like transformation. It is important for both relativistic and nonrelativistic statistical physics. It is contended that the results are essential for transport theory of superfluids when approximations are contemplated.

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PRESSURE-DRIVEN SUPERFLUID HELIUM FLOW

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From the Langer and Fisher thermal-fluctuation theory we have determined the pressure-velocity dependence for pressure-driven intrinsic superfluid helium flow to be $p \propto \exp(-\beta\rho_S/\rho v_S kT)$. Experiments of pressure-driven intrinsic flow through submicron pores in porous mica are in agreement with this relation.

Examinations of superfluid flow through a porous medium utilizing persistent currents in a superfluid gyroscope have shown the existence of an intrinsic critical superfluid-helium flow velocity[1] and the existence of an intrinsic energy-loss mechanism which permits the decay of these persistent currents.[2] In order to explain these two observations Langer and Fisher[3] (LF) have proposed a theory which permits the loss of energy from superfluid flow through the generation of excitations by thermal fluctuations. This description is at least functionally in agreement with experimental results. We have made measurements of superfluid flow, pressure driven through submicron pores, and find that it is possible to describe the properties of the flow over a wide range of parameters by an extension of the LF thermal-fluctuation description to our particular experimental arrangement.

In contrast to the decay of persistent superfluid currents flowing through small pores, pressure-driven flow reaches equilibrium when energy from the source compensates the energy loss from the superfluid flow. The balance between the rate of energy loss and rate of energy compensation is given by

$$V_T\rho_s v_s dv_s/dt = (\rho_s/\rho)pv_s A_T,$$

where p is the pressure across the pores, V_T is the total volume of the pores which have length l and total area A_T, $V_T = A_T l$, ρ_S is the superfluid density, and v_S the superfluid velocity. From this energy balance it follows that the pressure is given by $p = \rho l dv_S/dt$. If the mechanism for energy loss is due only to thermal excitations which cause 2π phase changes across the pores, we may use LF's expression[3]

$$dv_s/dt = (h/ml)V_p f_0 e^{-E_0/kT},$$

where V_p is the volume per pore, f_0 a basic fluctuation frequency, and E_0 the excitation energy. And in particular, if the excitation is assumed to be a vortex ring, then $E_0 \propto \beta\rho_S/\rho v_S$, where β may be a weak function of v_S and temperature T. These assumptions finally give for the equilibrium pressure-driven flow

$$p = \frac{h}{m}\rho V_p f_0 \exp\left(-\frac{\beta}{k}\frac{\rho_s}{\rho v_s T}\right).$$

Since the factors multiplying the exponential are independent of temperature, the fluctuation model predicts $v_S \propto \rho_S/T$ at constant p and $\ln p \propto 1/v_S$ at constant T.

The experimental apparatus (see Fig. 1) consisted of a copper can submerged in liquid helium which contained two identical coaxial capacitors — one open to the can, the other connected to the can only through porous mica.[4] Each capacitor was part of the tank circuit of an oscillator[5]; so the liquid-helium level in each capacitor was measured by monitoring the oscillator frequency. Flow measurements were made in two ways. For low pressure heads a small leak was established into the can and the pressure difference across the mica necessary for both capacitors

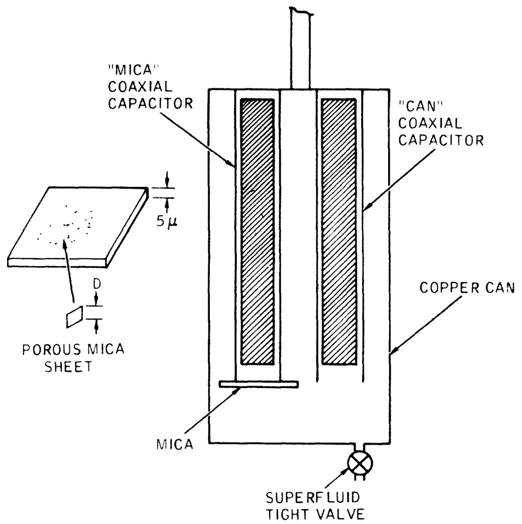

FIG. 1. Schematic of the experimental apparatus and characteristics of the porous mica.

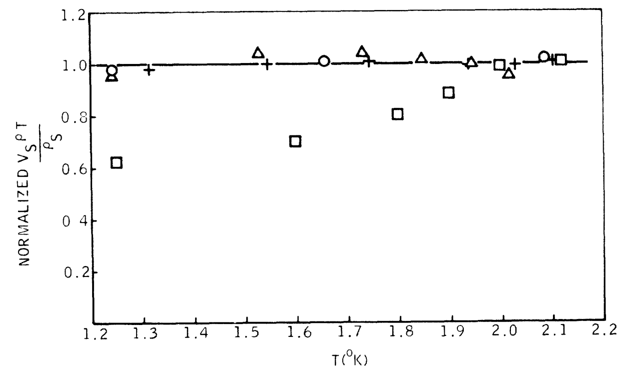

FIG. 2. Normalized $v_S \rho T/\rho_S$ as a function of temperature. Triangles, 100-Å pores at 10-cm helium head; crosses, 400-Å pores at 1500-μ helium head; circles, 800-Å pores at 1500-μ helium head; squares, 2000-Å pores at 10-cm helium head.

to fill at the same rate was determined. By varying the leak into the can it was possible to determine the superfluid-velocity dependence on pressure. For high pressure heads the can was partially filled and the pressure difference across the mica was monitored as the mica capacitor filled through the mica. The temperature characteristics of the system were monitored, differential temperatures to 5 ndeg and absolute temperatures to a microdegree, and the flow data were corrected for superfluid-flow-induced thermal effects. The measurements were made as a function of pore size down to 200 Å.

Measurements of the temperature dependence of the superfluid flow velocity at constant pressure have been made with 1-μ resolution from 1.3°K to the λ point for from 0 to 10^5-μ helium head. The normal-fluid flow under these pressure-driven conditions was found experimentally to be negligible except when the λ point was approached at the highest pressure head (i.e., 4% effect at 20 mdeg below the λ point under 10^5-μ head for 800-Å pores). Within experimental accuracy we found, for all pore sizes less than 2000 Å and for all pressures, $v_S \propto \rho_S/T$ over the entire temperature range as anticipated from the fluctuation theory. The measurements were made in two distinct temperature ranges, 1.3-2.1°K and 2.1°K-λ point, because the change in flow rate over the entire temperature range overwhelmed the sensitivity of the apparatus. Figure 2 shows normalized data from 1.3 to 2.1°K for various pore sizes at constant pressure. For 2000-Å pores the temperature dependence of v_S agreed with the ρ_S/T dependence of the smaller pores from the λ point down to only 2.0°K. Below this temperature the measured v_S deviated to a weaker temperature dependence. As the pore size was increased the deviation occurred at higher temperatures in qualitative agreement with Clow and Reppy. Under these conditions some geometry-dependent mechanism begins to affect v_S.

Between 2.1°K and a half millidegree below the λ point, the $1/T$ variation is too small to affect the experimental results and the velocity is essentially proportional to ρ_S, if ρ_S is measured from the depressed λ point.[6] From these high-temperature data we have estimated the magnitude of the velocity at 1-μ head. For these experiments the number of pores in the mica could be determined to 10%, while their spread in size was about 20%. Although considerable precautions were taken, the pores nevertheless slowly filled with contaminants (such as water vapor) during the experimental preparation. Thus the final number of open pores was usually less precisely known. However by assuming all the pores were effective, it was possible to place a lower limit on v_S from the measured volume flow. This estimate gives v_S(cm/sec) $=460\rho_S/\rho T$ for intrinsic superfluid-helium flow through 800-Å pores from 1.3°K to the λ point at 1-μ helium head. Hence we have found for our mica pore system that we observe intrinsic superfluid flow ($v_S \propto \rho_S/T$ at constant p) from 1.3°K to the λ point, from (0 ± 1)-μ to 10^5-μ helium head in all pore sizes from less than 2000 Å down to 200 Å.

In order to determine the pressure dependence

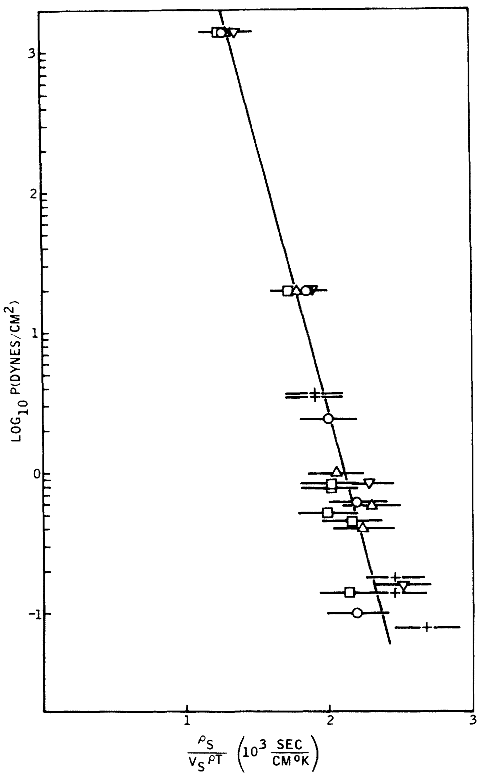

FIG. 3. Drive pressure versus $\rho_S/v_S\rho T$ for 800-Å pores. Crosses, 1.268°K; circles, 1.527°K; triangles, 1.789°K; squares, 1.966°K; inverted triangles, 2.091°K. The solid-line fit to the data is $p = 2\times10^8 \exp(-9.1 \times 10^3 \rho_S/v_S\rho T)$.

of v_S for this intrinsic flow, p vs v_S for 800-Å pores was systematically examined at several temperatures from 1.3 to 2.1°K. As shown in Fig. 3, the measurements were in agreement with the $\ln p$ dependence of $1/v_S$ expected from our extension of the LF theory with $f_0 = 5\times10^{27}$ and $\beta = 1.3\times10^{-12}$. Consequently, we find excellent agreement with a thermal-excitation description which has the excitation energy depending on ρ_S/v_S, as does the energy of a vortex ring.

This determination of β compares favorably with that determined from the persistent current data under equivalent experimental conditions. In that case an effective β must be used since it is found v_S is not strictly proportional to ρ_S/T and $\beta_{\rm eff} = 2.2\times10^{-12}$. However, these two experimental determinations are in violent disagreement with the theoretical estimate of $\beta = 50\times10^{-12}$. In addition to this discrepancy the free vortex ring of the LF model becomes larger than the pore size near the λ point. This implies that if the microscopic description of vortex-ring generation is to be maintained, the properties of the vortex rings in a pore must be quantitatively (but not qualitatively) different from the properties of free vortex rings. However, some indications exist in this direction from vortex-ring calculations[7] in pores in which the excitation energy is decreased due to the walls.

In conclusion we find excellent functional-dependence agreement with the LF thermal-fluctuation description extended to pressure-driven superfluid flow using a vortex-ring microscopic model for the thermal excitations. However, the quantitative agreement is not good. But more careful consideration of the microscopic model of vortex rings in pores may take care of this disagreement.

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[4]Since it is possible to generate a known number of uniform pores of well-defined size and shape in mica, this porous mica provides an excellent system for controlled superfluid-flow experiments. The porous mica was produced following the techniques developed by the General Electric Corporation group [see P. B. Price et al., J. Appl. Phys. 33, 3047 (1962); R. L. Fleischer et al., Rev. Sci. Instr. 34, 510 (1963)]. The mica (see Fig. 1) was 5 μ thick with 10^3 to 10^7 pores depending on the particular flow experiment. The pores had a diamond-shaped cross section. We denote the pore size by the arm length D of the diamond, since the pore cross-sectional area was very close to the area of a circle of diameter D. The pore-size spread for any sample was about 20 % and there was a slight taper along the length of the pores.

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