not both peaks of the "A_2." In addition the mass of this narrow resonance (1289 ± 10 MeV) is centered below 1300 MeV.

(2) Two independent spin and parity analyses of this $\rho^0\pi^-$ resonance favor the assignment $J^{PC} = 1^{-+}$ (25%, 40%) over $J^{PC} = 2^{++}$ (2%, 6%). This is again different from previous investigations of the "A_2" $\to \rho\pi$ either as single or double peaks.

If this narrow $\rho^0\pi^-$ resonance is indeed a $J^{PC} = 1^{-+}$ particle, then it will be difficult to accommodate it in the simple $(\bar{q}q)$ model for particle classifications where $P = C$ for all natural-parity states, such as the ρ meson ($J^{PC} = 1^{--}$). It may be necessary to invoke new models for particle classifications such as the four-dimensional $(q\bar{q})$ harmonic-oscillator model, or the existence of daughter trajectories in the Chew-Frautschi plot, or both.[10]

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[9]The normal is defined in the three-pion rest frame by $n = \pi_a^- \times \pi_b^-$, where the lower momentum π^- has been taken first. The axes are defined in this frame with the z direction in the direction of the incident K^- and the y direction normal to the A_2 production plane. Θ and Φ are then the polar and azimuthal angles. Maximum likelihood fits were made to the combined Θ and Φ distribution. Possible distributions are given by S. M. Berman and M. Jacob, Phys. Rev. 139, B1023 (1965).

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BROKEN-DUALITY MODEL FOR THE REACTION $pp \to \pi^+d$ *

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A model based on a broken (N_α, N_γ) exchange degeneracy is proposed to explain the structureless data for the $pp \to \pi^+d$ differential cross section. An effective Regge trajectory is determined which agrees with our prediction. Quantitative fits to the data are presented and physical aspects of the broken degeneracy are discussed.

The appropriate theoretical description of the accumulated data on the reaction $pp \to \pi^+d$ has remained as an unsolved problem during this period in which Regge-exchange models have accounted for a substantial fraction of two-body scattering reactions. The only systematic approach to the $pp \to \pi^+d$ data[1-4] proposed thus far is the empirical formula $\exp(-ap_\perp)$, which unfortunately has no sound theoretical basis.

In terms of exchange models, the u and t channels for $pp \to \pi^+d$ are $B = 1$ and $I = \frac{1}{2}$, and the nucleon is an obvious candidate for exchange. In fact, from the nucleon Regge pole we expect the integrated cross section to have an energy dependence

$$\sigma \sim s^{-3},$$

which is crudely compatible with the data. However, for nucleon exchange conventional Regge theory also predicts a differential-cross-section zero[5] at $u \simeq -0.15$ (GeV/c)^2 where $\alpha_N(u) = -\frac{1}{2}$. This strongly contradicts the data which show no such pronounced dip.

A possible resolution of the difficulty of formulating an exchange model for this reaction is suggested by duality considerations.[6,7] Proceeding from the initial supposition that the $pp\to\pi^+d$ s channel has no resonances, we need exchange-degenerate poles in the u channel.[7] The realization of this situation would be an exchange degeneracy of N_α $(\frac{1}{2}^+, \frac{5}{2}^+, \cdots)$ and N_γ $(\frac{3}{2}^-, \frac{7}{2}^-, \cdots)$ trajectories. The exchange of the two poles then would also explain the lack of a dip in the differential cross section since the amplitude zeros of the two trajectories are interlaced. An analogous situation has already been encountered in K^+p backward elastic scattering, where Λ_α and Λ_γ exchange-degenerate amplitudes successfully describe the structureless K^+p cross section.[7,8] The lack of analogy between $pp\to\pi^+d$ and the $\pi^+p\to\pi^+p$ dynamics[5] is presumably a reflection of the difference in duality requirements.

This suggestion for an (N_α, N_γ) exchange-degenerate model for the reaction $pp\to\pi^+d$ is somewhat of an idealization, since the trajectories obtained from the Chew-Frautschi plot of Fig. 1 are split apart: For N_α,

$$\alpha=-0.38+1.0u$$

and for N_γ,

$$\alpha=-0.8+1.0u. \tag{1}$$

Similarly, we might expect deviations from exact degeneracy for the residues. However, these departures themselves lead to interesting and testable consequences.

The most direct experimental consequence of a broken exchange degeneracy pertains to the energy dependence of the differential cross section, which is closely tied to the Regge-amplitude zeros at wrong-signature nonsense points. In the exchange-degenerate limit, the amplitudes are purely real and proportional to

$$\frac{1}{\Gamma(\alpha+\frac{1}{2})} \times\left[\frac{1+e^{-i\pi(\alpha-\frac{1}{2})}}{\cos\pi\alpha}+\frac{1-e^{-i\pi(\alpha-\frac{1}{2})}}{\cos\pi\alpha}\right]s^{\alpha-\frac{1}{2}},$$

where the first term represents the N_α contribution and the second, N_γ. As the degeneracy of trajectories and of residues is gradually broken, we expect the "effective" trajectory of the differential cross section to be bounded above by the N_α trajectory and below by the N_γ trajectory, since the relative sign is positive between N_α and N_γ contributions to the (dominant) real part of the amplitude. Furthermore, at $u\simeq-0.12$, where $\alpha(N_\alpha)=-\frac{1}{2}$, the N_α exchange contribution vanishes and the effective trajectory becomes that of the N_γ at this u value. On the other hand, at $u\simeq-0.8$, where $\alpha(N_\gamma)=-\frac{3}{2}$, the roles are reversed with the N_γ amplitude vanishing and the N_α becoming the effective trajectory. This pattern for the effective trajectory is displayed in the spin-mass plot of Fig. 1. If we use the trajectory values at the above two wrong-signature nonsense points, we estimate an approximate straight-line effective trajectory for the reaction $pp\to\pi^+d$ between $u=-0.1$ and $u=-0.7$ of

$$\alpha_{\rm eff}=-0.9+0.28u.$$

This effective trajectory is represented by the dashed line in Fig. 1. The broken-exchange-degenerate model therefore predicts little shrinkage for $0.1<-u<0.7$. However, for $0.7<-u<2.2$ and for $u>-0.1$ the shrinkage is predicted to be greater than that from trajectories of normal slope ~1 (GeV/c)^2. These different regimes of energy dependence in $pp\to\pi^+d$ can provide a straightforward test of the model.

Since the N_γ trajectory lies about a half unit below the N_α trajectory, the N_γ contribution will die out more rapidly with increasing energy. Therefore another experimental test of the model should be the observation of the N_α dip at $u\simeq-0.15$ progressively appearing at higher ener-

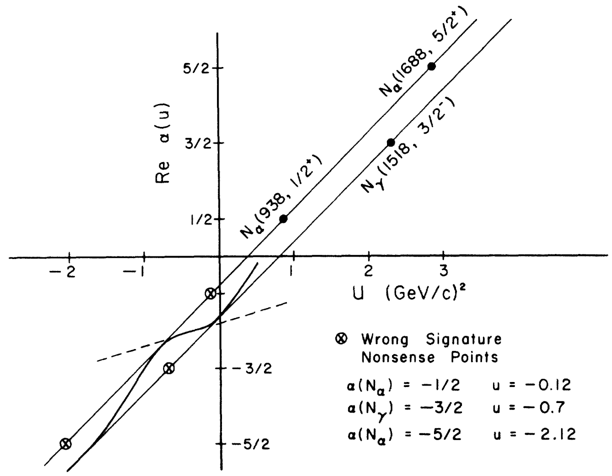

FIG. 1. Regge spin-mass plot for N_α and N_γ trajectories. The solid curve represents the "effective" Regge trajectroy expected for the reaction $pp\to\pi^+d$. The dashed line depicts the effective linear trajectory for the momentum-transfer range $0.1\le-u\le0.7$ (GeV/c)^2.

gies. In fact the 3.31-, 3.62-, and 21.1-GeV/c $pp\to\pi^+d$ data[1,2] do show a break in slope at this momentum transfer, which we would argue is a natural reflection of the N_α wrong-signature nonsense dip.

The splitting of the N_α and N_γ trajectories found from the Chew-Frautschi plot implies a nonzero imaginary amplitude for $pp\to\pi^+d$. The duality principle in turn suggests that s-channel resonances must be present to build this imaginary amplitude. The $pp\to\pi^+d$ data below 6 GeV/c [1,4,9] show a substantial amount of energy-dependent structure, which is characteristic of the direct-channel resonant behavior observed in more common two-body inelastic channels. In $pp\to\pi d$ and $\pi d\to\pi d$ the s-channel low-energy structure in the imaginary part has been calculated successfully using $NN_{1/2}{}^*$ and $N\Delta$ intermediate states. This latter interpretation provides an alternative to bona fide d^* resonances, although the two approaches might be reconciled. These "effective" d^* contributions couple strongly to πd and only weakly to pp.

The choice of energy variable for Regge-exchange amplitudes is somewhat ambiguous at nonasymptotic energies. Several frequently used forms are $s^{\alpha-1/2}$, $(s-\frac{1}{2}\sum_i m_i{}^2)^{\alpha-1/2}$, and $(s-t)^{\alpha-1/2}$. These choices differ only in nonleading powers, but make a substantial difference in phenomenological analyses of data at lower energies. The energy scale of $\sum_i m_i{}^2\sim 6$ (GeV)^2 for the reaction $pp\to\pi^+d$ is considerably larger than for most reactions that have been analyzed. As a consequence, we cannot expect to use experimental data on $pp\to\pi^+d$ below 5-GeV/c momenta for reliable determinations of trajectory parameters. Unfortunately the bulk of the present data on this reaction falls below 3.75 GeV/c, with the exception of a recent experiment at 21.1 GeV/c and some fixed-angle points. In order to make a crude check of the proposed model, we made an initial fit to data above 3 GeV/c using a single, effective, linear trajectory $\alpha=\alpha_0+\alpha_1u$ and the parametrization

$$\frac{d\sigma}{du}=[a_1e^{b_1u}+a_2e^{b_2u}]\left(\frac{s-t}{2}\right)^{2\alpha(u)-2}. \tag{2}$$

The data encompassed the u region $+0.25<u>-1.5$ (GeV/c)^2. The effective trajectory so obtained was

$$\alpha=-0.8+0.6u \tag{3}$$

which qualitatively agrees with our expectations from Fig. 1 for this u range. Of course, we must bear in mind that the 3.31- and 3.62-GeV/c data used in this analysis are in a momentum region that has s-dependent "resonancelike" structure.

Although a more sophisticated approach probably is not warranted with present data, we have nevertheless proceeded to consider the full kinematics for $pp\to\pi^+d$ Reggeization.[10] In general, six u-channel helicity amplitudes are involved. However, if we estimate the Regge amplitudes from the couplings to the $N_\alpha(\frac{1}{2}^+)$ exchange, the D-state deuteron coupling dominates near $u=0$ (recall that the D-state amplitude at $u=M_N{}^2$ is suppressed by kinematic factors of centrifugal barrier origin) and only the two $\lambda_d=0$ u-channel helicity amplitudes survive in this approximation. The resulting cross-section expression is

$$\frac{d\sigma}{du}=\frac{M_N{}^2}{32\pi s p_i{}^2}[|1+\cos\theta_u|\,|f_0{}^-+f_0{}^+|^2+|1-\cos\theta_u|\,|f_0{}^--f_0{}^+|^2], \tag{4}$$

where

$$f_0{}^\pm(s,\sqrt{u})=\frac{\gamma^\pm(\sqrt{u})}{\Gamma(\alpha+\frac{1}{2})}\,\frac{1+\tau e^{-i\pi(\alpha-\frac{1}{2})}}{\cos\pi\alpha}\left(\frac{s-t}{2}\right)^{\alpha-\frac{1}{2}} \tag{5}$$

for a single trajectory $\alpha(u)=\alpha_0+\alpha_1u$. The residue $\gamma^{(\tau P)}$ contains the kinematic singularities and obeys the MacDowell symmetry relation $\gamma^+(\sqrt{u})=\gamma^-(-\sqrt{u})$. The N_α residue can be parametrized as

$$\gamma^+(\sqrt{u})=ae^{bu}(1+u^{\frac{1}{2}}/M_N)u^{-\frac{1}{2}}\{[(M_d+M_N)^2-u][(M_d-M_N)^2-u][(u^{\frac{1}{2}}+M_N)^2-M_d{}^2][(u^{\frac{1}{2}}-M_N)^2-\mu^2]\}^{\frac{1}{2}}. \tag{6}$$

For an exchange-degenerate N_γ, a term similar to that above is included in $f_0{}^\pm$. The merit of using this more elaborate formalism for analyzing present data is that the exact kinematics contain nonlead-

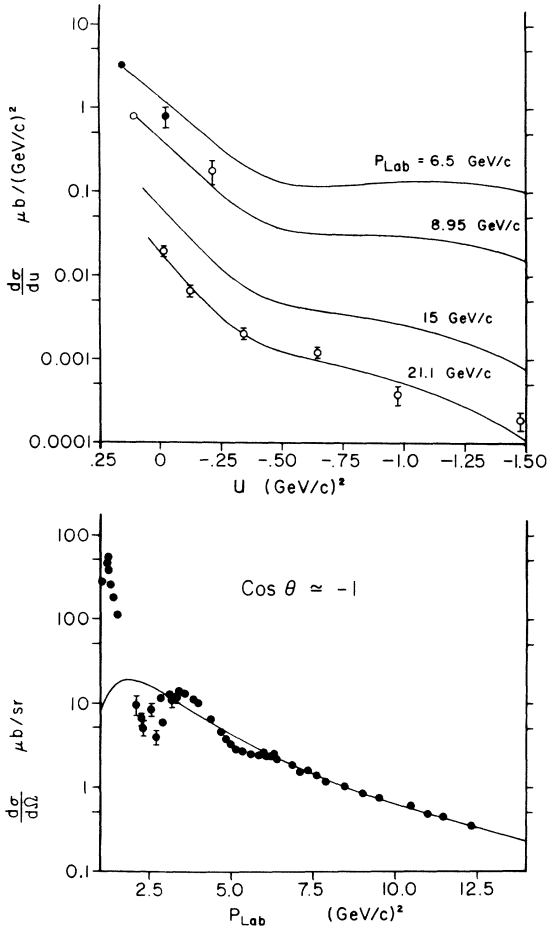

FIG. 2. (N_α, N_γ) broken–exchange-degeneracy fits to the $pp\rightarrow\pi^+d$ data. Data are taken from Refs. 1-4 and 9.

ing powers in energy. The dominance of the u-channel $\lambda_d=0$ helicity amplitude suggested above can be experimentally checked by polarization measurements.

Using Eqs. (4)-(6) and the trajectories of Eq. (1), we have fitted all $pp\rightarrow\pi^+d$ differential-cross-section measurements above 6 GeV/c. The results of the fit are shown in Fig. 2. The model reproduces the change in slope of the 21.1-GeV/c data[2] at $u\simeq-0.2$ (GeV/c)^2 and agrees with the backward cross sections at 6.5 and 9 GeV/c.[3] For the data[4] near $\cos\theta=-1$ the model correctly describes the energy dependence of the high-energy data and extrapolates through the mean of the fluctuations below 6 GeV/c. Further experiments between 6 and 20 GeV/c are badly needed to examine critically the validity of the model. The residue parameters obtained from this analysis were [in (GeV/c)$^{-2}$], for N_α and for N_γ, respectively,

$$a=-1.04 \text{ and } b=-1.36,$$

$$a=8.1 \text{ and } b=0.87. \tag{7}$$

It is not possible to make any reasonable quantitative comparison of the residue parameters with the exchange-degeneracy concept when the trajectories are as widely separated as in this case. The breakdown of exchange degeneracy can be investigated further through the time-reversed reaction $\bar{p}d\rightarrow p\pi^-$ at backward angles. With exact exchange degeneracy we would predict the equality

$$d\sigma(pp\rightarrow\pi^+d)/du=d\sigma(\bar{p}d\rightarrow p\pi^-)/du$$

in the Regge energy region, but we expect a breakdown of the equality in the physical situation of nonexact degeneracy. There is one other interesting difference in the above two reactions in that $pp\rightarrow\pi^+d$ is fore-aft symmetric whereas $\bar{p}d\rightarrow p\pi^-$ should have only a backward peak.

Finally we draw attention to the empirical equality

$$d\sigma(pp\rightarrow\pi^+d)/du\simeq d\sigma(pp\rightarrow\rho^+d)/du$$

experimentally observed in the 21.1-GeV/c data.[2] This empirical relation suggests that the effective residues of πN and ρN to the nucleonic Regge exchanges are approximately equal. This in turn leads us to predict the relation

$$d\sigma(\pi^+p\rightarrow\pi^+p)/du\simeq d\sigma(\pi^+p\rightarrow\rho^+p)/du.$$

A similar approximate empirical equality has already been observed[11] for the Δ_δ exchange processes, namely

$$d\sigma(\pi^-p\rightarrow\pi^-p)/du\simeq d\sigma(\pi^-p\rightarrow\rho^-p)/du.$$

In making comparisons of these reactions at intermediate and low energies, considerable caution must be exercised to compare at fixed u, rather than fixed $\cos\theta$. For example, with equal ρN and πN couplings, we would obtain the ratio

$$\frac{d\sigma(pp\rightarrow\pi^+d)/du}{d\sigma(pp\rightarrow\rho^+d)/du}\approx4$$

for $\cos\theta=-1$ at 4 GeV/c because of the difference in kinematic cutoffs and the sharpness of the backward peak.[12]

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[11]E. W. Anderson et al., Brookhaven National Laboratory–Carnegie-Mellon collaboration, reported by G. Belletini, in Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, Austria, 1968, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 329.

[12]G. Smith (private communication) has informed us that recent data at $\cos\theta = -1$ in this momentum range give a ratio of magnitude comparable with this prediction.

SCATTERING AMPLITUDES IN QUANTUM ELECTRODYNAMICS AT INFINITE ENERGY

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We have studied the ee, $e\gamma$, and $\gamma\gamma$ elastic-scattering amplitudes contributed by certain infinite sets of diagrams in the high-energy limit. They found to be proportional to s, the square of the c.m.-system energy, multiplied by simple combinations of the Glauber forms of high-energy scattering.

We have previously presented a method of the infinite momentum frame for calculating Feynman amplitudes in which the variables $p^0 \pm p^3$ play an important role.[1] In this Letter we report some results of analyzing, using this method, the leading terms in the high-energy ee, $e\gamma$, and $\gamma\gamma$ elastic-scattering amplitudes contributed by certain infinite sets of diagrams.

Summing the diagrams with arbitrary numbers of photons exchanged between the particles moving with large and opposite momenta (see Fig. 1), we find that the results are very simple. For the ee scattering amplitudes one finds a Glauber form[2]:

$$M(e^-e^\mp) = \tfrac{1}{2} is\,\delta_{aa'}\delta_{bb'} m^{-2} F_\pm'(\vec{k}); \tag{1}$$

$$F_\pm'(\vec{k}) \equiv \int d^2b\, e^{-i\vec{b}\cdot\vec{k}}\left(e^{\pm i\chi(\vec{b})}-1\right) \equiv F_\pm(\vec{k}) - (2\pi)^2\delta(\vec{k}); \tag{2}$$

$$\chi(\vec{b}) \equiv -e^2\int d^2q\,(2\pi)^{-2}(\vec{q}^{\,2}+\lambda^2)^{-1} e^{i\vec{q}\cdot\vec{b}}, \tag{3}$$

where $s \to \infty$ is the square of the c.m.-system energy, λ is a fictitious photon mass, the Kronecker δ's in (1) indicate that the helicities are not flipped, and $\vec{k}$ is the momentum transfer, which lies in the (1, 2) plane. The kinematics is shown in Fig. 2(a).

The $e\gamma$ and $\gamma\gamma$ scattering amplitudes are found to be simple combinations of F_+ and F_-:

$$M(\gamma e^\mp) = -\tfrac{1}{2} is\,\delta_{bb'} m^{-1}\int d^2k_1\, d^2k_2\,(2\pi)^{-2}\delta(\vec{k}_1+\vec{k}_2-\vec{k}) I_{ij}(\vec{k}_1,\vec{k}_2)[F_+(\vec{k}_1)F_-(\vec{k}_2) - (2\pi)^4\delta(\vec{k}_1)\delta(\vec{k}_2)], \tag{4}$$

$$\begin{aligned} M(\gamma\gamma) = \tfrac{1}{2} is\,(2\pi)^{-6}\int d^2\vec{k}_1\, d^2\vec{k}_2\, d^2\vec{k}_3\, d^2\vec{k}_4\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4-\vec{k}) I_{ij}(\vec{k}_1+\vec{k}_2,\vec{k}_3+\vec{k}_4) I_{i'j'}(\vec{k}_1+\vec{k}_3,\vec{k}_2+\vec{k}_4) \\ \times[F_+(\vec{k}_1)F_-(\vec{k}_2)F_-(\vec{k}_3)F_+(\vec{k}_4) - (2\pi)^8\delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\delta(\vec{k}_4)]. \end{aligned} \tag{5}$$

All the δ functions appearing above are two-dimensional. The indices i and j denote, respectively, the final and the initial polarizations of the photon. The function I_{ij} is given by

$$\begin{aligned} I_{ij}(\vec{k}_1,\vec{k}_2) = (e^2/4\pi^2)\int_0^1 d\beta\, d\beta'\, dx\,\delta(1-\beta-\beta')\{4K_iK_j\beta\beta' x(1-x) - \tfrac{1}{2}\delta_{ij}\vec{K}^2[1-8\beta\beta'(x-\tfrac{1}{2})^2]\} \\ \times[m^2+x(1-x)\vec{K}^2]^{-1}, \quad \vec{K} \equiv \beta'\vec{k}_1 - \beta\vec{k}_2. \end{aligned} \tag{6}$$