SCATTERING OF POSITIVE IONS BY ELEMENTARY EXCITATIONS IN SUPERFLUID HELIUM*

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Measurements of the positive-ion mobility in He II are reported, and are analyzed in terms of scattering by elementary excitations.

The purpose of this Letter is to report accurate measurements of the mobility of positive ions in He II, and to discuss these in terms of possible models for the interactions of the ions with phonons, rotons, and He^3 impurities. The equilibrium drift velocity v_D of the ions under the influence of a uniform electric field E was measured as a function of field strength and temperature T by methods which have been briefly described in a previous paper.[1] A typical plot of v_D vs E, taken at 0.5°K, is shown in Fig. 1. It is easily seen that the quantity $P'(E)=eE/v_D$ is the average momentum per centimeter transferred from the ion to the excitation gas by scattering processes. In the low-field limit, P' is independent of E and is related to the more commonly used mobility μ by $P'=e/\mu$. The points in Fig. 2 are our measured values of P' for T ranging from 1.001°K to 0.426°K. Temperature was determined by means of a Speer carbon resistor[2] which was calibrated against the He^3 vapor pressure down to 0.37°K. We estimate a probable deviation of ±0.001°K from the 1962 He^3 scale.[3] The absolute uncertainty of our values of P' is about ±1%. They differ from the earlier results of Reif and Meyer[4] by amounts ranging from a few percent near 1°K to 50% at 0.5°K.

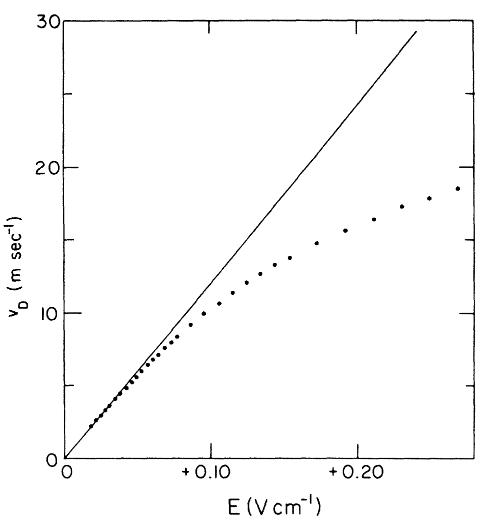

FIG. 1. Positive-ion equilibrium drift velocity v_D as a function of electric field E, at $T=0.5$°K.

The interpretation of P' in terms of the microscopic scattering processes has been shown by Baym, Barrera, and Pethick[5] to be elegantly simple. In the case of an elementary excitation $\vec{k}$ scattered by the ion with a differential scattering cross section $\sigma(k,\theta)$ and obeying $|\vec{k}'|\cong|\vec{k}|$, their result may be written

$$P'=-\frac{\hbar^2}{6\pi^2}\int_0^\infty k^4\frac{\partial n}{\partial\epsilon}v_g(k)\sigma_T(k)dk, \tag{1}$$

where n is the distribution function of the excitation under consideration, $v_g(k)$ is its group velocity, and $\sigma_T(k)$ is the momentum-transfer cross section defined by

$$\sigma_T(k)=\int(1-\cos\theta)\sigma(k,\theta)d\Omega. \tag{2}$$

Microscopic models of the interaction between the positive-ion probe and the elementary excitations in the superfluid may then be tested to some extent by calculating P' from Eqs. (1) and (2) and comparing the results with our data.

It has been shown by Baym, Barrera, and Pe-

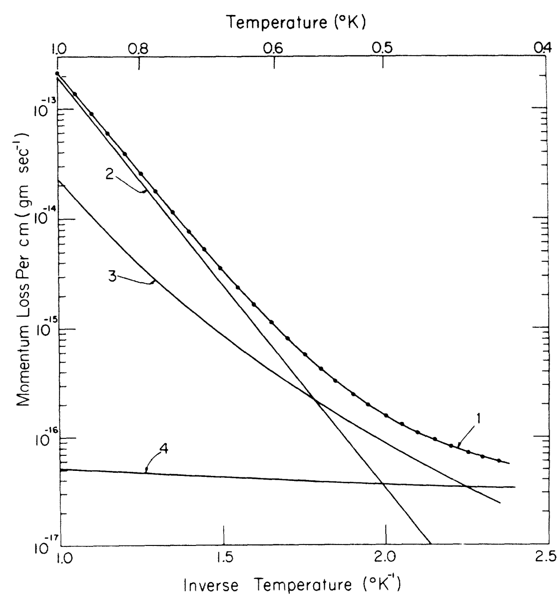

FIG. 2. The dots are our experimental points. Curve 1 is the best fit computed from Eq. (6), with curves 2, 3, and 4 showing the contributions of roton, phonon, and He^3 scattering, respectively.

thick[5] that treating the phonon-ion interaction as classical sound scattering by a deformable sphere provides a good explanation of the observed behavior of the negative-ion bubble below 0.5°K. The positive ion, which is expected to behave as a high-density object,[6,7] should however scatter sound like a classical hard sphere. By including s- and p-wave scattering we obtain in the long-wavelength limit

$$\sigma_T(k)=\frac{4\pi}{9}k^4a_+^{\,6}\frac{(11-4\alpha+2\alpha^2)}{(2+\alpha)^2}, \tag{3}$$

where a_+ is the effective radius of the positive ion and α = (mass of liquid displaced by ion)/(mass of ion). This yields from Eq. (1)

$$P_{\rm ph}'=\frac{2\times 8!}{27\pi}\zeta(8)\frac{(11-4\alpha+2\alpha^2)}{(2+\alpha)^2}\times\hbar\left(\frac{k_{\rm B}T}{\hbar s}\right)^8 a_+^{\,6}, \tag{4}$$

where ζ is the Riemann zeta function, $k_{\rm B}$ is the Boltzmann factor, and s is the velocity of sound in the liquid. From the effective mass measurements of Dahm and Sanders[7] one can estimate $\alpha=0.4\pm0.1$, and Eq. (4) becomes $P_{\rm ph}'=1.46\times10^{30}a_+^{\,6}T^8$.

For the He^3 impurities the obvious model leads to hard-sphere scattering. By treating the impurities as a Maxwell-Boltzmann gas, one then obtains a term $P_{\mathrm{He}^3}'=CT^{1/2}$. C is an unknown constant since in our experiment the natural He^3 concentration was not known. We do not expect this simple picture to provide a good description of the He^3 scattering processes, but it is adequate for our present purpose.

The most interesting case is the ion-roton interaction, since no detailed description of the scattering processes (nor indeed of the roton itself) are available. However, all that is needed to establish the temperature dependence of P_r' is the reasonable assumption that $\sigma_T(k)$ is a weak function of $k-k_0$. Because of the strong minimum at $k_0=1.91$ Å^{-1} in the roton energy spectrum Eq. (1) then yields to a good approximation

$$P_r'=\frac{\hbar k_0^{\,4}}{3\pi^2}\sigma_T(k_0)\exp(-\Delta/k_{\rm B}T)=0.474\times10^5\sigma_T(k_0)\exp(-\Delta/k_{\rm B}T), \tag{5}$$

where $\Delta/k_{\rm B}=8.65\pm0.04$°K is the roton energy gap derived from neutron scattering.[8]

Combining the scattering effects discussed above leads to the expression

$$P'=0.474\times10^5\sigma_T(k_0)\exp(-\Delta/k_{\rm B}T)+1.46\times10^{30}a_+^{\,6}T^8+CT^{1/2}, \tag{6}$$

which we fit to our data, using $\sigma_T(k_0)$, a_+, and C as variable parameters. The resulting best fit is in excellent agreement with the experimental data. This is shown by curve 1 in Fig. 2, with the separate roton, phonon, and He^3 contributions drawn in as curves 2, 3, and 4, respectively. Although the experimental points vary by four orders of magnitude over our temperature range, they all fall within 2% of curve 1, with a mean deviation of less than 1%. To get an idea of how sensitive this fit is to variations in the functional form of Eq. (6), we let Δ, the power of T in the phonon term, and the power of T in the He^3 term be variable parameters in turn. That is, we did four-parameter fits to our data using one of the above as our fourth variable parameter. The results were

$$\Delta/k_{\rm B}=8.67\pm0.04\text{°K}$$
$$P_{\rm ph}'\propto T^{7.5\pm1.0}$$
$$P_{\mathrm{He}^3}'\propto T^{1.0\pm0.7}. \tag{7}$$

Our experiment thus provides an independent measure of the roton energy gap which is in excellent agreement with the neutron-scattering result. One may also conclude that $\sigma_T(k)$ for rotons is indeed a weak function of $k-k_0$, as was assumed in deriving Eq. (5). The observed temperature dependence of the phonon term is in striking agreement with the T^8 behavior predicted by the Rayleigh scattering model. More work is clearly needed to accurately establish the temperature dependence of the He^3 scattering term, but one can conclude that it is quite weak, as expected.

One sees from the tests described above that Eq. (6) provides a satisfactory description of our observed temperature dependence. Fitting Eq. (6) to the data, of course, serves to determine the variable parameters. From this we obtain $a_+=5.0\pm0.1$ Å and $\sigma_T(k_0)=238\pm5$ Å^2. Although to some extent our value of the positive-ion radius must still be model dependent, it is probably more accurate than the previous estimates of 6.6 Å by Parks and Donnelly[9] and 5.8 Å by Dahm and Sanders.[7] Both of these values depend on some rather drastic assumptions, whereas our determination involves taking a sixth root and should hence be quite insensitive to details.

A satisfactory interpretation of $\sigma_T(k_0)$ probably requires a quantum-mechanical treatment of the ion-roton interaction. In lieu of such a detailed description, one may indulge in some interesting speculations of a more general nature. Since the roton de Broglie wavelength of ~3 Å is rather small compared with $[\sigma_T(k_0)/\pi]^{1/2}$ it is perhaps not a bad guess that the collision cross section is nearly geometrical. Although the roton carries a great deal of momentum and energy, its effective mass is much less than that of the ion. If we assume that the scattering is elastic, then $|\vec{k}'|=|\vec{k}|$ is satisfied. These considerations lead to a hard-sphere cross section $\sigma_T(k_0)\cong \pi(a_+ + a_r)^2$, where a_r is the effective collision radius of the roton. Using our phonon-scattering value of a_+ and the measured $\sigma_T(k_0)$, we then find $a_r = 3.7 \pm 0.2$ Å. This may be compared with the early estimate of $a_r = 4.0$ Å arrived at by Landau and Khalatnikov[10,11] from a calculation of the roton contribution to the normal-fluid viscosity which assumed hard-sphere roton-roton scattering. It is noteworthy that two such disparate approaches yield values of a_r which are in close agreement. One can surmise from this that the roton is localized within a region of radius ~3.7-4.0 Å, and that it interacts strongly with any disturbance which penetrates this region. It is worth pointing out that 3.7-4.0 Å is only slightly larger than the nearest-neighbor distance in liquid helium.[12] A roton might thus be pictured as a highly correlated motion of an energetic He^4 atom and its nearest neighbors only.

To sum up, it now appears that the experimental ion mobilities can be understood quantitatively in terms of quite simple models. The phonon-ion interaction has proved to be particularly amenable to such an approach, for both negative-[1-5] and positive-ion probes. While results for the roton-ion processes look promising, a more detailed theoretical description of the interactions is clearly needed. The He^3 terms still need to be investigated in detail, although some work in this direction has been done by Neeper and Meyer.[13]

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