LINEWIDTH OF THE RADIATION EMITTED BY A JOSEPHSON JUNCTION*

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A quasiparticle-current–fluctuation contribution to the radiation emitted by Josephson tunnel junctions is calculated. The result is combined with Stephen's result for the pair-current–fluctuation contribution and compared with experimental linewidth data for Sn-I-Sn and Pb-I-Pb junctions.

Recently Stephen has calculated a contribution to the linewidth of radiation emitted by a Josephson junction resulting from fluctuations of the pair current.[1] In this Letter we discuss a contribution to the linewidth due to quasiparticle-current fluctuations and compare the resulting theoretical expression for the linewidth with experimental data.[2]

When a Josephson junction is biased at a voltage V, the Josephson supercurrent is given by $I = I_1 \sin\varphi$, where $\dot{\varphi} = 2eV/\hbar$. If the bias voltage contains both a dc voltage V_0 and a modulating noise voltage $V_n(t)$, then

$$I = I_1 \sin[\omega_0 t + (2e/\hbar)\int^t V_n(t')dt'], \tag{1}$$

with $\omega_0 = 2eV_0/\hbar$. For the cases of interest, the linewidth of the radiation $\Delta\omega$ is determined primarily by the voltage fluctuations in the frequency range $(0, \Delta\omega)$, and $\Delta\omega$ is much smaller than the instantaneous Josephson frequency $\omega = 2eV/\hbar$. These voltage fluctuations are in turn directly related to the current fluctuations via the junction impedance. In this way, the high-frequency spectrum of the current [Eq. (1)] which generates the radiation arises from and reflects the low-frequency current fluctuations. The mean-square current fluctuation is related to a power spectrum $P_I(\omega)$ by

$$\langle \delta I^2 \rangle = \int_0^\infty P_I(\omega) d\omega. \tag{2}$$

In thermal equilibrium, the well-known fluctuation-dissipation theorem relates $P_I(\omega)$ to the junction conductance and we have the familiar Johnson-Nyquist result

$$P_I(\omega) = [\hbar\omega\sigma_1(\omega)/\pi] \coth(\tfrac{1}{2}\beta\hbar\omega), \tag{3}$$

where $\sigma_1(\omega)$ is the real part of the junction conductance at frequency ω. A direct calculation using the tunneling-Hamiltonian formalism shows that $\sigma_1(\omega)$ can be expressed in terms of the dc current-voltage characteristic $I(V)$ by

$$\sigma_1(\omega) = (e/2\hbar\omega)[I(\hbar\omega/e) - I(-\hbar\omega/e)]. \tag{4}$$

In the present case, however, the junction is biased by an externally applied dc potential V_0 and is operated in a steady-state but not thermodynamic equilibrium condition, so that the standard fluctuation-dissipation relation is no longer applicable.

In the last part of this Letter we derive the following expression[3,4] for the quasiparticle part of the power spectrum for a junction biased at voltage V_0:

$$P_{Iqp}(\omega) = (e/2\pi)\{I_{qp}(V_0 + \hbar\omega/e) \coth[\tfrac{1}{2}\beta(eV_0 + \hbar\omega)] + I_{qp}(V_0 - \hbar\omega/e) \coth[\tfrac{1}{2}\beta(eV_0 - \hbar\omega)]\}. \tag{5}$$

Here $I_{qp}(V_0 \pm \hbar\omega/e)$ is the quasiparticle part of the dc current at a voltage $V_0 \pm \hbar\omega/e$, so that just as Eq. (3) allowed the determination of the thermal-equilibrium power spectrum in terms of the observed I-V characteristic, Eq. (5) allows the quasiparticle part of $P_I(\omega)$ to be determined in terms of directly measurable properties of a junction.[5] Two limiting cases for this power spectrum are of particular interest. First, when the bias voltage V_0 goes to zero, the power spectrum reduces to Eq. (3). For the opposite case in which V_0 is large compared with $\hbar\omega/e$,

$$P_{Iqp}(\omega) = (e/\pi) I_{qp}(V_0) \coth(\tfrac{1}{2}\beta e V_0). \tag{6}$$

This is simply the expression for shot noise with an effective noise current $I_{qp}(V_0)\coth(\frac{1}{2}\beta eV_0)$. The factor $\coth(\frac{1}{2}\beta eV_0)$ arises because the noise current is the sum of the magnitudes of the forward and backward junction currents. For the low-frequency region of interest, it follows from an analysis of the Josephson-Maxwell equations[6] (and it is clear on physical grounds) that the junction impedance is simply the dynamic resistance $R_D = (dI/dV)^{-1}$, so that

$$\langle \delta V^2 \rangle = R_D^2 \int_0^\infty P_{Iqp}(\omega) d\omega. \tag{7}$$

Using Eq. (1), Eq. (3), and standard frequency-modulation noise theory,[7] we obtain for the quasiparticle contribution to the linewidth

$$\Delta\omega_{qp} = (2e/\hbar)^2 R_D^2 e I_{qp}(V_0) \coth(\tfrac{1}{2}\beta V_0). \tag{8}$$

Stephen's result[1] for the fluctuations due to pair transfer can be included by adding the pair-transfer "shot noise"

$$P_{Ip}(\omega) = (2e/\pi) I_p(V_0) \coth(\beta e V_0) \tag{9}$$

to the quasiparticle part of the current fluctuations, Eq. (6). Here $I_p(V_0)$ is the pair-transfer part of the dc current, which at finite temperatures consists of two terms,[8]

$$I_p(V_0) = I_1 \langle \sin\varphi \rangle + \sigma_1'(V_0) V_0 \langle \cos\varphi \rangle, \tag{10}$$

where the angular brackets imply time averages. The first term is the usual Josephson dc pair-transfer current, while the second term arises from pair-transfer processes which go via an intermediate state in which a thermally excited quasiparticle tunnels. At low temperature, $\sigma_1'(V_0)$ decreases with temperature in the same way as the normal quasiparticle conductance $\sigma_1(V_0)$. The factor of $2e$ entering Eq. (9) arises because pairs of electrons are coherently transferred. The expression for the linewidth including both the quasiparticle and the pair contributions is[9]

$$\Delta\omega = (2e/\hbar)^2 R_D^2 [e I_{qp}(V_0) \coth(\tfrac{1}{2}\beta e V_0) + 2e I_p(V_0) \coth(\beta e V_0)]. \tag{11}$$

In the present experiment $kT \gg eV_0$ so that the hyperbolic contangent factors can be expanded to obtain the simple relation

$$\Delta\nu = \Delta\omega/2\pi = (4\pi R_D^2 kT/V_0)(2e/h)^2 (I_{qp} + I_p). \tag{12}$$

Linewidths were measured experimentally using a sensitive superheterodyne receiver to detect the radiation. The local oscillator was phase locked to reduce its residual FM to less than 1 kHz. The linewidth of the amplified 30-MHz i.f. signal was measured using a rf spectrum analyzer. A 5-msec RC filter was placed in the cryostat on each of the four leads attached to the junction. These filters were necessary to eliminate external noise sources. The filter resistors in the current leads formed the 40-kΩ source resistance of the dc bias source. The linewidth of the emitted radiation was measured at a sequence of bias points on the first self-induced step. The dynamical resistance $(dI/dV)_{V_0}^{-1}$ was determined as a function of current by a detailed measurement of the I-V characteristic of the induced step. The voltage V_0 was determined from the frequency of the emitted radiation and the relation $2eV_0 = h\nu$. A Sn-I-Sn tunnel junction and a Pb-I-Pb tunnel junction with respective normal resistances of 0.825 and 2.5 Ω were used in the experiments reported here. The Josephson-current densities were approximately 1 A cm^{-2}. The dimensions were approximately 0.8 by 0.3 mm. The first self-induced step (determined by the 0.8-mm dimension) occurred near 20 μV with a corresponding radiated frequency near 10 GHz.[10]

The experimental results are compared with the theoretical expression [Eq. (12)] in Fig. 1. The linewidths were normalized to constant bias voltage and temperature using the functional dependences indicated by Eq. (12). The ratio of this normalized linewidth to the square of the dynamical resistance is plotted versus the total current. The scatter in the experimental data is larger than the uncertainties in the determination of the linewidth alone and presumably includes contributions from other experimental factors. The theory is represented by the solid curve. The experimental linewidth is in qualitative agreement with theory but is larger in magnitude by approximately a factor of 2. The linewidth at the temperatures indicated in Fig. 1 was observed to have a slightly stronger than linear dependence on the current. The dependence became extreme at the lowest temperature used (1.2 K), where the linewidth varied roughly as the fifth power of the current; these data are not presented in Fig. 1. In order to estimate the strength of the rf

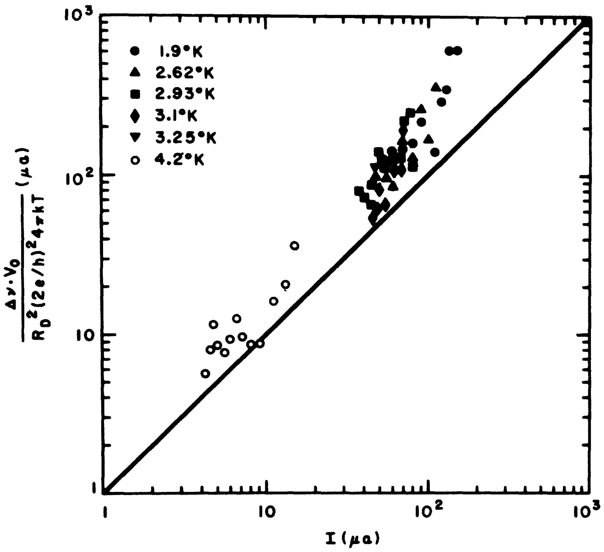

FIG. 1. The ratio of the linewidth to the square of the dynamical resistance versus current. The linewidth is normalized to a standard operating voltage and temperature. The solid symbols represent data from a Sn-I-Sn junction and the open symbols from a Pb-I-Pb junction. The solid curve represents the theoretical expression for the linewidth [Eq. (12)].

field in the Sn-I-Sn junction the I-V characteristics on the first self-induced step were compared with the analog-computer-generated curves of Werthamer and Shapiro.[11] The I-V curves near resonance resembled the lower half of a cusplike peak and showed no inflection point. The maximum pair current on the first self-induced step

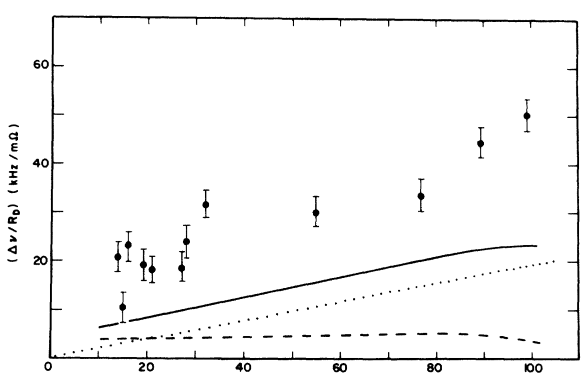

FIG. 2. The ratio of the linewidth to the dynamical resistance versus the dynamical resistance for a Sn-I-Sn tunnel junction at 2.93 K. The dotted, dashed, and solid curves represent, respectively, the calculated quasiparticle- and pair-current contributions to the linewidth, and their sum.

was proportional to the square of the temperature-dependent Josephson current amplitude for $T > 2.0$ K but was reduced for $T = 1.2$ K. We conclude that the rf fields in the junction lay in the range $0.1 < 2eV_{\rm rf}/\hbar\omega < 1$ for $T > 2.0$ K, and were larger for $T = 1.2$ K. We note that the theory of Ref. 1 was derived for the case of small fields ($2eV_{\rm rf}/\hbar\omega < 0.1$). The quantitative discrepancy between experiment and theory may therefore be a result of large rf fields and the presence of harmonic radiation in the junctions used in this experiment.

We find experimentally that the product of the pair current and the dynamical resistance is approximately constant on some self-induced steps. This allows us to separate approximately the quasiparticle and pair contributions to the linewidth by plotting the data in the form $\Delta\nu/R_D$ vs R_D. Such a plot is shown in Fig. 2 for the Sn-I-Sn junction at $T = 2.93$ K. The slope and intercept of the experimental curve measure the quasiparticle and pair current contributions to the linewidth, respectively. Except for the quantitative factor of approximately 2 the agreement is satisfactory.

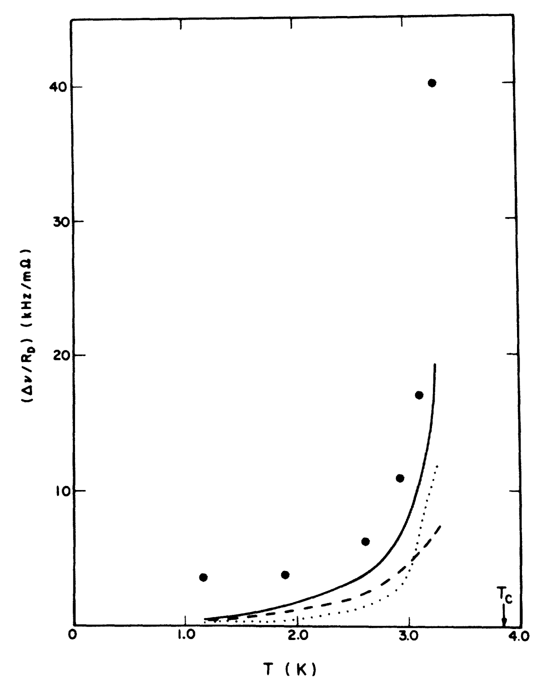

FIG. 3. Minimum value of the ratio of the linewidth to the dynamical resistance versus temperature for a 825-mΩ Sn-I-Sn tunnel junction. The dotted, dashed, and solid curves represent, respectively, the calculated quasiparticle- and pair-current contributions to the linewidth, and their sum.

The linewidth increased rapidly with temperature above $T/T_c \cong 0.75$ for the Sn-I-Sn junction. To illustrate the qualitative dependence of the linewidth on temperature, we have chosen somewhat arbitrarily to plot in Fig. 3 the minimum observed value of $\Delta\nu/R_D$ versus temperature for the Sn-I-Sn junction. For comparison the linewidth calculated from Eq. (12) is also plotted using the measured values of R_D, I_{qp}, and I_p. The increase in linewidth results primarily from the increase in quasiparticle current and the increase in the dynamic resistance.

The theoretical expression for the quasiparticle part of the current power spectrum [Eq. (5)] was obtained as follows: To second order in the tunneling matrix elements, $P_{Iqp}(\omega)$ can be written in the form

$$\mathrm{Re}\langle I(t)I(0)\rangle = \int_0^\infty P_{Iqp}(\omega)\cos\omega t d\omega = \mathrm{Re}[\exp(-ieV_0t/\hbar)\langle I_+(t)I_-(0)\rangle + \exp(ieV_0t/\hbar)\langle I_-(t)I_+(0)\rangle], \tag{13}$$

where

$$I_+(t) = I_-{}^\dagger(t) = ie\exp[i(H_1+H_2)t/\hbar]\sum_{kqs}T_{kq}C_{ks}{}^\dagger C_{qs}\exp[-i(H_1+H_2)t/\hbar], \tag{14}$$

and the angular brackets now denote thermal averages over H_1+H_2. H_1 and H_2 are the Hamiltonians describing the two metals which make up the junction, and T_{kq} is the tunneling matrix element which transfers an electron from a state q, s on side 1 to a state k, s on side 2. To this same order in the tunneling interaction, the quasiparticle dc current is given by the Kubo relation

$$I_{qp}(V_0) = (2\pi/e)[-\pi^{-1}\mathrm{Im}G_{-+}(eV_0/\hbar)] \equiv (2\pi/e)S_{-+}(eV_0/\hbar), \tag{15}$$

where $G_{-+}(eV_0/\hbar)$ is the Fourier transform of the current-current Green's function

$$G_{-+}(t) = -i\langle[I_-(t)I_+(0)]\rangle\theta(t). \tag{16}$$

The correlation functions which determine the power spectrum can be expressed directly in terms of the spectral weight $S_{-+}(\omega)$ defined by Eq. (15). For the first term of Eq. (13) one finds

$$\mathrm{Re}\exp(-ieV_0t/\hbar)\langle I_+(t)I_-(0)\rangle$$
$$= \int_0^\infty d\omega\cos\omega t\{n(\omega+eV_0/\hbar)S_{-+}(\omega+eV_0/\hbar) + [n(\omega-eV_0/\hbar)+1]S_{+-}(\omega-eV_0/\hbar)\}, \tag{17}$$

where $n(\omega)$ is $(e^{\beta\hbar\omega}-1)^{-1}$. Expressing the second term of Eq. (13) in a similar manner and extracting the power spectrum, one obtains

$$P_{Iqp}(\omega) = S_{-+}(\omega+eV_0/\hbar)[2n(\omega+eV_0/\hbar)+1] + S_{+-}(\omega-eV_0/\hbar)[2n(\omega-eV_0/\hbar)+1]. \tag{18}$$

This would just be another formal relationship except that the Kubo relation Eq. (15) shows that the spectral weight S_{-+} is directly determined by the observable quasiparticle I-V characteristic. Using the Kubo relation to replace S_{-+} by I_{qp}, we obtain Eq. (5) for the quasiparticle part of the current power spectrum.

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[9]In the zero-temperature limit $\Delta\omega$ approaches $(8e^3/\hbar^2)R_D^{\,2}I(V_0)$, which is a factor of 2 smaller than the result obtained in Ref. 1. In that paper, an additional contribution to the fluctuations arises from the reservoirs which were formally introduced to simulate the leads to the junction. We believe that the correct representation of the external coupling does not contain these fluctuations. M. J. Stephen concurs (private communication).
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SUPPRESSION OF THE KONDO MANY-BODY SCATTERING EFFECT

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An experimental study of the electrical resistivity in dilute Cu:Mn doped with Pt impurities is presented. The results indicate a suppression of the Kondo divergence in the conduction-electron scattering amplitude as a result of the spin-orbit interaction at the Pt impurities. The results are interpreted in terms of a reduction in the spin "memory" time.

In 1964, Kondo[1] demonstrated the existence of a logarithmic divergence in perturbation theory for the scattering of conduction electrons from magnetic impurities in metals. Since that time, there has been considerable research directed toward the goal of understanding the physical origin of the many-body scattering as well as toward a determination of the ground state of this coupled many-body system. Although we still do not have a satisfactory theory for temperatures less than the divergence temperature[2]

$$T_K = (D/k)\exp[-1/|J|\rho], \tag{1}$$

existing experimental and theoretical work[3] indicates that the ground state is a many-body singlet in which nonperturbative spin correlations are built up in the electron gas in the vicinity of the impurity. These spin correlations may be viewed as fundamentally arising from an indirect electron-electron interaction[4] via the magnetic impurity. A first electron scatters from the impurity leaving the spin flipped so that subsequent scattering of a second electron depends on the fact that the first event has already taken place. From this point of view, the question of the "memory" of the impurity spin in the intermediate state takes on special significance. If this memory is made sufficiently short either perhaps by intrinsic coupling of the impurity to the conduction electrons (i.e., spin fluctuation effects[5]) or by direct relaxation of the impurity spin to the lattice, one may expect a suppression of the indirect interaction and the associated logarithmic divergence with the result that the many-body spin correlations will not form. In this Letter we present electrical-resistivity data which yield experimental evidence of such suppression of the Kondo effect in the system Cu:Mn due to spin-orbit scattering from (heavy) Pt impurities.

The temperature dependence of the resistivity for Cu:Mn (110 ppm of Mn) doped with 0, 0.1, 0.3, 0.5, 1, 2, and 5 at.% Pt has been studied between 1.5 and 4.2°K. Figure 1 shows the results for the lowest concentrations. Platinum was chosen because of its relatively good solubility in Cu as well as its high Z and thus large value of