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SCATTERING AMPLITUDES IN QUANTUM ELECTRODYNAMICS AT INFINITE ENERGY

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We have studied the ee, $e\gamma$, and $\gamma\gamma$ elastic-scattering amplitudes contributed by certain infinite sets of diagrams in the high-energy limit. They found to be proportional to s, the square of the c.m.-system energy, multiplied by simple combinations of the Glauber forms of high-energy scattering.

We have previously presented a method of the infinite momentum frame for calculating Feynman amplitudes in which the variables $p^0\pm p^3$ play an important role.[1] In this Letter we report some results of analyzing, using this method, the leading terms in the high-energy ee, $e\gamma$, and $\gamma\gamma$ elastic-scattering amplitudes contributed by certain infinite sets of diagrams.

Summing the diagrams with arbitrary numbers of photons exchanged between the particles moving with large and opposite momenta (see Fig. 1), we find that the results are very simple. For the ee scattering amplitudes one finds a Glauber form[2]:

$$M(e^-e^\mp)=\tfrac{1}{2}is\,\delta_{aa'}\delta_{bb'}m^{-2}F_\pm'(\vec{\mathrm{k}}); \tag{1}$$

$$F_\pm'(\vec{\mathrm{k}})\equiv\int d^2b\,e^{-i\vec{\mathrm{b}}\cdot\vec{\mathrm{k}}}(e^{\pm i\chi(\vec{\mathrm{b}})}-1)\equiv F_\pm(\vec{\mathrm{k}})-(2\pi)^2\delta(\vec{\mathrm{k}}); \tag{2}$$

$$\chi(\vec{\mathrm{b}})\equiv -e^2\int d^2q\,(2\pi)^{-2}(\vec{\mathrm{q}}^2+\lambda^2)^{-1}e^{i\vec{\mathrm{q}}\cdot\vec{\mathrm{b}}}, \tag{3}$$

where $s\to\infty$ is the square of the c.m.-system energy, λ is a fictitious photon mass, the Kronecker δ's in (1) indicate that the helicities are not flipped, and $\vec{\mathrm{k}}$ is the momentum transfer, which lies in the (1, 2) plane. The kinematics is shown in Fig. 2(a).

The $e\gamma$ and $\gamma\gamma$ scattering amplitudes are found to be simple combinations of F_+ and F_-:

$$M(\gamma e^\mp)=-\tfrac{1}{2}is\,\delta_{bb'}m^{-1}\int d^2k_1\,d^2k_2\,(2\pi)^{-2}\delta(\vec{\mathrm{k}}_1+\vec{\mathrm{k}}_2-\vec{\mathrm{k}})I_{ij}(\vec{\mathrm{k}}_1,\vec{\mathrm{k}}_2)[F_+(\vec{\mathrm{k}}_1)F_-(\vec{\mathrm{k}}_2)-(2\pi)^4\delta(\vec{\mathrm{k}}_1)\delta(\vec{\mathrm{k}}_2)], \tag{4}$$

$$M(\gamma\gamma)=\tfrac{1}{2}is\,(2\pi)^{-6}\int d^2\vec{\mathrm{k}}_1\,d^2\vec{\mathrm{k}}_2\,d^2\vec{\mathrm{k}}_3\,d^2\vec{\mathrm{k}}_4\,\delta(\vec{\mathrm{k}}_1+\vec{\mathrm{k}}_2+\vec{\mathrm{k}}_3+\vec{\mathrm{k}}_4-\vec{\mathrm{k}})I_{ij}(\vec{\mathrm{k}}_1+\vec{\mathrm{k}}_2,\vec{\mathrm{k}}_3+\vec{\mathrm{k}}_4)I_{i'j'}(\vec{\mathrm{k}}_1+\vec{\mathrm{k}}_3,\vec{\mathrm{k}}_2+\vec{\mathrm{k}}_4)$$
$$\times[F_+(\vec{\mathrm{k}}_1)F_-(\vec{\mathrm{k}}_2)F_-(\vec{\mathrm{k}}_3)F_+(\vec{\mathrm{k}}_4)-(2\pi)^8\delta(\vec{\mathrm{k}}_1)\delta(\vec{\mathrm{k}}_2)\delta(\vec{\mathrm{k}}_3)\delta(\vec{\mathrm{k}}_4)]. \tag{5}$$

All the δ functions appearing above are two-dimensional. The indices i and j denote, respectively, the final and the initial polarizations of the photon. The function I_{ij} is given by

$$I_{ij}(\vec{\mathrm{k}}_1,\vec{\mathrm{k}}_2)=(e^2/4\pi^2)\int_0^1 d\beta\,d\beta'\,dx\,\delta(1-\beta-\beta')\{4K_iK_j\beta\beta'x(1-x)-\tfrac{1}{2}\delta_{ij}\vec{\mathrm{K}}^2[1-8\beta\beta'(x-\tfrac{1}{2})^2]\}$$
$$\times[m^2+x(1-x)\vec{\mathrm{K}}^2]^{-1},\quad \vec{\mathrm{K}}\equiv\beta'\vec{\mathrm{k}}_1-\beta\vec{\mathrm{k}}_2. \tag{6}$$

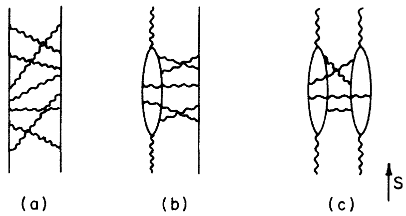

FIG. 1. Examples of diagrams with many photons exchanged between the particle on the left and that on the right. (a) ee scattering. (b) $e\gamma$ scattering. (c) $\gamma\gamma$ scattering.

Since the Glauber form occurs often in theories of high-energy scattering, the above results are not unexpected. In (4) and (5), each external photon appears as an e^- and an e^+, which scatter independently against the $e^\pm$ moving in the opposite direction. The function I_{ij} is effectively a form factor describing the composition of the photon during the scattering.[3]

The diagrams of lowest orders in e^2 and in the same category as those shown in Fig. 1 have been analyzed in detail by Cheng and Wu.[4] The terms to the lowest orders in e^2 in (1), (4), and (5) can be shown easily to be the same as their results. However, we would like to emphasize that our investigation by no means includes the elegant mathematical analysis of Ref. 4 as a special case.[5] Our method, which sacrifices much mathematical rigor for its simiplicity, cannot be a substitute for the elaborate procedure in achieving the rigorous results of Ref. 4.

In this Letter we shall only discuss briefly the basic features of our calculation, which seems to be very simple compared with the existing nontrivial quantum electrodynamic calculations. A paper is being prepared to illustrate our methods in detail both for pedagogical purposes and to include results of analyzing more general diagrams.

The crucial simplifying step in our calculation is to factor out the leading s dependence before the loop integrals are carried out. This is easily done using the variables $p^0 \pm p^3 \equiv p_{0\pm 3}$ discussed in Ref. 1. In the infinite-s limit, each diagram which contributes to the leading terms consists of two parts joined by photon lines. One part includes internal lines with infinite p_{0+3}, which is proportional to $\sqrt{s}$, describing particles moving in the positive 3-direction with infinite momenta. The other part includes lines with infinite p_{0-3} describing those moving in the negative 3-direction with infinite momenta. The momenta of the photons joining these two parts are finite.

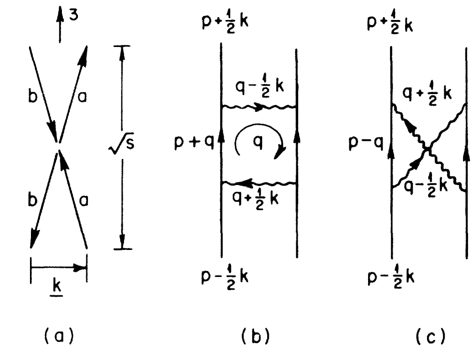

FIG. 2. (a) The kinematics of $a+b \to a+b$. (b), (c) e^--e^- scattering with two photons exchanged. Notice that p has only 0 and 3 components and k has only 1, 2 components.

For each part one writes down a factor which is the product of coupling constants, propagators, etc. These factors transform in a well-defined way under Lorentz transformations.

To analyze the part with infinite p_{0+3}, we choose a reference frame moving so fast along the +3 direction that, in this frame, the momentum variables p' become finite. Recall that, under a Lorentz transformation, p changes to p' according to

$$p_{0\pm 3} = p_{0\pm 3}' e^{\pm\lambda},$$

$$p^{1,2} = p'^{1,2}. \tag{7}$$

We can choose the standard frame defined by $e^\lambda = \sqrt{s}$. Thus the s dependence becomes explicit in the factor pertaining to this part of the diagram. Notice that, upon boosting the standard frame to infinite momentum, the $0+3$ component of any vector or tensor picks up a factor $\sqrt{s}$ while the $0-3$ component picks up $1/\sqrt{s}$. Thus the $0+3$ and $0-3$ are referred to as the "big" and "small" components, respectively. For the part of the diagram with infinite p_{0-3}, we simply change the +3 in what we just said to −3. With the s dependence explicitly exhibited, one then ignores all but the leading term. The algebra is thus greatly simplified.

To illustrate our approach, we shall sketch the derivation of (1). First, consider the fourth-order e^-e^- scattering. The electron on the left of Fig. 2(b) moves with infinite p_{0+3}. We have for this part a factor

$$\bar{u}_{a'}(p+\tfrac{1}{2}k)\gamma^\mu(\not{p}+\not{q}+m)\gamma^\nu u_a(p-\tfrac{1}{2}k)[(p+q)^2-m^2+i\epsilon]^{-1}. \tag{8}$$

Let us choose the standard frame with $p_{0+3}'=1$. The leading term comes from the big component of (8), i.e., $\mu=\nu=0+3$. Since $(\gamma_{0+3})^2=0$, only γ_{0-3} can survive between the two (γ_{0+3})'s. Thus, (8) becomes

$$\tfrac{1}{4}s\bar{u}_{a'}(p'+\tfrac{1}{2}k)\gamma_{0+3}\gamma_{0-3}\gamma_{0+3}u_a(p'-\tfrac{1}{2}k)(s^{1/2}q_{0-3}-m^2-\vec{q}^2+i\epsilon)^{-1}=s^{1/2}\delta_{aa'}m^{-1}(q_{0-3}+i\epsilon)^{-1}. \tag{9}$$

By symmetry, the electron line on the right of Fig. 2(b) gives

$$s^{1/2}\delta_{bb'}m^{-1}(q_{0+3}+i\epsilon)^{-1}. \tag{10}$$

The two-photon propagators give

$$[(q+\tfrac{1}{2}k)^2-\lambda^2]^{-1}[(q-\tfrac{1}{2}k)^2-\lambda^2]^{-1}. \tag{11}$$

Figure 2(c) leads to the same contribution as that of Fig. 2(b) except that the q in (9) is changed to $-q$. Adding the two diagrams, we find that a factor

$$(q_{0-3}+i\epsilon)^{-1}+(-q_{0-3}+i\epsilon)^{-1}=-2\pi i\delta(q_{0-3}) \tag{12}$$

results. Thus, the q_{0-3} in (11) may be set to zero, and q_{0+3} disappears in (11) with it. Now the q_{0+3} denominator of (10) can be replaced by

$$\tfrac{1}{2}(-2\pi i)\delta(q_{0+3}). \tag{13}$$

Collecting terms and performing the loop integrals $[(2\pi)^4 i]^{-1}\frac{1}{2}\int dq_{0+3}dq_{0-3}d^2q$, we have the fourth-order e^-e^- scattering amplitude

$$-\tfrac{1}{4}is\delta_{aa'}\delta_{bb'}m^{-2}e^4\int d^2q(2\pi)^{-2}[(\vec{q}+\tfrac{1}{2}\vec{k})^2+\lambda^2]^{-1}[(\vec{q}-\tfrac{1}{2}\vec{k})^2+\lambda^2]^{-1}$$
$$=\tfrac{1}{2}is\delta_{aa'}\delta_{bb'}m^{-2}[-\tfrac{1}{2}\int d^2b\,e^{i\vec{k}\cdot\vec{b}}\chi(\vec{b})^2]. \tag{14}$$

There are two important features illustrated above. First, only γ_{0+3} appears at the vertices on the electron line on the left, and only γ_{0-3}, on the right. Second, as a result of adding the diagrams in Figs. 2(b) and 2(c) $\delta(q_{0\pm3})$ appears. The first feature clearly remains when additional photons are exchanged between the electrons. The numerator of (9) then involves additional factors of $\gamma_{0-3}\gamma_{0+3}$, which can be reduced by the identity $\gamma_{0+3}\gamma_{0-3}\gamma_{0+3}=4\gamma_{0+3}$. The second feature also persists when many photons are exchanged provided one adds up diagrams with photon vertices permuted in all possible ways. It becomes a matter of careful counting to show that a factor

$$\frac{1}{N!}\prod_{j=1}^{N-1}\delta(q_{j\,0-3})\delta(q_{j\,0+3})(2\pi i)^2 \tag{15}$$

results when N photons are exchanged. Equations (12) and (13) give the case $N=2$. When terms with $N=1$ to $N=\infty$ are summed, one obtains (1). Equations (4) and (5) are obtained in the same way with a bit more involved algebra.

We would like to emphasize that it is crucial to include the diagrams with crossed photon lines in addition to the ladder diagrams. For the ee ladder diagram with N photons exchanged, one obtains the leading term

$$\sim s(\ln s)^{N-1}\tfrac{1}{2}i\delta_{aa'}\delta_{bb'}m^{-2}. \tag{16}$$

Formally summing over N, one would expect an $s/\ln s$ behavior, which might look like what is expected from a Regge cut. The spurious $\ln s$ terms are manifestly absent when diagrams with crossed photon lines are included, as is required by gauge invariance.

It is clear that we have summed only certain restricted sets of diagrams. It is quite possible that the s dependence becomes different when more sets of diagrams are included. This question is under study.

The important question of the legitimacy of taking the limit $s\to\infty$ before performing the loop integrals is not settled. However, the evidence for the correctness of our procedure is provided by the fact that no subtraction was needed in our calculation and that the gauge invariance of our results can be verified explicitly. Furthermore, the physical meaning of our procedure is clear and above all, the remarkable simplicity makes it valuable.

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[2]R. J. Glauber, in Lectures in Theoretical Physics, edited by Wesley E. Brittin et al. (Interscience Publishers, Inc., New York, 1959), Vol. 1, p. 315.

[3]The factor $I(\vec{k}_1, \vec{k}_2)$ is related to the $\mathcal{I}_\gamma(\vec{k}_1, \vec{k}_2)$ defined by H. Cheng and T. T. Wu [Phys. Rev. Letters 22, 666 (1969); Phys. Rev., to be published (papers I-IV)], through $\mathcal{I}_\gamma(k_1, k_2) = \frac{1}{2}[I(\vec{k}_1+\vec{k}_2, 0) - I(\vec{k}_1, \vec{k}_2)]$. We are grateful to Professor S. Adler for showing us the preprints of the Phys. Rev. papers.

[4]Cheng and Wu, Ref. 3.

[5]In Paper IV of Ref. 3 there is a relatively short derivation of their lowest order photon impact factor using the variables $p_0 \pm p_3$. This derivation may be viewed as a special case of our analysis.

THEOREM ON THE FORM FACTORS IN K_{l3} DECAY

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We derive a formula for the matrix element $\langle K | \partial_\mu \mathfrak{F}_a^{\ \mu} | \pi \rangle$ correct up to second order in both the momentum transfer t and the breaking of SU(3)⊗SU(3) symmetry. This leads to a prediction that ξ is very small compared with unity. Our result is independent of any assumption concerning the structure of the symmetry-breaking Hamiltonian.

It has become increasingly apparent that it is interesting to discuss the strong interactions in terms of an approximate SU(3)⊗SU(3) symmetry.[1,2] That is, it is meaningful to write the strong-interaction Hamiltonian as $H = H_0 + \epsilon H'$, where H_0 is SU(3)⊗SU(3) invariant, H' simultaneously breaks SU(3) and SU(3)⊗SU(3), and ϵ is small enough that a perturbation expansion makes sense. The symmetric limit ($\epsilon = 0$) is to be understood as one in which the octet of pseudoscalar mesons is massless, allowing the axial-vector currents to be conserved without requiring the presence of SU(3)⊗SU(3) multiplets of particles. The practical advantage to adopting this picture of the strong interactions is that an expansion in powers of ϵ provides a systematic way of keeping track of corrections to SU(3) and partial conservation of axial-vector current, since $\epsilon H'$ is responsible for both the mass splittings among SU(3) multiplets and the small "extrapolation errors" encountered in any application of partial conservation of axial-vector current.

In this Letter we show how applying the above ideas allows one to calculate some symmetry-breaking terms in form factors for processes such as K_{l3} decay. Specifically, we prove the following theorem:

Theorem. – Let $\mathfrak{F}_b^{\ \mu}$, $b = 1, \cdots, 8$, denote one of the vector currents and $|M_a(p)\rangle$, $a = 1, \cdots, 8$, a covariantly normalized pseudoscalar-meson state with momentum p^μ. Further, let us expand the matrix elements of $\partial_\mu \mathfrak{F}_b^{\ \mu}$ as follows:

$$\langle M_a(p) | \partial_\mu \mathfrak{F}_b^{\ \mu}(0) | M_c(p') \rangle \equiv a_0 + a_1 t + a_2 t^2 + \cdots, \tag{1}$$

where $t = (p-p')^2$. Then, it can be shown that

$$a_0 = -(m_a^{\ 2} - m_c^{\ 2}) f_{abc} + O(\epsilon^3),$$

$$a_1 = -\frac{1}{2}\left[\frac{f_c}{f_a} - \frac{f_a}{f_c}\right] f_{abc} + O(\epsilon^2), \tag{2}$$

where f_{abc} are the structure constants of SU(3), m_a is the mass of the ath pseudoscalar meson, and f_a is defined by the matrix element of the axial current between a pseudoscalar-meson state and the vacuum,

$$\langle M_a(p) | \mathfrak{F}_a^{\ \mu 5}(0) | 0 \rangle \equiv -ip^\mu / 2f_a. \tag{3}$$