SCATTERING THEORY OF SMALL-PARTICLE ABSORPTION: CYCLOTRON AND MAGNETOPLASMA RESONANCE*

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The excitation of a small ellipsoidal sample in an electromagnetic cavity is treated quantitatively as a two-wave Rayleigh-scattering problem. Application to the special case of a solid-state magnetoplasma shows that magnetoplasma resonance occurs when $\kappa' = -(1/L-1)$, with strength proportional to $1/\kappa''$, and approximate half-width κ'', where $\kappa = \kappa' + i\kappa''$ is the net bulk dielectric constant of the material comprising the ellipsoid, and L is the mks depolarization factor. New methods of deducing transport parameters from magnetoplasma resonance are outlined.

Magnetoplasma resonance in a solid was first reported by Dresselhaus, Kip, and Kittel[1] (DKK) as a complication in microwave cyclotron-resonance experiments on small disks of n-type Ge and Si. The problem is that as carrier concentration exceeds a rather small value, the cyclotron-resonance line is first shifted, then replaced, by magnetoplasma effects. These effects must be clearly understood and corrected for if accurate effective masses are to be deduced. DKK[1,2] introduced a treatment known as the depolarization method, which proceeds by modifying the equations of motion of the carriers, to take into account the uniform depolarization fields set up inside the sample by carriers "accumulating" on its surfaces. This leads to an effective small-sample conductivity used to compute observed absorption. The method has been applied to two-carrier systems by Michel and Rosenblum[3] (MR) and generalized by Lax and Mavroides[4] to include tensor effective masses. One disadvantage of this calculational procedure is the implication that depolarization effects are connected with changes in the fundamental dynamics of the microscopic systems comprising the sample, so that knowledge of intrinsic constitutive parameters is of little use in interpreting experiment. Also, it is difficult in practice to acquire insight into systems described this way, especially when there are two or more carrier types, as in many solids of potential interest.

In this Letter we present an alternative treatment which restores the utility of bulk sample conductivity, automatically includes depolarization effects, results in a transparent formula for line shape, and yields the remarkably simple result that magnetoplasma resonance in a small ellipsoid occurs simply when the net bulk dielectric constant approaches $-(1/L-1)$, where L is the (MKS) depolarization factor, $\frac{1}{3}$ for a sphere. Several new relations are deduced which may prove useful in determining transport parameters from magnetoplasma-resonance data.

In the typical experiment, a small sample is placed on the center line of a rectangular or circular cavity at a position of maximum electric field, $\vec{E}_{MW}$, which is perpendicular to the applied dc magnetic field $\vec{B}$. We shall assume the sample is an ellipsoid of revolution about $\vec{B}$, with volume $V = \frac{4}{3}\pi a^2 b$. Provided the sample is sufficiently small, and merely a perturbation on the cavity, the experiment can be treated as the simultaneous scattering of two oppositely directed transverse plane waves traveling along the direction of $\vec{B}$ with essentially the guided wavelength and properly chosen polarization and phase.

The power P extracted from these two waves can be calculated accurately if conditions for Rayleigh scattering[5] by a single wave are met. Consider first that $\vec{B} = 0$ and that the sample is made from isotropic material having complex permittivity ratio κ_e and complex permeability ratio κ_m. The necessary conditions for Rayleigh scattering are

$$|k_0 b| \ll 1, \tag{1a}$$

$$|k_i b| \ll 1, \tag{1b}$$

where k_0 and k_i are the complex propagation constants for transverse plane waves in the material outside the sample and inside it, respectively. In the isotropic case, as is well known,

$$k^2 = (\omega^2/c^2)\kappa_e \kappa_m. \tag{2}$$

When the Rayleigh conditions hold, each wave extracts power $\frac{1}{2}P$ proportional to the electric dipole extinction cross section

$$\Sigma(\text{Ellipsoid}) = \frac{|k_0|^3 V}{2\pi}\left(\frac{1}{L}\right)^2 \frac{\kappa_e''}{[\kappa_e' + \gamma_L]^2 + [\kappa_e'']^2}, \tag{3}$$

where $\gamma_L = 1/L - 1$, L is the depolarization factor transverse to the symmetry axis of the ellipsoid,[6] and $\kappa_e = \kappa_e' + i\kappa_e''$ is defined by $\kappa_e = \epsilon/\epsilon_0 + i\sigma/\omega\epsilon_0$. (We assume MKS units, $e^{-i\omega t}$ time dependence, and the familiar constitutive relations $\vec{D} = \epsilon\vec{E}$ and $\vec{J} = \sigma\vec{E}$.) Total power extracted from each wave is given by $\sum |\vec{S}|$, where $\vec{S}$ is the Poynting vector of the wave.

When the sample is placed at a position of maximum microwave magnetic field, P is again measured by 2Σ, except that κ_m replaces κ_e and the interaction is magnetic dipole in character, rather than electric. Thus, Eq. (3) can be used to study magnetic materials.

Equation (3) is easily applied to anisotropic materials by proper arrangement of experimental conditions to satisfy the definitive feature of Rayleigh scattering: that an essentially uniform external field [Eq. (1a)] produce an essentially uniform interior field [Eq. (1b)] of the same polarization type. This requires identification of the principal waves of the medium: plane waves which propagate with at least one field vector ($\vec{E}_p$ or $\vec{H}_p$) having a simple polarization state transverse to the propagation vector $\vec{k}_p$. Relation (3) applies provided (1) the cavity field at the sample has the same polarization state as the transverse vector of the principal wave, (2) κ_e is replaced by the permittivity or permeability associated with the transverse vector, (3) the Rayleigh conditions (1a) and (1b) hold using $|\vec{k}_i| = |\vec{k}_p|$, and (4) the symmetry axis of the ellipsoid coincides with the principal direction $\vec{k}_p$. When the exciting field is a superposition of principal transverse fields, cross sections associated with each type of principal wave are calculated separately and added, with weight factors derived from the decomposition of the exciting field . Erroneous shifts in resonance can be predicted if one of the cross sections is incorrectly calculated, say in violation of (1b).

The Rayleigh cross section (3) exhibits two kinds of resonances pertinent to discussion of cyclotron and magnetoplasma resonance. Briefly, the first type occurs when the denominator is essentially stationary and the numerator κ'' goes through a local maximum, as happens at the cyclotron-resonance magnetic field. The second type occurs when $\kappa' + \gamma_L$ passes through zero while κ'' is relatively stationary and not too large; this turns out to be the condition for uniform magnetoplasma resonance in the particle. Line shapes of the two types are clear: Cyclotron resonance has the shape of the peak in κ_e''; magnetoplasma resonance is Lorentzian in κ', to the extent that κ'' is essentially constant over the linewidth. This second type of resonance is centered at $\kappa' = -\gamma_L$, has strength proportional to $1/\kappa''$ evaluated at resonance, and has linewidth, in κ', approximated by $2\kappa''$ evaluated at resonance.

To demonstrate partially the validity of this analysis we apply it to p-type Ge, the two-component system studied by MR in a small ellipsoidal sample having $\gamma_L \simeq 1.4$. Figure 1 shows the variation with magnetic field of κ', the real part of the principal dielectric constant for circularly polarized waves, computed for various total hole concentrations. (The dashed curves are κ'', the imaginary part.) There are three independent contributions to the bulk dielectric constants shown: a constant lattice contribution κ_L of about 20, a light-hole contribution, and a heavy-hole contribution. The positive values of magnetic field correspond to one sense of circular polarization; the negative values, to the opposite sense. The hole contributions are calculated using a simple Drude model, the known effective-mass ratios 0.044 and 0.334, the heavy-to-light-hole concentration ratio 350, and reasonable values for the relaxation times of light and heavy holes ($\omega\tau = 5$ and 10, respectively).

At low hole concentrations κ' is mostly κ_L. This is typified in the uppermost curve, for 10^{12} holes/cm^3. The variation in κ' due to holes is barely observable and κ' certainly does not approach -1.4. Therefore, only the first type of resonance in Σ can be expected. In fact, MR observed two cyclotron resonances, precisely at the positions LH and HH where the (unplotted) peaks in the imaginary part κ'' occur.

The next lower curve shows what happens when the concentration is increased by about an order of magnitude to 2.4×10^{13} holes/cm^3. The hole contributions to κ' are now large enough that κ'

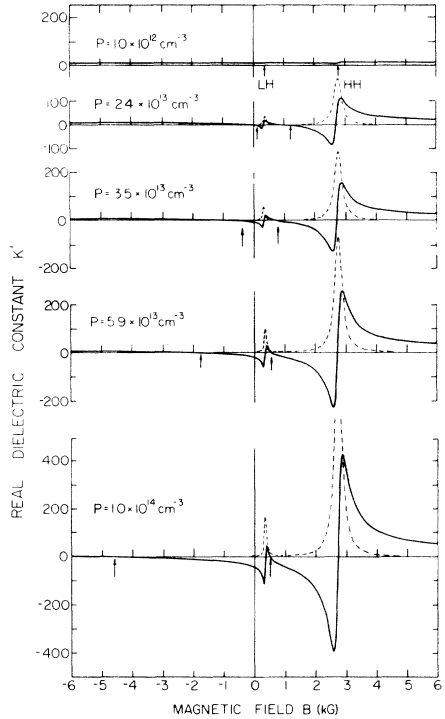

FIG. 1. Theoretical behavior of the real part κ' of the dielectric constant of p-type Ge, for 24-GHz waves circularly polarized transverse to the applied magnetic field. P is total hole concentration. The two arrows labeled LH and HH mark the light-hole and heavy-hole cyclotron resonance fields. The other arrows mark the fields at which $\kappa' = -1.4$, the condition derived in this paper for magnetoplasma resonance of a small ellipsoid. The dashed curves are κ'', the corresponding imaginary part.

passes through -1.4 at two fields, marked by arrows, where κ'' (dashed curve) is small. These fields are just those at which magnetoplasma resonance is observed experimentally. As concentration increases, down the figure, the -1.4 points both move toward negative field values. Inspection of the slope of κ' near $\kappa' = -1.4$ reveals that with increasing hole concentration the low-field resonance (in this case, 0.4 kG $\lesssim B_r \lesssim$ 1.0 kG) becomes less sensitive to small changes in particle shape (i.e., L) while the high-field resonance ($B_r \lesssim -1.0$ kG) becomes more sensitive.

Figure 2 shows how well the scattering theory predicts the fields B_r at which resonances are observed. The points are the data of Michel and Rosenblum while the lines are computed using the Rayleigh formula, Eq. (3). Cyclotron resonance is correctly predicted at low concentrations, as is the initial shift with increasing con-

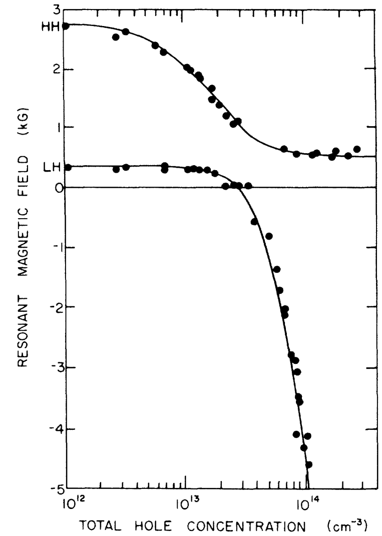

FIG. 2. Prediction of small-sample absorption maxima by Rayleigh extinction cross section. Points are the data of Michel and Rosenblum on p-type Ge. For hole concentrations above 3×10^{12} cm^{-3} the theory lines are given by $\kappa' = -1.4$, where κ' is the real part of the bulk dielectric constant of the material.

centration. Above 3×10^{12} holes/cm^{-3} magnetoplasma resonance dominates and the theory line follows from the very simple condition $\kappa' = -1.4$. The lower curves of the previous figure illustrate why the upper resonance of Fig. 2 asymptotically approaches a field slightly greater than the light-hole cyclotron-resonance field lh. The observation by MR that increasing hole concentration drastically weakens the upper resonance of Fig. 2 relative to the lower is now easily explained: The upper resonance is progressively muted because κ'' becomes so large at B_r (see dashed curve at bottom of Fig. 1) that conditions for a prominent resonance of the second type are no longer met. We note that at higher concentrations the second Rayleigh condition [Eq. (1b)] is violated for magnetic fields near the maxima in κ''; this means that the uniform fields assumed in depolarization analysis do not really exist.

Application of this scattering analysis to a one-component system reveals that in many cases simple features of magnetoplasma-resonance data directly yield transport parameters, including carrier concentration, effective mass m^*, mobility μ, and lattice dielectric constant κ_L. The basic condition is that the resonance line be well removed from the maxima in κ_e' and κ_e'', which, for a system of electrons, takes the form $B+B_c \gg 1/\mu$, where $B_c \equiv -\omega m^*/e$ is the cyclotron-resonance field. Assuming this condition the Drude dielectric constants[7] are well approximated by $\kappa' \simeq \kappa_L - ne/\omega\epsilon_0(B+B_c)$ and $\kappa'' \simeq ne/\omega e_0\mu(B+B_c)^2$. Then, provided $ne\mu/\epsilon_0\omega \gg \kappa_L$, magnetoplasma resonance in a sphere is expected at B_r given by

$$B_r = -B_c + ne/\omega\epsilon_0(\kappa_L+2), \qquad (4)$$

a relation independent of mobility μ. Evidently, a plot of experimental B_r vs n will yield a straight line whose slope gives κ_L and whose projected field-intercept $-B_c$ gives effective mass m^*. Remembering that the line is Lorentzian in κ', we compute the half-power fields $B_\pm$ from the relation $\kappa'(B_\pm) = \kappa'(B_r) \pm \kappa''(B_r)$ and find

$$\text{Linewidth} \equiv B_+ - B_- = 2/\mu. \qquad (5)$$

Thus, magnetoplasma-resonance linewidth is a direct measure of mobility, independent of all the other parameters, when $B_r + B_c \gg 1/\mu$. We further find that the cross section at resonance $\Sigma(B_r)$ obeys the relation

$$\frac{\Sigma(B_r)}{6|k_0a|^3} = \frac{ne\mu}{\epsilon_0\omega(\kappa_L+2)^2} = \frac{\mu(B_r+B_c)}{\kappa_L+2}, \qquad (6)$$

which should prove useful in testing consistency of values obtained from (4) and (5), or in studying the product $n\mu$. We note that the necessary conditions $B+B_c \gg 1/\mu$ and $ne\mu/\epsilon_0\omega \gg \kappa_L$ are frequently attainable in moderate- to high-mobility materials. These simple properties of magnetoplasma resonance are apparently new and have not been exploited experimentally.

The scattering formalism is being expanded to include the dimensional resonances of a magnetoplasma sphere.

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