OBSERVATIONS OF ENHANCED RESISTIVITY IN THE WAVE FRONT OF A LASER-PRODUCED PLASMA INTERACTING WITH A MAGNETIC FIELD*

F. Schwirzke
Naval Postgraduate School, Monterey, California 93940

and

R. Tuckfield
Gulf General Atomics Incorporated, San Diego, California
(Received 23 September 1968; revised manuscript received 24 March 1969)

A laser-produced plasma has been used in a new approach to the problem of investigating the mechanisms associated with the turbulent energy dissipation is a collisionless laboratory plasma. The measurements indicate that the two-stream instability enhances the resistivity in the wave front by more than two orders of magnitude and thus determines the width of the wave front to a value of about $d \approx 13c/\omega_{pe}$.

The creation of a dense high-temperature plasma by means of a giant-pulse laser is of prime interest to the field of plasma physics. Such a hot plasma which expands across a magnetic field also uncovers new extensive possibilities for the investigation of instabilities and collisionless damping mechanisms in the wave front. The expansion of a laser-produced plasma in a magnetic field has been discussed by Bhadra.[1] It has been shown[2] that the initial expansion phase of the $\beta \gg 1$ plasma can be described by the hydrodynamic theory of Dawson.[3] The plasma temperature T decreases rapidly with increasing radius and most of the energy content of the plasma shows up as kinetic expansion energy. Obviously, the next problem is then to investigate the interaction of the expanding plasma with a confining magnetic field. This interaction is of special importance if the purpose is to confine a thermalized hot plasma in a stable magnetic-field configuration. Surface currents in the wave front can thermalize the expansion energy, again by resistive effects. An expanding plasma of infinite conductivity would repeatedly bounce off the compressed B where $\beta \approx 1$. With finite resistivity, diffusion across B will also occur. Hence, the bounce radius and the characteristic width for pressure gradient in the wave front are determined by the effective resistivity. If the density in the spherically expanding plasma has been reduced such that the collision frequency becomes small, instabilities may develop in the front and contribute effectively to the energy dissipating process. The problem of stability within the wave front of a steady-state solitary wave propagating across the magnetic field has been analyzed by Sagdeev.[4]

The two-stream instability will start where the current-associated mean ordered velocity of the electrons in the wave front becomes larger than the mean thermal velocity. The current is in the direction perpendicular to B and to the normal of the wave front. The growth time of the two-stream instability, with the growth rate $\gamma \approx \omega_{pe} \times (m/M_i)^{1/3}$, is rather small in comparison with the propagation time of the wave which is on the order of $\tau \approx 10(c/\omega_{pe}V_A)$, where V_A is the Alfvén velocity and $d \approx 10c/\omega_{pe}$ is of the order of the characteristic width of the front when the resistivity of the plasma is determined by the presence of the instability. ω_{pe} is the plasma frequency, c the velocity of light, m and M_i the electron and ion masses. If $\gamma\tau \gg 1$ a steady state will exist within the wave front and the onset of the instability can be studied locally by considering the perturbed motion of the ions and electrons by means of the equations of the two-fluid theory. The oscillation frequency of the perturbed motion of the electrons is of the order ω_{pe}. In the considered density range $n_e \gtrsim 10^{12}$ cm^{-3} and with $B \lesssim 1500$ G we have $\omega_{pe} \gg \omega_{ce}$, where ω_{ce} is the electron cyclotron frequency. Hence, the effect of the magnetic field on the perturbed motion can be neglected. The wavelength of the high-frequency disturbance is characteristically much smaller than the radius of the plasma. Under these circumstances the perturbation can be described as a plane wave which propagates in θ direction, perpendicular to B and perpendicular to r. The ions are considered at rest in the reference frame which moves with the speed of the wave front. Assuming a small perturbation v_e the electron velocity can be written $V_e = V_{e\theta}(r) + v_e$, where $V_{e\theta}(r)$ is the unperturbed mean velocity of the electrons in θ direction. The density and potential perturbations, n_e and

φ, are defined by similar expressions. Thus the linearized equations of motion become

$$i(\omega+kV_{e\theta})n_e v_e = \nabla e\varphi - \nabla(T_e/n_0)n_e,$$

$$i\omega M_i v_i = -\nabla e\varphi.$$

$n_0 = n_0(r)$ is the unperturbed r-dependent plasma density in the wave front. The electron continuity equation becomes

$$i(\omega+kV_{e\theta})n_e + ikn_0 v_{e\theta} + (d/dr)(n_0 v_{er}) = 0.$$

Assuming negligible unperturbed motion of the ions in θ direction, $V_{i\theta}=0$, the continuity equation for the ions takes the form

$$i\omega n_i + ikn_0 v_{i\theta} + (d/dr)(n_0 V_{ir}) = 0,$$

and Poisson's equation becomes

$$4\pi(n_i - n_e) = k^2\varphi - d^2\varphi/dr^2.$$

Since the characteristic length of the high-frequency perturbation is smaller than the width of the wave front, $dn_0/dr \ll dn_e/dr$, v_e and φ can be eliminated to yield an equation for the perturbed electron density n_e:

$$\frac{T_e}{m}\frac{d^2 n_e}{dr^2} + \left[(\omega+kV_{e\theta})^2 - \frac{T_e}{m}k^2 - \frac{\omega_{pe}^2}{1-(\omega_{pi}/\omega)^2}\right]n_e = 0.$$

$n_e(r)$ shows an oscillatory structure if the expression in the brackets becomes positive. Considering a stationary profile propagating across the magnetic field, which is common to shock theory, the current-associated electron velocity $V_{e\theta}$ will have a maximum where the pressure gradient is steepest (Fig. 1). At the turning point $d^2n/dr^2 = 0$. If the wavelength of the disturbance is larger than the Debye length, $1/k \gg \lambda_D$, then the frequency of the unstable mode of oscillation $\omega < \omega_{pe}$ and the dispersion relation can be written approximately as[4]

$$\omega^2 \approx \omega_{pi}^2 k^2 (T_e/m - V_{e\theta}^2) \times[\omega_{pe}^2 - k^2(V_{e\theta}^2 - T_e/m)]^{-1}.$$

ω becomes imaginary and the two-stream instability appears if there is a region inside the wave front in which $V_{e\theta}^2 > T_e/m$. If B is oriented

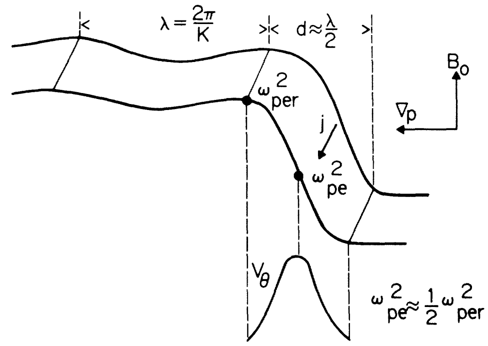

FIG. 1. Structure of the wave front (schematic).

in z direction and the steady-state wave front propagates in the r direction across the magnetic field, the unperturbed current is given by $j = -enV_{e\theta} = (c/B)dp_e/dr$. Hence, $V_{e\theta} = -(c/4\pi me) \times dB/dr$. Substituting $\omega_{pe}^2 = 4ne^2/m$ and $K = |(1/B) \times dB/dr|$, where K is inversely proportional to the width of the front d, the criterion for the onset of the two-stream instability becomes

$$d < (\pi c/\omega_{pe})(2/\beta)^{1/2}, \tag{1}$$

where $\beta = 8\pi nT_e/B^2$. Criterion (1) will be satisfied at a certain radius since n and T_e decrease rapidly during the expansion of the laser-produced plasma: $n_r \approx 3N/4\pi r^3$, $T_r \approx T_0(r_0/r)^2$. N is the total number of target particles which have been ionized by the giant-laser pulse. T_0 and r_0 are the temperature and radius of the plasma when it ceases to absorb laser energy. After the initial sharp drop of the temperature due to the adiabatic expansion the temperature increases again due to the resistive energy dissipation by the surface currents in the front. If we want to compare the instability criterion (1) with the measured width d of the front of the expanding plasma, we have to introduce the ω_{pe} and β values which exist within the wave front where the pressure gradient has a maximum. From Fig. 1 we can assume that at the maximum of the pressure gradient $\omega_{pe}^2 \approx \frac{1}{2}\omega_{per}^2$ and $\beta \approx \frac{1}{2}\beta_r$, where ω_{per}, β_r, and d are the quantities which can be determined rather easily in the experiment at a given radius. Thus the criterion for instability becomes

$$d < 2\pi(2/\beta_r)^{1/2}(c/\omega_{per}). \tag{2}$$

The laser used was a Korad K2Q system with a neodymium-doped glass rod and a Pockels cell. A 150-MW 20-nsec pulse was focused on the tip of a Pyrex fiber of 30 μ diam by means of a 10-cm lens producing an image size of ~300-μ full width at half-maximum intensity. Each laser pulse was monitored by placing a Fresnel plate in the beam and reflecting ~2% of the total power of the output onto a magnesium oxide diffuser, which was viewed by a photodiode, as shown in Fig. 2. Both energy and power could be monitored by this means on a Tektronix 519 scope.

Alignment of the target at the focal point of the lens on the optic axis was accomplished by means of a small cw laser and a referenced telescope. By placing an exposed Polaroid print at the downstream vacuum-box window, the resulting burn pattern gave verification of the correct target position. The total number of target atoms exposed to the laser pulse can be calculated from the evaporated length of the fiber. Typically, about 1 J was absorbed by $N = 10^{16}$ target particles. The magnetic-field coils produced a dc mirror field which could be varied from 0 to 1500 G in the central region of the target with a mirror ratio of 2 to 1.

Diagnostics used to measure the plasma produced were fixed diamagnetic loops on the axis; a 1-mm-diam movable magnetic probe; an image-converter framing camera; and a photomultiplier for viewing the total light. The exit beam was analyzed by placing a magnesium oxide diffuser and photodiode arrangement at the downstream exit window in the same way that the beam was analyzed at the upstream end. Charged-particle detectors were placed at various distances from the target orthogonal to the beam and target and also downstream to measure expansion velocities and ion-saturation currents. The velocity of the expanding plasma (10^7-10^8 cm/sec) depends on the absorbed laser energy. The observed velocities are comparable with the ones obtained in θ pinches which have been used so far exclusively to study collisionless mechanisms in a laboratory plasma. Also a Space Technology Laboratories image-converter camera viewed the expanding plasma.[2] The photographs showed that for fibers smaller in diameter than about 100 μ the initial expansion of the plasma was spherical.

Figure 3 shows oscilloscope traces of the total light emitted from the plasma and magnetic signals at different radii. The upper traces are from the photomultiplier showing the time behavior of the light emission. The characteristic time scale for three-body volume recombination is, in the density and temperature range considered, $R > 0.5$ cm, much larger than the duration of the second light pulse. The radial dependence of the magnetic signal was measured by means of a 1-mm-diam magnetic probe. At $R = 0.5$ the diamagnetic probe signal (recorded in positive, upwards direction) corresponds to $\beta \geqslant 1$. An increase of B is observed at $R = 1$, 1.5, and 2 cm before the diamagnetic signal is recorded, i.e., the expanding plasma compresses B like a piston and thus produces an increase of the magnetic field. At $R = 1.5$ cm a steep front exists as indicated in the sudden increase of the diamagnetic signal. Assuming a homogeneous distribution of

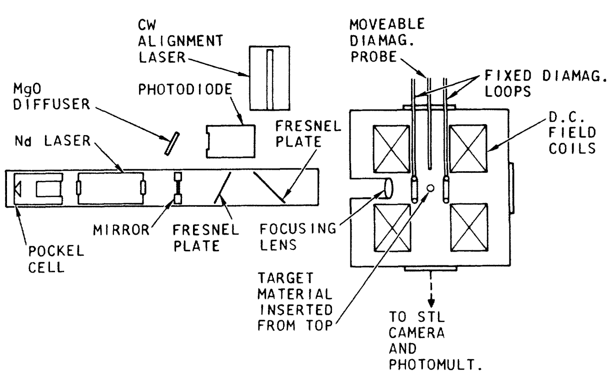

FIG. 2. Experiment with 200-MW Nd-laser system.

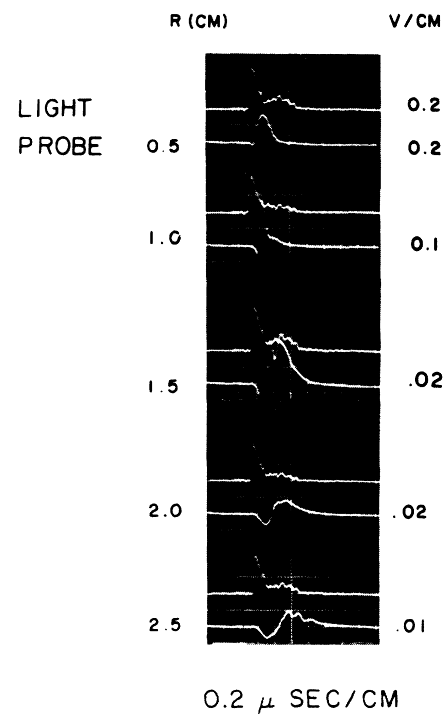

FIG. 3. Movable magnetic probe and total light photomultiplier signals at various radii, $B = 1500$ G.

$N=10^{16}$ electrons and an ionization level of $Z=1$ over a sphere of $R=1.5$ cm, the density would be $n_\gamma=7\times10^{14}$ cm^{-3}. In reality a density profile exists and a reasonable estimate gives $n_\gamma\sim3\times10^{14}$ cm^{-3} and $\omega_{per}\approx10^{12}$. Introducing these values into criterion (2) we find that the two-stream instability should occur if the observed width of the front becomes $d<0.46$ cm. From the measured expansion velocity $V=10^7$ cm/sec and the rise time of the diamagnetic signal in Fig. 3 we observe a width of $d=0.4$ cm $=13.3c/\omega_{per}$, i.e., the criterion for instability is fulfilled.

That a collisionless mechanism is contributing to the energy dissipation process in the wave front is indicated by the discrepancy in the determination of the temperature from the magnetic probe signal and from the effective conductivity which can be estimated from the measured width of the front. From the observed magnetic probe signal, $\beta_\gamma=8\pi n_\gamma T/B^2=0.33$, with $n_\gamma\approx3\times10^{14}$ cm^{-3}, follows a temperature of $T_{obs}\approx31$ eV. On the other hand, the width of the front is determined by the effective conductivity σ. The distance for which B penetrates into the plasma is given by the magnetic Reynolds number $R_M=4\pi V\sigma d/c^2=1$. Hence, σ can be determined[5] from the measured expansion velocity $V=10^7$ cm/sec and $d=0.4$ cm:

$$\sigma_{eff}=c^2/4\pi Vd=1.8\times10^{13}\ \text{sec}^{-1}.$$

The uncertainty in the experimentally measured V, d, and the β signal is about ±15%.

We can compare this value of σ_{eff} with Spitzer's formula[6] which is based on the Coulomb collision frequency only,

$$\sigma=\frac{2(2T)^{3/2}}{\pi^{3/2}m^{1/2}Ze^2c^2\ln\Lambda}.$$

Assuming a Coulomb logarithm of $\ln\Lambda\approx10$ the measured conductivity σ_{eff} appears to correspond to an effective temperature of $T_{eff}\approx1$ eV only. In conclusion, the observed conductivity is about 170 times smaller than the "collisional" one based on the temperature of $T\approx31$ eV,

$$\frac{\sigma_{eff}}{\sigma_{coll}}=\left(\frac{T_{eff}}{T_{obs}}\right)^{3/2}\approx\frac{1}{170}.$$

This result strongly suggests that the two-stream instability contributes to the effective resistivity and thus influences the width of the wave front. It should be noted that the observed ratio $\sigma_{eff}/\sigma_{coll}$ is within the expected possible range. The influence of the instability on the conductivity $\sigma=ne^2/m\nu$ can be estimated by assuming that in the turbulent plasma state the effective ν is of the order of the growth rate of the instability, $\nu_{eff}\lesssim\gamma$. Hence, the range of possible values is given by $1>\sigma_{eff}/\sigma_{coll}=\nu_{coll}/\nu_{eff}\gtrsim\nu_{coll}/\gamma$ where in the example considered $\nu_{coll}/\gamma\gtrsim1/790$.

---

*Research at the Naval Postgraduate School partially supported by the Naval Air Systems Commands and the Air Force Office of Scientific Research.

[1]D. K. Bhadra, Phys. Fluids 11, 234 (1960).

[2]R. G. Tuckfield and F. Schwirzke, Plasma Phys. 11, 11 (1969).

[3]J. M. Dawson Phys. Fluids 7, 981 (1964).

[4]R. Z. Sagdeev, in Reviews of Plasma Physics, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4.

[5]R. Kh. Kurtmullaev et al., in Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Culham, England, 1965 (International Atomic Energy Agency, Vienna, Austria, 1966), Paper No. CN-21/218.

[6]L. Spitzer, Jr., Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1962), 2nd ed.

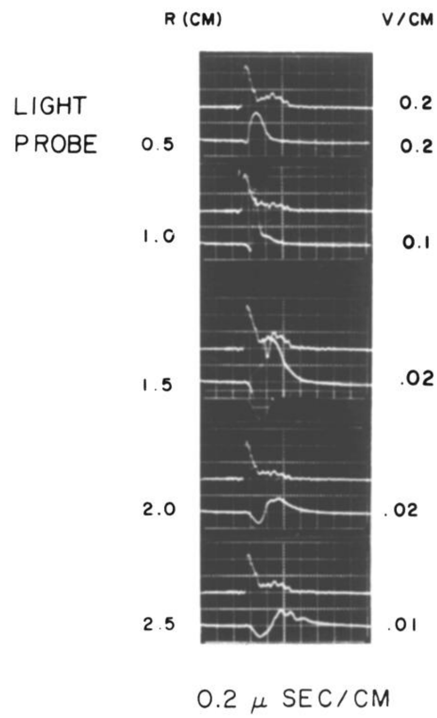

FIG. 3. Movable magnetic probe and total light photomultiplier signals at various radii, $B = 1500$ G.