the physical meaning of our procedure is clear and above all, the remarkable simplicity makes it valuable.

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[1]S. J. Chang and S. Ma, Phys. Rev. 180, 1506 (1969). See also the original work of S. Weinberg, Phys. Rev. 150, 1313 (1966). The usefulness of variables $p_0 \pm p_3$ has been known to many people for a long time in various applications.

[2]R. J. Glauber, in Lectures in Theoretical Physics, edited by Wesley E. Brittin et al. (Interscience Publishers, Inc., New York, 1959), Vol. 1, p. 315.

[3]The factor $I(\vec{k}_1, \vec{k}_2)$ is related to the $\mathcal{I}_\gamma(\vec{k}_1, \vec{k}_2)$ defined by H. Cheng and T. T. Wu [Phys. Rev. Letters 22, 666 (1969); Phys. Rev., to be published (papers I-IV)], through $\mathcal{I}_\gamma(k_1, k_2) = \frac{1}{2}[I(\vec{k}_1 + \vec{k}_2, 0) - I(\vec{k}_1, \vec{k}_2)]$. We are grateful to Professor S. Adler for showing us the preprints of the Phys. Rev. papers.

[4]Cheng and Wu, Ref. 3.

[5]In Paper IV of Ref. 3 there is a relatively short derivation of their lowest order photon impact factor using the variables $p_0 \pm p_3$. This derivation may be viewed as a special case of our analysis.

THEOREM ON THE FORM FACTORS IN K_{l3} DECAY

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We derive a formula for the matrix element $\langle K|\partial_\mu \mathfrak{F}_a{}^\mu|\pi\rangle$ correct up to second order in both the momentum transfer t and the breaking of SU(3)⊗SU(3) symmetry. This leads to a prediction that ξ is very small compared with unity. Our result is independent of any assumption concerning the structure of the symmetry-breaking Hamiltonian.

It has become increasingly apparent that it is interesting to discuss the strong interactions in terms of an approximate SU(3)⊗SU(3) symmetry.[1,2] That is, it is meaningful to write the strong-interaction Hamiltonian as $H = H_0 + \epsilon H'$, where H_0 is SU(3)⊗SU(3) invariant, H' simultaneously breaks SU(3) and SU(3)⊗SU(3), and ϵ is small enough that a perturbation expansion makes sense. The symmetric limit ($\epsilon = 0$) is to be understood as one in which the octet of pseudoscalar mesons is massless, allowing the axial-vector currents to be conserved without requiring the presence of SU(3)⊗SU(3) multiplets of particles. The practical advantage to adopting this picture of the strong interactions is that an expansion in powers of ϵ provides a systematic way of keeping track of corrections to SU(3) and partial conservation of axial-vector current, since $\epsilon H'$ is responsible for both the mass splittings among SU(3) multiplets and the small "extrapolation errors" encountered in any application of partial conservation of axial-vector current.

In this Letter we show how applying the above ideas allows one to calculate some symmetry-breaking terms in form factors for processes such as K_{l3} decay. Specifically, we prove the following theorem:

Theorem. – Let $\mathfrak{F}_b{}^\mu$, $b = 1, \cdots, 8$, denote one of the vector currents and $|M_a(p)\rangle$, $a = 1, \cdots, 8$, a covariantly normalized pseudoscalar-meson state with momentum p^μ. Further, let us expand the matrix elements of $\partial_\mu \mathfrak{F}_b{}^\mu$ as follows:

$$\langle M_a(p)|\partial_\mu \mathfrak{F}_b{}^\mu(0)|M_c(p')\rangle \equiv a_0 + a_1 t + a_2 t^2 + \cdots, \tag{1}$$

where $t = (p-p')^2$. Then, it can be shown that

$$a_0 = -(m_a{}^2 - m_c{}^2) f_{abc} + O(\epsilon^3),$$

$$a_1 = -\frac{1}{2}\left[\frac{f_c}{f_a} - \frac{f_a}{f_c}\right] f_{abc} + O(\epsilon^2), \tag{2}$$

where f_{abc} are the structure constants of SU(3), m_a is the mass of the ath pseudoscalar meson, and f_a is defined by the matrix element of the axial current between a pseudoscalar-meson state and the vacuum,

$$\langle M_a(p)|\mathfrak{F}_a{}^{\mu 5}(0)|0\rangle \equiv -ip^\mu/2f_a. \tag{3}$$

Before discussing the proof of this theorem, we shall devote the next few paragraphs to a discussion of its theoretical and experimental implications.

Adopting conventional notation, the matrix element for $K \to \pi$ + leptons is expressed in terms of two form factors $F_+(t)$ and $F_-(t)$. In practice, this matrix element is usually parametrized in terms of two constants,

$$\xi \equiv F_-(0)/F_+(0) \text{ and } \lambda_+ \equiv m_\pi^{\,2}[d\ln F_+(t)/dt]_{t=0}.$$

The theorem quoted above provides one relation between ξ and λ_+ which is accurate to $O(\epsilon^2)$. In a straightforward manner one finds it to be

$$\xi \equiv \frac{1}{2}\left[\frac{f_\pi}{f_K} - \frac{f_K}{f_\pi}\right] - \frac{m_K^{\,2} - m_\pi^{\,2}}{m_\pi^{\,2}}\lambda_+ + O(\epsilon^2). \tag{4}$$

If we take, as seems to be indicated by the energy dependence of K_{l3} decay, $\lambda_+ \approx 0.02$ and $f_\pi \approx 1.3 f_K$ (as determined by the ratio of $K_{\mu 2}$ to $\pi_{\mu 2}$ rates) we find that the two terms on the right-hand side of Eq. (4) nearly cancel, giving a (poorly determined) value for ξ of $\xi \approx 0.1$.

Experimentally[3] such a small positive value of ξ is compatible with the observed $K_{e3}/K_{\mu 3}$ branching ratio, but it is in serious disagreement with recent measurements of muon polarization in $K_{\mu 3}$ decay which tend to give values for ξ around -1. The discrepancy between these two different types of experiments may be due to an unexpectedly large t dependence in F_+ or F_-, or to a breakdown of μ-e universality (which would require a complete change in our picture of the weak interactions), or may cease to exist as more experiments are conducted. In any case it is crucial to know whether the value of ξ is small and positive or is around -1. Clearly if the value of ξ is about -1, in disagreement with Eq. (4), then we will have to reconsider either (i) the whole idea of SU(3) $\otimes$ SU(3) symmetry or (ii) the Gell-Mann–Cabibbo theory of semileptonic decays, which relates the weak current to the (almost) conserved SU(3) $\otimes$ SU(3) currents.

There is an additional point worth making with regard to the experimental situation, and it is that Lee[4] has calculated the form factors F_+ and F_- using a phenomenological Lagrangian possessing a broken SU(3) $\otimes$ SU(3) symmetry. His form factors (necessarily) satisfy our theorem, and if one accepts his particular interpretation of the "vector-dominance" hypothesis, one predicts a rapidly varying F_-; hence, one obtains a possible explanation of the discrepancy between the different measurements of ξ.

Another point worth making is that we can further divide the symmetry-breaking Hamiltonian $\epsilon H'$ into $\epsilon_1 H'' + \epsilon_2 H'''$, where H'' breaks SU(3) $\otimes$ SU(3) and SU(3) but not SU(2) $\otimes$ SU(2) which is broken by H'''. This is an interesting thing to do since the extreme smallness of the pion mass leads one to suspect that $\epsilon_2 \ll \epsilon_1$. Neglecting terms of order ϵ_2 entirely one has the usual soft-pion theorem of Callan and Treiman and Mathur, Okubo, and Pandit[5] which can be written as

$$F_+(m_K^{\,2}) + F_-(m_K^{\,2}) = f_\pi/f_K + O(\epsilon_2) \tag{5}$$

to remind us of the order of the terms being neglected. If one now considers Eq. (2), we see that it gives us a relation correct to second order in the total symmetry-breaking Hamiltonian $\epsilon_1 H'' + \epsilon_2 H'''$. We can combine these two results to obtain a result correct up to terms of order $\epsilon_2^{\,2}$ and $\epsilon_1\epsilon_2$. Assuming that ϵ_2 is really small compared with ϵ_1, this should be better than either Eq. (2) or (5).

To do this, one simply changes the argument of the form factors in Eq. (5) from $t = m_K^{\,2}$ to $t = m_K^{\,2} - m_\pi^{\,2}$. As we shall show, one then has

$$F_+(m_K^{\,2} - m_\pi^{\,2}) + F_-(m_K^{\,2} - m_\pi^{\,2}) \equiv f_\pi/f_K + O(\epsilon_1\epsilon_2) + O(\epsilon_2^{\,2}). \tag{6}$$

In order to show that Eq. (6) holds, one first expands in powers of ϵ_1 and ϵ_2 using the fact that $F_+(0) \equiv 1 + O(\epsilon_1^{\,2}) + O(\epsilon_2^{\,2}) + O(\epsilon_1\epsilon_2)$. A comparison with Eq. (2) then shows that there are no errors of order ϵ_1 or ϵ_2 in Eq. (6). On the other hand, Eqs. (5) and (6) differ only by terms of order ϵ_2 [remember $m_\pi^{\,2} = O(\epsilon_2)$] and, therefore, Eq. (6) cannot contain terms of order $\epsilon_1^{\,2}$ which gives the result that it is accurate to $\epsilon_1\epsilon_2$ and $\epsilon_2^{\,2}$. Unfortunately, the point $t = m_K^{\,2} - m_\pi^{\,2}$ is outside the physical decay region; so in order to test Eq. (6) one will have to use some model to extrapolate the form factors.

Let us finally turn to the proof of our theorem. Since the result for a_0 in Eq. (2) is just the Ademollo-Gatto theorem,[6] we need only concern ourselves with the calculation of a_1 up to terms of order ϵ^2.

The essential point in this calculation is that the pseudoscalar mesons dominate the divergences $\partial_\mu \mathcal{F}_a{}^{\mu 5}$ of axial-vector currents except for symmetry-breaking terms of order ϵ. As a result one can write

$$\int d^4x d^4y\, e^{+ip\cdot x} e^{-ip'\cdot y} \langle 0|T(\partial_\lambda \mathcal{F}_a{}^{5\lambda}(x)\partial_\sigma \mathcal{F}_c{}^{5\sigma}(y)\partial_\mu \mathcal{F}_b{}^\mu(0))|0\rangle$$
$$\equiv -\frac{1}{4f_a f_c}\frac{m_a{}^2}{(p^2-m_a{}^2)}\frac{m_b{}^2}{(p'^2-m_c{}^2)}\langle M_a(p)|\partial_\mu \mathcal{F}_b{}^\mu(0)|M_c(p')\rangle + O(\epsilon^2), \tag{7}$$

where the right-hand side is the on-mass-shell matrix element considered as a function of $t=(p-p')^2$. The reason that the correction terms to this expression are of order ϵ^2 is that the divergence of the vector current $\partial_\mu \mathcal{F}_b{}^\mu$ is itself of order ϵ, and this multiplies the terms of order ϵ coming from the corrections to pole dominance.

Having established Eq. (7) one simply follows the standard procedure of integrating by parts, replacing the derivatives ∂_λ, ∂_σ, and ∂_μ by explicit factors of momentum. This procedure leads, as is usual, to a long string of equal-time commutators. Fortunately, not all of the resulting terms are relevant to our result, since we are only interested in the coefficient a_1 as defined in Eq. (1). Effectively, this means that we may ignore any terms in the expansion of the right-hand side of Eq. (7) which are independent of t or of order t^2, etc. Also, since we are free to take $p^2=p'^2=0$ in Eq. (7) we can ignore terms which vanish at this point.

If one uses these facts and works through a considerable amount of algebra one finds that the relevant part of the time-ordered product appearing in Eq. (7) contains only the terms

$$-if_{abc}p_\lambda p_\sigma' M_c{}^{\lambda\sigma}(-p'), \quad -if_{cba}p_\lambda p_\sigma' M_a{}^{\lambda\sigma}(p), \quad +iN_{acb}(p-p'), \tag{8}$$

where we have defined

$$\delta_{ab}M_a{}^{\lambda\sigma}(q) \equiv \int d^4x\, e^{+iq\cdot x}\langle 0|T(\mathcal{F}_a{}^{5\lambda}(x)\mathcal{F}_b{}^{5\sigma}(0))|0\rangle \tag{9}$$

and

$$N_{acb}(q) \equiv \int d^4x\, e^{iq\cdot x}\langle 0|T(\Sigma_{ac}(x)\partial_\mu \mathcal{F}_b{}^\mu(0))|0\rangle \tag{10}$$

with

$$2i\Sigma_{ac}(x) \equiv \delta(x_0)[\partial_\mu \mathcal{F}_a{}^{5\mu}(x), \mathcal{F}_b{}^{50}(0)] + a \to b. \tag{11}$$

In obtaining Eq. (8) we have assumed that all time-ordered products are redefined so as to be covariant in order that we may ignore Schwinger terms. To complete the proof, we need only note that since $\partial_\mu \mathcal{F}_a{}^\mu$ and Σ_{ab} are both of order ϵ, the term N_{acb} is of order ϵ^2 and thus can be ignored. We may now determine the coefficient a_1 in Eq. (1) if we know the constant C_a in the term $M_a{}^{uv}(0) \equiv C_a g^{uv}$. Evaluating C_a is easily enough accomplished by writing

$$q_\mu q_\nu M_a{}^{\mu\nu}(q) = C_a q^2 + O(q^4) = \int d^4x\, e^{+iq\cdot x}\langle 0|T(\partial_\mu \mathcal{F}_a{}^{5\mu}(x)\partial_\nu \mathcal{F}_a{}^{5\nu}(0))|0\rangle - i\langle 0|\Sigma_{aa}(0)|0\rangle. \tag{12}$$

Noting that to order ϵ^2 the T product is dominated by its meson pole we get

$$C_a q^2 + O(q^4) = \frac{+im_a{}^4}{(q^2-m_a{}^2)(2f_a)^2} - i\langle 0|\Sigma_{aa}(0)|0\rangle + O(\epsilon^2). \tag{13}$$

Finally equating coefficients of q^2 in Eq. (13)[7] yields $C_a = -i(2f_a)^{-2} + O(\epsilon^2)$ which, together with Eqs. (7) and (8) gives the value of a_1 quoted in Eq. (2).

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[1]An extensive list of references on chiral symmetry may be found in S. Weinberg, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 253.

[2]Our particular approach to SU(3)⊗SU(3) symmetry is explaned in R. Dashen, to be published, and R. Dashen and M. Weinstein, to be published.

[3]For a review of the experimental situation, see J. Cronin, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 281.

[4]B. W. Lee, Phys. Rev. Letters 20, 617 (1968). References to other current-algebra–type calculations of ξ are given in Ref. 1. These calculations satisfy our theorem, if one assumes [in order to guarantee the existence of the usual SU(3) multiplets of particles] that the vacuum is SU(3) symmetrical in the limit $\epsilon \rightarrow 0$. In particular this means that for the hypothetical 0^+ κ meson, f_κ is assumed to be of order ϵ and the magnitude of m_κ remains finite as $\epsilon \rightarrow 0$. It should be noted, however, that a small-mass κ meson might make second-order effects large and limit the applicability of our theorem.

[5]C. Callan and S. Treiman, Phys. Rev. Letters 16, 153 (1966); V. Mathur, S. Okubo, and L. Pandit, Phys. Rev. Letters 16, 371 (1966).

[6]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[7]The cancellation of the constant terms on the right-hand side of Eq. (13) follows from the first-order mass formula derived in Ref. 2.

SUPPRESSION OF $A_2 \rightarrow K\bar{K}$ IN THE VENEZIANO MODEL

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Relations between coupling constants derived from the Veneziano amplitude for $\pi K \rightarrow \rho K$, combined with a previous result, allow us to predict the (observed) suppression of the $K\bar{K}$ decay mode of the A_2 meson.

In a recent publication[1] we have obtained several results pertaining to the decays of various mesons within the framework of the Veneziano model.[2] One of the results [Eq. (8′) of Ref. 1] was predictive of the decay rates of the spin-2^+ A_2 meson[3]:

$$\Gamma_{A_2 \rightarrow \rho\pi}\Gamma_{A_2 \rightarrow \eta\pi} = 2700 \pm 1100 \text{ MeV}^2. \tag{1}$$

In this Letter we extend the considerations of Ref. 1 to include K-mesonic systems and are thereby able to present a more comprehensive picture of the decay systematics of the A_2 meson.

We consider the process

$$K(p_1) + \pi^a(p_2) \rightarrow K(p_3) + \rho^b(p_4) \tag{2}$$

described by the amplitude

$$M^{ba} = \epsilon_{\mu\nu\kappa\lambda} e^\mu p_1{}^\nu p_2{}^\kappa p_3{}^\lambda \{A^{(+)}\delta^{ba} + \tfrac{1}{2}[\tau^b, \tau^a]A^{(-)}\}, \tag{3}$$

where $A^{(\pm)}$ permits the Veneziano representation

$$A^{(\pm)}(s,t,u) = \beta\Gamma(1-\alpha(t)) \left[\frac{\Gamma(1-\alpha_{K^*}(s))}{\Gamma(2-\alpha_{K^*}(s)-\alpha(t))} \pm \frac{\Gamma(1-\alpha_{K^*}(u))}{\Gamma(2-\alpha_{K^*}(u)-\alpha(t))}\right]. \tag{4}$$

The su term is absent in order to eliminate any $I=\frac{3}{2}$ resonances. $\alpha(t)$ denotes (quasi) degenerate ρ-ω-f^0-A_2 trajectory[4,5] [$\alpha(t) \simeq 0.50 + 0.9t$] and $\alpha_{K^*}(s)$ denotes the degenerate $K^*(890)$-$K_N(1420)$ trajectory [$\alpha_{K^*}(s) \simeq 0.28 + 0.9s$].