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GRIFFITHS' THEOREMS FOR THE FERROMAGNETIC HEISENBERG MODEL

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It is shown that for the quantum mechanical Heisenberg ferromagnet, the correlation is not necessarily a monotone increasing function of interactions.

Griffiths[1-3] has shown that for Ising ferromagnets of arbitrary spin, the correlation is nonnegative and a monotone increasing function of the interactions. It is natural to enquire whether the corresponding assertion is correct for the Heisenberg ferromagnet and its various approximations.

In another paper these questions will be discussed in detail, but here it will be shown that Griffiths' theorem II, which asserts that the correlations are monotonic functions of the bond energies, is not true in general for the quantum mechanical Heisenberg ferromagnet. The proof consists in producing a counter example, which is obtained by considering the simplest nontrivial spin configuration. For this counter example, there exist domains of the bond energies for which the correlations decrease as the positive bond energies increase.

The partition function in the absence of an external field is

$$Z=\mathrm{Tr}(\exp\sum_{i<j}J_{ij}\vec{\sigma}_i\cdot\vec{\sigma}_j), \tag{1}$$

where $\vec{\sigma}_i$, $i=1,\cdots,n$, are a collection of Pauli spin matrices satisfying the usual commutation and anticommutation relations:

$$[\sigma_{ip},\sigma_{jq}]_-=2i\delta_{ij}\epsilon_{pqr}\sigma_{jr},$$
$$[\sigma_{ip},\sigma_{iq}]_+=2\delta_{pq}. \tag{2}$$

The correlation functions $\langle\vec{\sigma}_k\cdot\vec{\sigma}_l\rangle$ are defined as usual by

$$\langle\vec{\sigma}_k\cdot\vec{\sigma}_l\rangle=Z^{-1}\,\mathrm{Tr}(\vec{\sigma}_k\cdot\vec{\sigma}_l\exp\sum_{i<j}J_{ij}\vec{\sigma}_i\cdot\vec{\sigma}_j)$$
$$=\partial(\log Z)/\partial J_{kl}. \tag{3}$$

If we define

$$P_{kl}=\tfrac{1}{2}(1+\vec{\sigma}_k\cdot\vec{\sigma}_l), \tag{4}$$

it is well known (Dirac[4]) that the P_{kl} have the same algebraic properties as the set of transpositions of S_n, the symmetric group on n objects. Hence a representation of $\vec{\sigma}_i$ is a representation of S_n, because every element of the latter can be generated from the transpositions alone, and with the interpretation of $\vec{\sigma}_i$ as spin operators this representation is 2^n dimensional. If we replace in (3) a term $\vec{\sigma}_i\cdot\vec{\sigma}_j$ in the exponent by $2P_{ij}$ this introduces a multiplicative factor into Z, and so does not alter the value of $\langle\vec{\sigma}_k\cdot\vec{\sigma}_l\rangle$. We denote the altered expression for Z by Z^*.

Our assertion is that

$$\partial^2(\log Z^*)/\partial J_{kl}\partial J_{rs} \ngtr 0 \quad (5)$$

in general, even when all $J_{kl} \geq 0$.

To prove this consider a lattice with only three spin locations, with $J_{12}, J_{23} \neq 0$, $J_{13} = 0$.

The operators P_{12}, P_{23} are represented by a $2^3 = 8$ dimensional representation of S_3, and this reducible representation decomposes into four 1-dimensional and two 2-dimensional irreducible representations according to

$$8 = \{3\}^4 + \{2, 1\}^2. \quad (6)$$

For these representations we have

$$\{3\}: P_{12} = P_{23} = (1),$$

$$\{2, 1\}: P_{12} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P_{23} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, \quad (7)$$

and the eigenvalues of $J_{12}P_{12} + J_{23}P_{23}$ are therefore

$$\{3\}: J_{12} + J_{23},$$

$$\{2, 1\}: \pm(J_{12}^2 - J_{12}J_{23} + J_{23}^2)^{1/2}. \quad (8)$$

Hence we have

$$Z^* = 4[e^{2(J_{12} + J_{23})} + \cosh 2(J_{12}^2 + J_{23}^2 - J_{12}J_{23})^{\frac{1}{2}}], \quad (9)$$

and it follows that

$$\frac{\partial^2(\log Z^*)}{\partial J_{12}\partial J_{23}} = -4J_{12}J_{23} + \cdots < 0 \quad (10)$$

for J_{12}, J_{23} sufficiently small and positive (Scrader[5]). Hence Griffiths' theorem II cannot be true when posed in its most general form, although it is not known whether additional conditions can be imposed to restore it. This result has the important consequence that a simple proof of the existence of a phase transition cannot be constructed for models with long-range (ferromagnetic) bonds in an arbitrary number of dimensions, once it is known that a phase transition exists in a sufficiently simple case. For the Heisenberg model, in contrast to the Ising model, even this fact is not known to be true, so that the type of argument used by Griffiths for the Ising model does not go through for two reasons.

It is clear that further investigation is necessary to understand the reason for the breakdown of this important theorem in such a simple situation, and to see whether there are in more realistic situations, when intuition seems to require it to be true, additional conditions to make it work.

In the more detailed account referred to earlier it will be shown that Griffiths' theorem I, which asserts that $\langle\vec{\sigma}_k \cdot \vec{\sigma}_l\rangle \geq 0$, is true, even when an external magnetic field is present.

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SUPPRESSION OF A "DRIFT-TYPE" INSTABILITY IN A MAGNETOPLASMA BY A FEEDBACK TECHNIQUE

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This paper presents the experimental details and the results obtained when using a feedback system to suppress drift instabilities present in a magnetoplasma.

One of the problems in fusion research in recent years has been the possible correlation between anomalous diffusion and the presence of finite-amplitude instabilities in the plasma. Most fusion experiments are designed so that the effect of such a correlation is minimized. The usual method of achieving this object is by using complex containing magnetic fields. Another way, however, has been proposed,[1] in which a feedback method is employed in order to stabilize the instability in the plasma system. The method essentially relies on being able to detect