Our assertion is that

$$\partial^2(\log Z^*)/\partial J_{kl}\partial J_{rs} \ngtr 0 \tag{5}$$

in general, even when all $J_{kl} \geq 0$.

To prove this consider a lattice with only three spin locations, with $J_{12}, J_{23} \neq 0$, $J_{13} = 0$.

The operators $P_{12}$, $P_{23}$ are represented by a $2^3 = 8$ dimensional representation of $S_3$, and this reducible representation decomposes into four 1-dimensional and two 2-dimensional irreducible representations according to

$$8 = \{3\}^4 + \{2, 1\}^2. \tag{6}$$

For these representations we have

$$\{3\}: P_{12} = P_{23} = (1),$$

$$\{2, 1\}: P_{12} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P_{23} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, \tag{7}$$

and the eigenvalues of $J_{12}P_{12} + J_{23}P_{23}$ are therefore

$$\{3\}: J_{12} + J_{23},$$

$$\{2, 1\}: \pm(J_{12}^2 - J_{12}J_{23} + J_{23}^2)^{1/2}. \tag{8}$$

Hence we have

$$Z^* = 4[e^{2(J_{12} + J_{23})} + \cosh 2(J_{12}^2 + J_{23}^2 - J_{12}J_{23})^{\frac{1}{2}}], \tag{9}$$

and it follows that

$$\frac{\partial^2(\log Z^*)}{\partial J_{12}\partial J_{23}} = -4J_{12}J_{23} + \cdots < 0 \tag{10}$$

for $J_{12}$, $J_{23}$ sufficiently small and positive (Scrader[5]). Hence Griffiths' theorem II cannot be true when posed in its most general form, although it is not known whether additional conditions can be imposed to restore it. This result has the important consequence that a simple proof of the existence of a phase transition cannot be constructed for models with long-range (ferromagnetic) bonds in an arbitrary number of dimensions, once it is known that a phase transition exists in a sufficiently simple case. For the Heisenberg model, in contrast to the Ising model, even this fact is not known to be true, so that the type of argument used by Griffiths for the Ising model does not go through for two reasons.

It is clear that further investigation is necessary to understand the reason for the breakdown of this important theorem in such a simple situation, and to see whether there are in more realistic situations, when intuition seems to require it to be true, additional conditions to make it work.

In the more detailed account referred to earlier it will be shown that Griffiths' theorem I, which asserts that $\langle\vec{\sigma}_k \cdot \vec{\sigma}_l\rangle \geq 0$, is true, even when an external magnetic field is present.

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[5]The same example has been found by Dr. R. Scrader, private communication.

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# SUPPRESSION OF A "DRIFT-TYPE" INSTABILITY IN A MAGNETOPLASMA BY A FEEDBACK TECHNIQUE

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This paper presents the experimental details and the results obtained when using a feedback system to suppress drift instabilities present in a magnetoplasma.

One of the problems in fusion research in recent years has been the possible correlation between anomalous diffusion and the presence of finite-amplitude instabilities in the plasma. Most fusion experiments are designed so that the effect of such a correlation is minimized. The usual method of achieving this object is by using complex containing magnetic fields. Another way, however, has been proposed,[1] in which a feedback method is employed in order to stabilize the instability in the plasma system. The method essentially relies on being able to detect the presence of an instability and then arranging a feedback of the signal with the correct amplitude and phase such that an external constraint is applied to the plasma in order to damp out the instability. This technique has already been demonstrated fairly successfully on the flute instability[2] and has also been predicted for the drift instability.[3] In this paper we report some experimental results obtained from employing this method on a "driftlike" instability.[4]

The success of any feedback method depends on how well the feedback constraint has phase coherence with the detected instability. Therefore, care has to be taken that the time delay in the external feedback loop is not longer than the coherence time of the instability and also that the detector and the exciter are not farther apart than a spatial coherence length.

The experiments we performed fell into two categories: (i) those establishing that a feedback method does in fact work for driftlike instabilities and concerned with the characteristics of a given feedback system, i.e., the effect of loop gain and phase shift in the loop, and (ii) the effect of suppression and enhancement of the instability on the steady-state properties of the arc together with establishing the effect of one or more separate feedback systems.

The plasma used in our experiments was a hollow-cathode arc discharge running in argon.[5] Typical parameters of the plasma are an electron temperature of ~5 eV, a density $\sim 10^{13}$ cm$^{-3}$, and an inverse scale length $(1/n_0)dn_0/dr \approx 0.70 \pm 0.05$ cm$^{-1}$ for an axial magnetic field of 1 kG. Most of the feedback experiments were performed using an instability whose frequencies lay in the range 5.0-8.0 kHz. The basic properties of this instability were established by measuring the azimuthal variation of phase, the radial variation of amplitude, and the axial wavelength, etc. The instability was found to have an axial wavelength longer than the apparatus and an azimuthal mode number which was predominantly $m = +1$. The instability was identified as a radial electric-field–driven "drift-type" instability.[4]

One of the noise characteristics of the low-frequency instabilities ($\omega \ll \Omega_i$, the ion cyclotron frequency) is that the ions take part in the cooperative motion of the wave, and in order that these ion density fluctuations were detected, an ion-biased probe was used for detection of the instability. The feedback loop consisted of a wide-band amplifier with variable gain, a phase shifter capable of phase shifts of up to 450°, and a system of power amplifiers and similar phase shifters arranged so that power could be supplied to up to four plate electrodes situated at the edge of the plasma. The signals on these plates were phased such that separate azimuthal modes $m = 0, \pm1, \pm2$ could be fed back into the plasma. The best stabilization was achieved for each separate feedback loop when it was linked to only one driving plate. The effect of the feedback was observed by displaying on a spectrum analyzer the output from a separate axially moving ion-biased probe. Figure 1 shows the effect of the feedback when suppressing or enhancing the instability. Figure 1(a) shows the instability without feedback; Fig. 1(b) illustrates the effect of the feedback for optimum suppression of the instability; Fig. 1(c) shows the enhancement achieved when inserting a phase shift of 180° in the feedback loop after it had been set for the optimum suppression.

A detailed study was made of the effect of gain and phase shift in the feedback loop on the observed amplitude of the instability. This is illustrated in Fig. 2. Figure 2(a) shows the output

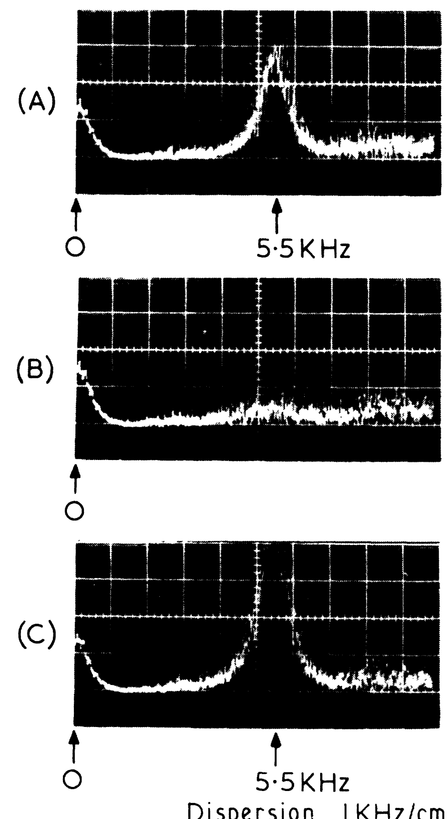


FIG. 1. Spectrum-analyzer output showing the instability at 5.5 kHz for (a) no feedback, (b) optimum feedback for suppression, and (c) optimum feedback for enhancement.

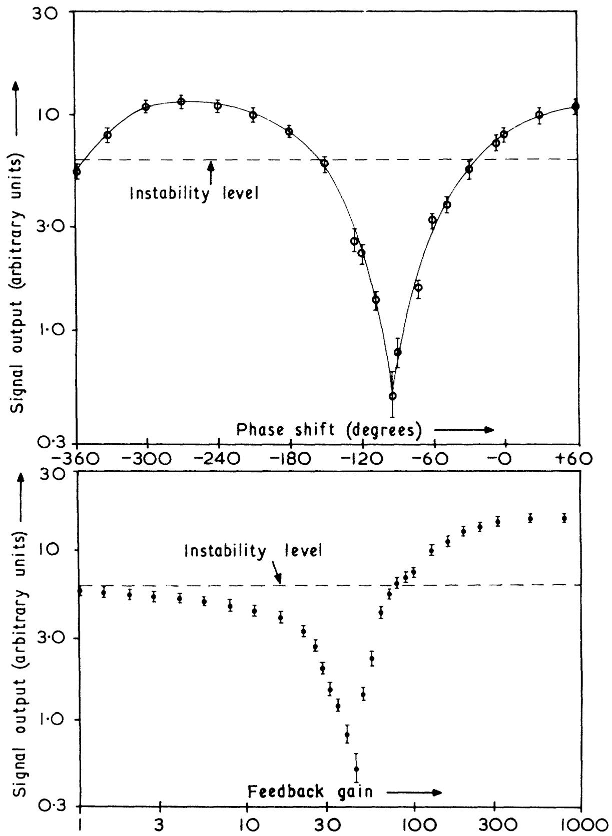


FIG. 2. The variation of the instability amplitude with both gain and phase in the feedback loop.

proportional to the ion density fluctuations as a function of the phase change in the feedback loop set at the optimum gain level (i.e., that gain at which it was possible to get maximum suppression). It is seen that there is a minimum at a relative phase shift of $-90°$, while there is a maximum and a net gain in the instability level at a phase shift of $+90°$. Figure 2(b) shows the density fluctuations as a function of gain when the phase is set for a minimum level (i.e., $-90°$). It is seen that at this phase setting, below a certain loop gain the feedback has a stabilizing effect while above it the system becomes progressively more unstable. At the setting of gain and phase in the feedback loop for a minimum level we checked, by moving the detecting ion-biased probe both radially and axially, that the signal was always at the same level. In some cases a single feedback loop was found to be incapable of completely suppressing the instability. In these cases suppression was found to be improved by increasing the number of separate feedback loops.

Once the conditions for the suppression of the instability had been found, the effect on the density and the temperature profile was studied, when the instability was present and when it was "switched off." These profiles were measured using a radially moving double probe.[6] These measurements are shown in Figs. 3(a) and 3(b)

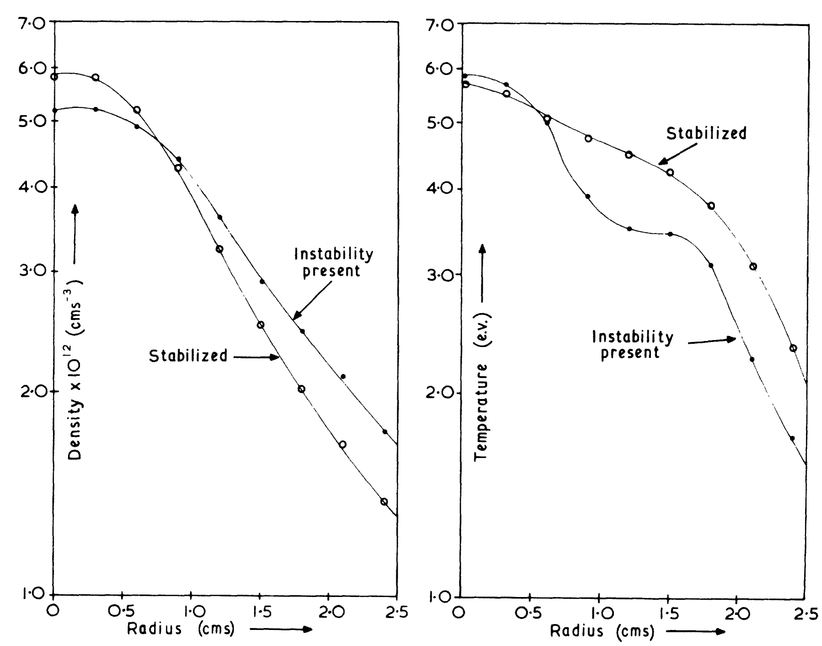


FIG. 3. The effect of the removal of the instability on the radial profiles of density and temperature.

for a single feedback loop system.

The theory has been worked out for a generalized feedback network concerned with an applied electric field from external plates.[1] However, this theory cannot be readily used in this case since the exciter plates are in good electrical contact with the plasma and so direct density perturbations as well as electric field perturbations must be included in the theory in order to describe these results. The main effects of the removal of the instability were that the density profile was "sharpened up" and that an inhomogeneity in the temperature profile was removed. The radial position of this temperature inhomogeneity corresponded to the maximum in the amplitude of the instability. When the instability was stabilized, it was seen that the density was higher at the center and fell off faster in the "wings" rather than when the instability was present. This is caused by the fact that the current in the arc column is stabilized at a constant value. The effect of this constraint is to make the total current density $J=e\int 2\pi n(r)V_{\parallel}(r)rdr = \text{const}$, where $e$ is the electronic charge, $n(r)$ is the density, and $V_{\parallel}(r)$ the parallel electron velocity at a particular radius $r$ of the column. Assuming $V_{\parallel}(r)$ falls off in a monotonic manner, any decrease in the cross field diffusion constant $D_{\perp}$ would manifest itself by increasing the density at the center and decreasing it in the wings. This is observed experimentally when the instability is suppressed. Therefore it is inferred that suppression of this instability reduces the cross-field diffusion rate, but in the absence of a suitable theory to describe the dynamics of the arc column it is not possible to ascribe a value to the change in the diffusion constant.


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## QUASIELASTIC RAYLEIGH SCATTERING IN NEMATIC LIQUID CRYSTALS*


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Using a light-beat laser spectrometer, we observe the relaxation of the thermally excited fluctuations of anisotropy in the nematic phase of $p$-azoxyanisole. We identify the two low-frequency purely dissipative modes recently predicted. A first study of the angular dependence of the width of these modes allows a determination of three among the six viscosity coefficients introduced by Leslie.


In 1948 Chatelain[1] made the first investigation on the intensity of the light scattered by a nematic monocrystal; his most interesting result was the observation of a very strong depolarization in small-angle scattering. De Gennes[2] has recently explained these results in terms of low-amplitude thermally excited fluctuations of the nematic direction of molecular alignment (i.e., the "director"). A computation of the dynamical behavior of these fluctuations shows that they are dominated, for the light-scattering properties, by two purely dissipative low-frequency modes.[3] In this Letter we report on the first observation of these modes, by spectral analysis of the quasi-elastic scattered light from a nematic liquid monocrystal.

Let us recall the main results of the theoretical study[3]: $\vec{n}_0$ is the average nematic director, and $\delta\vec{n}(\vec{q})$ its fluctuations as a function of wave vector $\vec{q}$. $\delta\vec{n}$ can be decomposed into two uncoupled components: (a) $\delta\vec{n}_1$ in the $(\vec{q},\vec{n}_0)$ plane, a combination of "bending" and "splay" deformations[4]; and (b) $\delta\vec{n}_2$ perpendicular to the $(\vec{q},\vec{n}_0)$ plane, a combination of "bending" and "twist" deformations.[4] For the usual nematic materials, these modes are purely relaxational with Lorentzian spectral densities of half-width $u_{S1}$ and $u_{S2}$ (angular-frequency units). Assuming that pi-

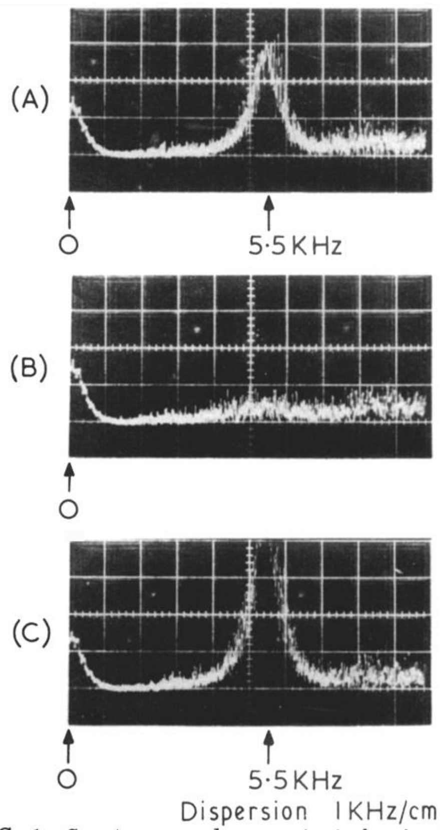


**FIG. 1.** Spectrum-analyzer output showing the instability at 5.5 kHz for (a) no feedback, (b) optimum feedback for suppression, and (c) optimum feedback for enhancement.