INELASTIC ELECTRON SCATTERING AT HIGH ENERGIES

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Field-theoretic model calculations for inelastic electron scattering show that (1) scaling laws must be weakly violated; (2) there is no evidence for $\sigma_T/\sigma_S=0$ or ∞; and (3) the angular spread in production amplitudes must increase as $|Q^2|$ increases and is roughly proportional to $|Q^2|$ for large $|Q^2|$, where Q is the four-momentum of the virtual photon. Some experimental tests are suggested.

We consider here the inelastic scattering of a high-energy electron,

$$e^-+p\rightarrow e^-+B, \tag{1}$$

where B is a hadron system with total mass $\sqrt{s}$. We shall assume one-photon exchange—i.e., we have in essence the scattering of a virtual photon of four-momentum Q by a proton. Let us denote, in the laboratory system, the initial and the final energies of the electron by E and E', and the scattering angle of the electron by θ. Then at high energies

$$Q^2\sim -2EE'(1-\cos\theta)\leq 0. \tag{2}$$

Furthermore,

$$s=m_p^{\,2}+2m_p\nu+Q^2, \tag{3}$$

where m_p is the mass of the proton and $\nu=E-E'$. Throughout this Letter, we shall assume s to be large (a few BeV^2 or more), while the ratio $-Q^2/2m_p\nu$ may take on any value less than 1. This is the deep inelastic region.

Let us begin with the simpleminded assumption that $|Q^2|$, when large as compared with the hadron masses, is the only important scale—i.e., the hadron masses can be set to zero. Under this assumption, the total cross section is a function of s and Q only. For dimensional reasons, the general form for the total hadronic cross sections of a virtual photon on a proton is

$$\sigma_i(s,Q^2)\sim a_i(s/|Q^2|)/|Q^2|, \tag{4}$$

where $i=T,S$ refers to transverse and longitudinal photons, respectively. Equation (4) leads directly to

$$W_1\sim F_1(s/|Q^2|), \tag{5}$$

and

$$W_2\sim \nu^{-1}F_2(s/|Q^2|), \tag{6}$$

where W_1 and W_2 are the usual structure functions for electroproduction.

Photoproduction corresponds to the special case $|Q^2|=0$. We learn from the experimental data of DESY[1] and the Stanford Linear Accelerator Center (SLAC)[2] that the total hadronic-photoproduction cross section is nearly a constant, very roughly 100 μb, at energies between 2 and 7.5 BeV. This property is not related to the massless nature of the photon since we know that πp and pp total cross sections are also nearly constant at high energies. Thus we expect, at high energies, the total hadronic cross sections of a virtual photon of fixed Q^2 on a proton to be nearly constant. This means that for large $s/|Q^2|$,

$$\sigma_i(s/|Q^2|)\sim a_i(\infty)/|Q^2|, \tag{7}$$

where $a_i(\infty)$ is a finite constant. From (7) we obtain

$$F_1(s/|Q^2|)\sim \text{const}\, s/|Q^2|. \tag{8}$$

and

$$F_2(s/|Q^2|)\sim \text{const}. \tag{9}$$

Equations (6) and (9) are in rough agreement with the preliminary SLAC data, as reported by Panofsky.[3]

It is the purpose of this Letter to study the simpleminded assumption about scales. Our argument above shows that, in any model constructed for inelastic electron scattering, the scaling laws like (4), (5), and (6) are always obtained as long as the model has no divergent quantities when the hadron masses are set to ze-

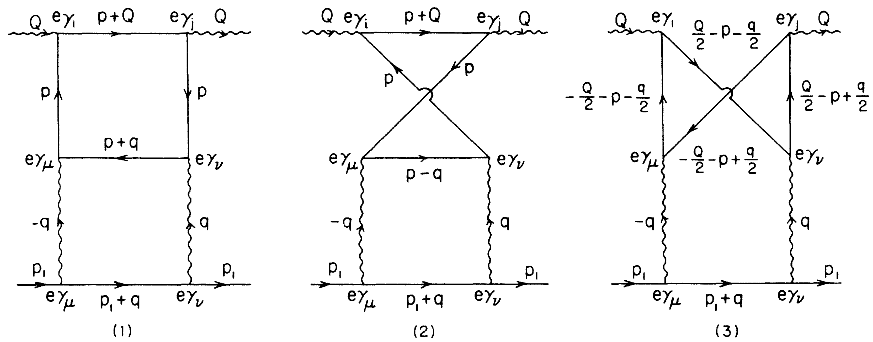

FIG. 1. Sixth-order diagrams for Compton scattering of a virtual photon.

ro. However, there is no reason to believe that such is in fact the case. In particular, there is no a priori basis to exclude logarithmic functions like $\ln(|Q^2|/\lambda^2)$ or $\ln(s/\lambda^2)$ from entering the expression for $\sigma_i(s, Q^2)$, where λ is some hadron mass. We have to turn to model calculations for clues.

As emphasized above, the near constancy[1,2] of the total hadronic-photoproduction cross section at high energies is crucial for obtaining (7), (8), and (9). Thus a relevant model calculation must yield this constancy in a natural manner. Recent studies[4] in quantum electrodynamics show that, to appropriate orders, Delbrück and Compton scattering processes do have this property. We therefore use the generalization of these considerations in quantum electrodynamics to virtual photons as a model for inelastic electron scattering. More precisely, we study the three Feynman-Dyson diagrams in Fig. 1 by the method previously given.[5] We find that

$$\sigma_T(s, Q^2) \sim \tfrac{4}{3}\alpha^3 |Q^2|^{-1}\{[\ln(|Q^2|/\lambda^2)]^2 + \tfrac{1}{3}\ln(|Q^2|/\lambda^2) + O(1)\}, \qquad (10)$$

and

$$\sigma_S(s, Q^2) \sim 8/3\alpha^3 |Q^2|^{-1}[\ln(|Q^2|/\lambda^2) - \tfrac{4}{3}], \qquad (11)$$

for $s \gg |Q^2| \gg m^2, \lambda^2$, where m is the fermion mass and λ is the mass of the exchanged vector mesons. In (10), one of the two logarithmic factors can be suppressed by introducing an electromagnetic form factor for the fermion, but the other logarithmic factor remains in general. Calculation of Compton scattering in the second order also shows that, in the limit $|Q^2| \to \infty$, with $|Q^2|/s$ held finite, logarithmic factors enter into the expressions for σ_T and σ_S. Thus our calculations show that no general principle exists to exclude the logarithmic factors, and the above simpleminded assumption about the scale cannot be correct.

We note that the fermion mass m does not appear in (10) and (11). It is therefore perhaps not unreasonable to take λ to be the ρ mass. If[3] $|Q^2| = 2.3$ (BeV/c)^2, $\ln(|Q^2|/\lambda^2)$ is only about 1.4. Thus we expect, at SLAC energies, to be able to see experimentally violations of (6) and (9) by measuring cross sections at larger $|Q^2|$.[6]

We conclude with a brief discussion of the angular distribution of the outgoing hadrons. To be specific, consider the electroproduction of ρ^0,

$$e^- + p \to e^- + p + \rho^0. \qquad (12)$$

When the mass of the p-ρ^0 system is large, ρ^0 is produced by the diffraction of the virtual photon. In the limit of photoproduction ($Q^2 = 0$), the angular distribution for ρ^0 is found[7] to be approximately e^{8t}, where t is the square of the momentum transfer. This angular distribution is strongly peaked in the forward direction. By the above arguments, the angular distribution of ρ^0 in the

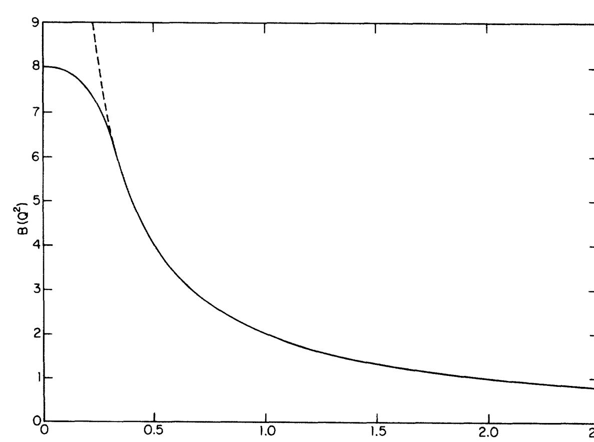

FIG. 2. Widening of angular distribution for the electroproduction of ρ^0. {The ρ^0 distribution is assumed to be of the form $\exp[B(Q^2)t]$.}

p-ρ^0 system must be less strongly peaked if $|Q^2|$ is larger. If we assume an exponential angular distribution of the form $\exp[B(Q^2)t]$, $B(Q^2)$ is a decreasing function of $|Q^2|$. A rough guess about $B(Q^2)$ is shown in Fig. 2. In general, the difficulty in making large transverse momentum transfers[8] at high energies is much less in electroproduction processes when the virtual photon is far off the mass shell. This point is verified in the model calculation, and experimental confirmation of this point will be of interest.

Details of the considerations in this Letter will be submitted to The Physical Review shortly.

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FINITE-ENERGY SUM RULES AND INFINITE SEQUENCE OF DAUGHTER TRAJECTORIES*

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It is shown that an infinite set of daughter trajectories is required to saturate finite-energy sum rules consistently. In the presence of this infinite set, the sum rules by themselves do not constrain the energy dependence of the trajectory function.

In a recent paper Mohapatra[1] has argued that, contrary to usual assumptions,[2,3] Regge trajectories are not linear functions of s, the square of the energy. The Veneziano representation,[4] on the other hand, has such an assumption built into it in a fundamental way.[5] Mohapatra shows that if finite-energy sum rules (FESR)[6] are to be consistent with analyticity at $t=0$ and saturation with a finite number of resonance trajectories, the trajectory function $\alpha(s)$ must be proportional to $(s\ln s)^{1/2}$. Kugler[7] has also presented arguments for a square-root trajectory; he interprets as resonances the loops in an Argand diagram obtained from a partial-wave analysis of a t-channel Regge pole.

In this note we show that the assumptions made by Mohaptra lead to inconsistencies unless there are an infinite number of daughter trajectories.[8] His results, as well as those of Kugler, can be interpreted as showing that if the sum over all daughter resonances is replaced by a finite set of effective leading trajectories, these trajectories must have the $(s\ln s)^{1/2}$ energy dependence. Since these effective trajectories do not satisfy the t dependence of the FESR, their usefulness is questionable.[9] In addition, we conclude that, in the presence of an infinite set of daughter trajectories, finite-energy sum rules by themselves are insufficient to determine the dependence of $\alpha(s)$ on s. Additional information such as provided by hypotheses like duality, linear trajectories, absence of cuts and intersecting trajectories, smoothness of appropriately parametrized residue functions, negligible background integral, absence of ghosts, or maximal simplicity of the resonance spectrum are needed to obtain a unique solution.

We start with Eq. (5) of Mohapatra[1] which we write in the form

$$I(N,t)=\int_0^N ds'\,\Gamma_{J(s')}P_{J(s')}(z)$$
$$=\beta(t)N^{\alpha(t)+1}+\beta'(t)N^{\alpha'(t)+1}+\cdots, \quad (1)$$

where the presence of secondary terms on the right of (1) is important. We have redefined Γ_J to include several s-dependent factors and used