Under these conditions all wave forms showed the same general behavior. Amplification was linear over about three orders of magnitude (in intensity), then nonlinear effects supervened and shaped the pulses into "π pulses" as predicted by McCall and Hahn.[6] Figure 2(a) shows the π pulse obtained when a smoothed step wave is used as input.

An interesting feature of the wave form in Fig. 2(a) is the damped periodic tail.[7] This results in part from the coherence which is assumed to characterize the spin-phonon interaction in this treatment. No such wave form can be deduced from a rate-equation model. In order to test experimentally for the presence of phenomena of this kind we therefore examined the shape of the "holes" burnt out at different times during the avalanche. We did this by applying a 90° microwave pulse to the spin system, thus turning the spectral distribution $M_z(\omega)$ into the xy plane, where it generates a free-induction signal corresponding to the Fourier transform of $M_z(\omega)$. In the early stages of burnout (~5 to 6 μsec after initial inversion) the induction signals had the form of a simple decay trace.[8] At higher burnout levels the induction signal took on a periodic form as shown in Fig. 2(b). Since the shape of the hole depends on the power spectrum of the acoustic waves which cause the burnout, trace 2(b) thus strongly suggests that acoustic pulses of the same general form as those shown in Fig. 2(a) develop during the later stages of the avalanche.

It is difficult to establish a close connection between Figs. 2(a) and 2(b) because (i) the shape of the hole is proportional to the acoustic power spectrum only in the limit of small M_z changes, and (ii) the hole records the total history of the avalanche rather than the acoustic spectrum at any given stage. Fortunately we were able to modify the experiment in such a way as to provide a more reliable observation of the acoustic power spectrum in the later stages of avalanche development. At a time ~12 μsec after the inverting pulse the Zeeman field was suddenly switched to a point in the wings of the line outside the central hole. The acoustic waves present in the avalanche at the time when the field was switched were thus able to burn out a portion of unused line. 3 μsec later a 90° pulse was applied to discover the form of the new hole produced. The resulting induction trace[9] [Fig. 2(c)] represents the effect of the acoustic field existing 12 μsec after the initial inversion and thus corresponds to a better defined time sampling of the acoustic field than trace 2(b). It should also provide, via its Fourier transform, a better indication of the acoustic power spectrum, since the degree of burnout was only half as great as that associated with trace 2(b). In order to make an accurate measurement of the power spectrum one would of course prefer to work with only minimal levels of burnout. The similarity between Figs. 2(b) and 2(c) suggests, however, that no qualitative differences would be observed even under more ideal conditions.

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[8] This suggests the presence of a hole consisting of a single smooth dip (as, e.g., for a Gaussian or Lorentzian function). Unfortunately we could not make a precise determination of the hole shape, since we could not (on account of instrumental transients) record the induction signal in the first 150 nsec following the 90° pulse.

[9] It was verified that this trace was not due to burnout during the main avalanche, by switching the Zeeman field immediately after the inverting pulse instead of waiting for 12 μsec. The avalanche then presumably develops in the normal manner, generating acoustic waves and burning a hole at a position corresponding to the new (displaced) center of the resonance line. No induction signal could be obtained from the wings of the line under these conditions.