backward production of charged bosons the signal-to-nonresonant background ratios are approximately the same.

It is a pleasure to acknowledge the generous cooperation and valuable assistance of the AGS staff in the setting up and running of this experiment. We also wish to acknowledge the important contributions made to this experiment by the staffs of the BNL On-Line Data Facility and Instrumentation Division (especially J. Fischer for design and construction of the wire chambers), and the Physics Design Groups at BNL and Carnegie-Mellon University. We are particularly grateful to A. Abrahamson, E. Bihn, R. Rothe, and J. Smith for their invaluable assistance throughout the experiment.

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[1]E. W. Anderson, E. J. Bleser, H. R. Blieden, G. B. Collins, D. Garelick, J. Menes, F. Turkot, D. Birnbaum, R. M. Edelstein, N. C. Hien, T. J. McMahon, J. Mucci, and J. Russ, Phys. Rev. Letters 20, 1529 (1968), and 21, 1030(E) (1968).

[2]Particle Data Group, University of California Radiation Laboratory Report No. UCRL 8030 (revised), 1968 (unpublished).

[3]C. C. Shih and B. L. Young (private communication) find that our A_1^- enhancement is not easily understood in terms of a simple backward multiperipheral calculation for the $pp\pi$ final state.

[4]We label the background coefficients $a(u)$ with the values of u calculated from the beam energy, the average laboratory angle of the data being fitted, and the ρ mass.

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[6]M. Deutschmann et al., Phys. Letters 19, 608 (1965). The square of the four-momentum transfer $t = (p_1-p_4)^2$ in this experiment.

[7]For $M_{B^-}^2 < 3.6$ GeV^2 the data plotted in Fig. 1(a) are replotted in Fig. 2.

[8]We obtain χ^2 probability of approximately 5% when the high-mass ($2.0 \le M^2 \le 7.1$ GeV^2) data shown in Fig. 2 are fitted with a smooth structureless curve.

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STUDY OF THE PRODUCTION OF LOW-MASS $K^*\pi$ SYSTEMS IN 12.7-GeV/c K^+p COLLISIONS*

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We have studied the production and decay characteristics of low-mass $K^{*0}(890)\pi^+$ systems produced in the reaction $K^+p \to K^+p\pi^+\pi^-$ at 12.7 GeV/c. Comparisons between these data and two diffractive production models are presented

We report on an investigation of the low-mass $K^+\pi^+\pi^-$ enhancement (Q bump) produced in the reaction[1]

$$K^+p \to K^+p\pi^+\pi^-. \tag{1}$$

This study is based on a 5-events/μb exposure of the Brookhaven National Laboratory 80-in. liquid-hydrogen bubble chamber to rf-separated 12.7-GeV/c K^+ mesons produced at the alternating-gradient synchroton.

The $K^+\pi^+\pi^-$ mass spectrum below 2 GeV/c in Reaction (1) consists predominately of $K^{*0}(890)\pi^+$ events.[2] In this note we compare the characteristics of this subset of Reaction (1) to the predictions of two diffractive models: (1) the particle exchange model of Ross and Yam (RY model),[3] which coherently sums the amplitudes for Feynman diagrams (A), (B), and (C) in Fig. 1; and (2) a double-Regge-exchange model[4] which considers just diagrams (B) and (C) (DR model).[5,6]

In the RY formulation we employed the usual parametrization of the three diagrams in Fig. 1.[7] No form factors were introduced at the dissociation vertices. The total cross sections σ (in mb) and the diffractive slopes β (in GeV^{-2}) which we used in our calculations are as follows: $\sigma_A = 18$ and $\beta_A = 7$ for diagram (A); $\sigma_B = 30$ and $\beta_B = 7$ for diagram (B); and, $\sigma_C = 18$ and $\beta_C = 5$ for diagram (C).

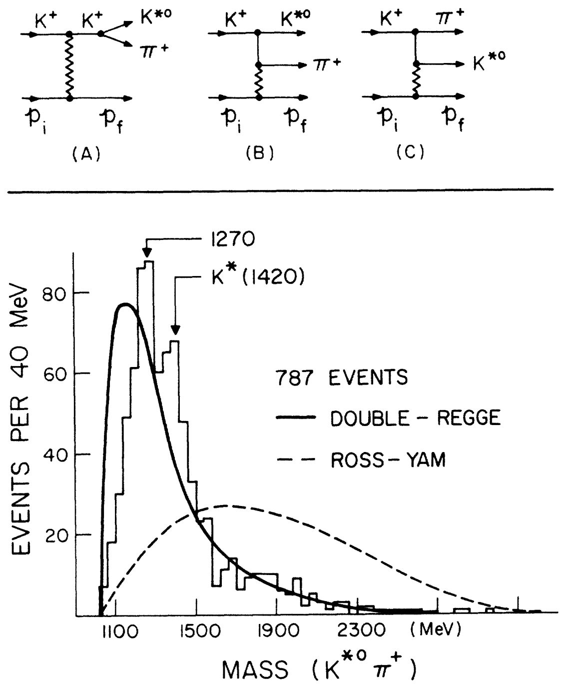

FIG. 1. The $K^*\pi$ mass distribution for the final state $K^{*0}\pi^+p$. Three Feynman diagrams (A), (B), and (C) are sketched at the top of the figure; the wavy lines represent diffraction scattering.

For the double-Regge-exchange calculation we employed the following amplitudes for diagrams (B) and (C), respectively,

$$M_B = N_B (S_{\pi p})^{\frac{1}{2}\beta_B t} P_\pi(\alpha_\pi)(S_{K^*\pi}/S_0)^{\alpha_\pi}, \qquad (2)$$

$$M_C = N_C (S_{K^*p})^{\frac{1}{2}\beta_C t} P_{K^*}(\alpha_{K^*}) \times (S_{K^*\pi}/S_0)^{\alpha_{K^*}}, \qquad (3)$$

where, using the particle symbol to represent its four-momentum, $t=(p_i-p_f)^2$, $S_{\pi p}=(\pi+p_f)^2$, $S_{K^*p}=(K^*+p_f)^2$, and $S_{K^*\pi}=(K^*+\pi)^2$. The parameters α_π and α_{K^*} are the Regge trajectories for π and K^*, respectively: We use $\alpha_\pi = t_\pi - m_\pi^2$ and $\alpha_{K^*} = t_{K^*} - m_{K^*}^2 + 1$, where $t_\pi = (K-K^*)^2$, $t_{K^*} = (K-\pi)^2$, and m_π and m_{K^*} are the π and K^* masses, respectively. $P_\pi(\alpha_\pi)$ and $P_{K^*}(\alpha_{K^*})$ are the Regge propagators for π and K^* exchange:

$$P(\alpha) = \pm \frac{1 \pm e^{-i\pi\alpha}}{\sin\pi\alpha}\,\frac{1}{\Gamma(\alpha+1)}, \qquad (4)$$

where the plus sign refers to α_π and the minus sign to α_{K^*}. The residue functions were assumed to be real constants and have been absorbed into the normalization factors N_B and N_C; the constant S_0 was taken to be 1.0 GeV^2.[8] The parameters β_B and β_C were set at the values specified in the previous description of the RY model.

The amplitudes M_B and M_C were added coherently and the ratio N_C/N_B was varied to obtain the best fit to our data.[9] The DR-model fit to be discussed below represents a calculation employing a mix of 40% of amplitude M_B to 60% of amplitude M_C.[10]

Because we employed exponential forms for the differential elastic-scattering cross sections in both the RY and DR models, we have imposed the following restrictions upon our data: $-t < 0.5$ GeV^2, $(S_{\pi p})^{1/2} > 2$ GeV, and $(S_{K^*p})^{1/2} > 2.5$ GeV. These same cuts were also used in calculating the model predictions.[11]

Four independent variables are sufficient to describe the three-body final state $pK^{*0}\pi^+$ at a particular beam momentum. We have chosen for our purposes the quantities $S_{K^*\pi}$, t, $\cos\theta$, and φ, where θ and φ are the polar and azimuthal (Jackson) angles of K^{*0} in the $K^{*0}\pi^+$ rest frame. In Fig. 1 we display the $K^{*0}\pi^+$ mass distribution $[M(K^*\pi)]$ along with the predictions of the RY and DR models normalized to the experimental data. Although the DR model yields a better fit to the data than does the RY model, there is nevertheless evidence for the presence of additional structure at 1270 MeV and at the $K^{*+}(1420)$ mass. Therefore, one can only conclude that neither model satisfactorily reproduces the $K^{*0}\pi^+$ mass spectrum.

The proton-to-proton momentum transfer spectra as a function of $K^{*0}\pi^+$ mass are presented in Fig. 2. The mass intervals employed (in GeV) are the following: Fig. 2(a), $M(K^*\pi) < 1.2$;

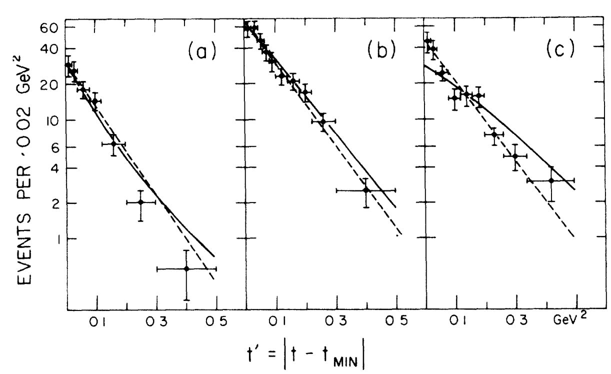

FIG. 2. The momentum-transfer spectra for three $K^*\pi$ mass regions: (a) less than 1.2 GeV, (b) between 1.2 and 1.4 GeV, (c) between 1.4 and 2.0 GeV.

Fig. 2(b), $1.2 < M(K^*\pi) < 1.4$; Fig. 2(c), $1.4 < M(K^*\pi) < 2.0$. Here the RY model fits the experimental data best although both models show the same general features: (1) a steeper slope for low $K^{*0}\pi^+$ masses than the input values, and (2) a decrease in this slope with increasing $K^{*0}\pi^+$ mass.[12]

Figure 3 displays the distribution of the polar and azimuthal angles in the $K^{*0}\pi^+$ rest frame for two mass intervals: $M(K^*\pi) < 1.3$ GeV in Figs. 3(a) and 3(b); and $1.3\text{ GeV} < M(K^*\pi) < 2.0$ GeV in Figs. 3(c) and 3(d). The superiority of the RY model fit is apparent; this is particularly interesting in light of the free parameter involved in our application of the DR model.[13]

We can summarize our results as follows:

(1) The RY model is in good agreement with our data except for the $K^{*0}\pi^+$ mass distribution. The introduction of absorptive corrections or form factors is required if this distribution is also to be fitted.

(2) Although the DR model provides a superior description of the average shape of the $K^{*0}\pi^+$ mass spectrum, it fails to describe the other aspects of the reaction very satisfactorily—particularly the K^{*0} decay angles in the Q rest frame. This agreement could presumably be enhanced through the introduction of nonconstant residue functions, but the model would then lose much of its simplicity.

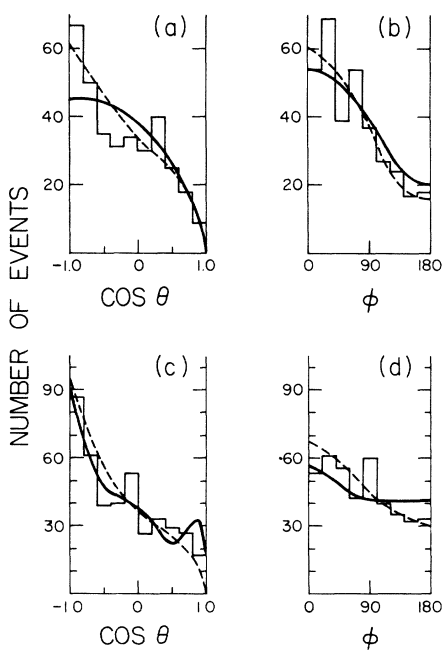

FIG. 3. The distributions in $\cos\theta$ and ϕ are shown for two regions of $K^*\pi$ mass: (a), (b) less than 1.3 GeV; (c), (d) between 1.3 and 2.0 GeV. Solid curves represent the predictions of the DR model and the dashed curves the predictions of the RY model (see text).

We conclude with the observation that agreement between diffractive models and the experimental characteristics of low-mass enhancements need not imply that a resonance interpretation for such low-mass enhancements is to be rejected; on the contrary, these two explanations may just be alternative descriptions of the same phenomenon.[14]

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[1]For references to previous studies of the Q region and to problems associated therewith, see G. Goldhaber, in Meson Spectroscopy, edited by C. Baltay and A. H. Rosenfeld (W. A. Benjamin, Inc., New York, 1968). See also B. French, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968).

[2]The $K^+\pi^+\pi^-$ events in the immediate vicinity of the Q mass are approximately 70% $K^{*0}\pi^+$ and 30% $K^+\rho^0$. In this paper the K^{*0} band is defined to span 800 to 1000 MeV and contains only a small amount of ρ^0 contamination.

[3]M. Ross and Y. Y. Yam, Phys. Rev. Letters 19, 546 (1967).

[4]F. Zachariasen and G. Zweig, Phys. Rev. 160, 1326 (1967); Chan Hong-Mo et al., Nuovo Cimento 49A, 157 (1967); N. F. Bali et al., Phys. Rev. Letters 19, 614 (1967), and Phys. Rev. 163, 1572 (1967). The multi-Regge model, as originally formulated by these authors, is applicable where all the four-momentum transfers are small and where all the invariant submasses are large (e.g., $s_{K^*\pi}$, $s_{\pi p}$, and s_{K^*p} in our case). It should therefore be most successful under such restrictions. Using the notion of duality, however, the multi-Regge model has been extended [see E. L. Berger, University of California Report No. UCRL-18472, 1968 (unpublished)] to cases where one of the invariant sub-

masses is small (e.g., $S_{K^*\pi}$ in our case) in the hope that it might then describe the average (rather than the detailed) behavior of the data. For a discussion of the validity of this approach, see R. Lipes et al., Phys. Rev. Letters 22, 433 (1969).

[5]The concept of duality as applied to the double-Regge-exchange model implies that diagrams (B) and (C) should contain an average description of contributions from type-(A) diagrams (private communication from R. Thews and G. Zweig). We have consequently ignored diagram (A) in our DR model calculation.

[6]The RY model and versions of the DR model which consider only diagram (B) have previously been compared with data from a wide variety of experiments. For details see J. C. Park et al., Phys. Rev. Letters 20, 171 (1967); F. Bomse et al., Phys. Rev. Letters 20, 1519 (1968); M. L. Ioffredo et al., Phys. Rev. Letters 21, 1212 (1968); E. L. Berger, Phys. Rev. Letters 21, 701 (1968); S. U. Chung, R. L. Eisner, N. Bali, and D. Luers, Phys. Rev. (to be published); S. U. Chung, V. E. Barnes, R. L. Eisner, D. Luers, and I. O. Skillicorn, Bull. Am. Phys. Soc. 14, 41 (1969); J. Andrews et al., Yale University Report No. 2726-540, 1969 (unpublished); J. G. Rushbrooke and J. R. Williams, Phys. Rev. Letters 22, 248 (1969); and C. Y. Chien et al., University of California Report No. UCLA 1031, 1969 (unpublished).

[7]We assumed that K^*p scattering [diagram (C)] was the same as K^+p scattering. For the diffractive scattering parts of the three diagrams we employed exponentials in the momentum-transfer variables. For diagram (A) we used the slope in t observed for K^+p elastic scattering at 13 GeV/c. For the other two diagrams we employed the average of the slopes in t observed for π^+p elastic scattering [diagram (B)] and K^+p elastic scattering [diagram (C)] in the energy range appropriate to our study. The results of the RY formulation are not very sensitive to small variations to these parameters. For further details, see H. Yuta, University of Rochester Report No. UR-875-271, 1969 (unpublished).

[8]We have investigated the effect of small variations in the parameter S_0 in the vicinity of 1.0 GeV^2 and found that the agreement between the DR model and our experimental data was not very sensitive to such variation. For simplicity, therefore, we have set $S_0 = 1.0$ GeV^2 in this Letter. We have also investigated other forms for the Regge amplitude and obtained results similar to those presented in this letter. For further details see, H. Yuta, University of Rochester Report No. UR-875-271, 1969 (unpublished).

[9]In fitting the DR model to our experimental distributions we have especially emphasized the decay-angle distributions since the fit to the t distribution is directly affected by the values of β_B and β_C, and these parameters were not varied in the fitting process.

[10]The relative amplitudes quoted in the text were calculated from the square root of the ratio of the phase-space integral of $|M_C|^2$ to the integral of $|M_B|^2$.

[11]We did not require any special restrictions on t_π or t_{K^*} because for the cut on t which we imposed upon our data, the sum of t_π and t_{K^*} is completely correlated with the mass of the $K^{*0}\pi^+$ system $(t_\pi + t_{K^*} = t - S_{K^*\pi} + m_{K^*}^2 + m_K^2 + m_\pi^2)$. Because of this correlation, both t_π and t_{K^*} tend to be simultaneously small in the Q region, and consequently merely restricting one of them to small values does not enhance the effect of either diagram (B) or (C) in a truly significant manner. Such a cut, however, greatly restricts one's ability to differentiate between the contributing diagrams, and we have therefore chosen to analyze the data sample described in the text because these events were most sensitive to the differences between diagrams (B) and (C). (An analysis of our data under the additional restriction of $t_\pi < 0.5$ GeV^2 yielded consistency with the conclusions reached in the text, but with the lesser sensitivity expected for such a kinematically restricted sample.)

[12]This mass dependence of the slope of the proton-to-proton momentum-transfer distribution has been previously reported. See, for example, J. Bartsch et al., Phys. Letters 27B, 336 (1968).

[13]Since neither model satisfactorily reproduces the $K^{*0}\pi^+$ mass distribution we have normalized each model separately to each experimental distribution in carrying out our comparisons. Except for this normalization, no other free parameters were employed in the RY model. For the DR model we not only examined the effect of varying the relative amounts of diagrams (B) and (C), but also investigated changing the sign of the interference term (the best fits were obtained using the positive sign); we also varied the magnitude of S_0 (see Ref. 8). Nevertheless, we were unable to obtain a satisfactory fit of the DR model to the data. (For example, in Fig. 2 a χ^2 of 68 for 34 degrees of freedom was obtained for the DR model in contrast to a χ^2 of 40 for the RY model).

[14]In this situation we expect to observe only an approximate agreement between the data at low mass and the DR model. See, for example, G. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968). We point out that if the concept of duality is to be regarded as valid for the reaction being presented in this note then it must include Pomeranchuk "scattering."