to the asymptotic Regge behavior is inherently ambiguous.

We can, of course, think of still other possible definitions of "duality" so as to make duality a property of the Veneziano model. Unfortunately, these definitions amount to little more than saying that duality is a property "which the Veneziano model has." One such possible definition for a meromorphic amplitude of x and y which is Regge behaved in x in all directions but one (for finite y) is that it has duality if it is Regge behaved in y (for finite x) in all directions but one. With this definition, however, duality equals crossing symmetry together with Regge behavior and poles.

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[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2]See, e.g., Phys. Today 22, No. 3, 59 (1969).

[3]In any sector of the complex s plane that excludes a finite angle around the positive real axis.

[4]Examples of definitions of weak duality are found in R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968); W. R. Frazer, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 415; G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968), and Phys. Rev. 176, 2112 (1968); C. Goebel, Phys. Rev. Letters 21, 383 (1968).

[5]A short list of papers in which strong duality is either defined or implied (aside from Ref. 1) follows: C. Schmid, Phys. Rev. Letters 20, 689 (1968); H. Harari, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 195; C. Lovelace, Phys. Letters 28B, 265 (1968); N. N. Khuri, Phys. Rev. 176, 2026 (1968); H. Harari, Phys. Rev. Letters 20, 1395 (1968); J. Rosner, Phys. Rev. Letters 21, 950 (1968), and 22, 689 (1969).

[6]Veneziano (Ref. 1) has discussed modifications which make his amplitude approximately satisfy finite-energy sum rules.

[7]If the right-hand side of (3) is replaced by an arbitrary constant $c \geq 2$ then our argument will still go through, even though its form will appear a little more cumbersome. In fact, (3) or its replacement need really be true only asymptotically in order for our argument to work.

[8]Higher Transcendental Functions, edited by A. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 1, p. 114.

[9]Higher Transcendental Functions, edited by A. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 1, p. 77, Eq. (15). If $\mathrm{Re}(xb) > 0$ then A_1 and A_2 separately tend to infinity and their leading terms cancel, so that (4) remains.

[10]It can also be shown directly that in the region $0 < \epsilon \leq \arg x \leq 2\pi - \epsilon$, A_1 tends to zero relative to A_2; see T. M. MacRobert, Proc. Edinburgh Math. Soc. 42, 84 (1923).

[11]See R. P. Boas, Entire Functions (Academic Press, Inc., New York, 1954), Th 1.4.2.

[12]We hope to be able to return to this general problem in a future publication.

SELECTION RULE FOR P-WAVE NONLEPTONIC HYPERON DECAYS AS A CONSEQUENCE OF DUALITY

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A selection rule for p-wave amplitudes of nonleptonic hyperon decays is derived from the duality principle. It predicts $P(\Sigma_-^-) = 0$ and $P(\Omega^- \to \Xi\pi) = 0$.

In the present Letter we shall explore consequences of duality in nonleptonic decay amplitudes of the hyperons. Assuming that the duality should prevail in the nonleptonic decay amplitudes too, we shall derive a selection rule that forbids the p-wave decays of $\Sigma^- \to n\pi^-$, $\Omega^- \to \Xi^0\pi^-$, and $\Omega^- \to \Xi^-\pi^0$. Experimentally $P(\Sigma_-^-) = 0$ has been well established while the other predictions should be compared with future experiment.

The duality means that resonance poles in different channels are strongly correlated with each other in such a way that a total amplitude of, for instance, two-body scattering is obtained by summing up resonances in one or two channels out of three. All the poles in the other channel are produced dynamically as bound states. For those processes which lack resonances in one channel, like the elastic $\pi^+\pi^-$ scattering, one can construct a total amplitude in the pole approximation only with a series of poles in either of the two other channels. A single Veneziano-Lovelace representation may work there.[1,2] Then what happens for the processes which lack resonances in two channels out of three? One cannot find a representation incorporating the duality for these processes; a series of poles in one channel alone can never exist dynamically. It implies that those amplitudes are zero apart from the Pomeranchukon contribution. This is an important property derived immediately from the duality principle, upon which our selection rule rests.

Before going into details, we notice that there is no standard theory at all for the p-wave decays of the hyperons, and that a clean symmetry argument is unlikely to work there. So it seems to be worthwhile to explore a selection rule based upon dynamics. We shall treat the nonleptonic hyperon decays like a scattering of

$$S+B\rightarrow\pi+B$$

(where S denotes octet spurion and B, baryon) with the four-momentum of the spurion being zero. For the p-wave decays S is the strangeness-carrying member of a normal scalar octet with $J^{CP}=0^{++}$. This is a quasihadronic process like the photoproduction for which we have so far no evidence for a fixed singularity characteristic of weak amplitudes. Moreover, it is free from possible background contributions associated with the Pomeranchukon exchange. We therefore believe that all properties of the duality model for hadron scattering can be transfered to our amplitudes. We shall, of course, assume as usual that there is no exotic resonance like the $\underline{10}^*$ or $\underline{27}$ baryon resonances, or $\underline{10}$, $\underline{10}^*$ or $\underline{27}$ meson resonances.

For the process $S+B\rightarrow\pi+B$, the trajectories associated with the octet partners of the π and the A_1 (axial-vector meson) and their family trajectories may, at first sight, look to contribute in the t channel. But we shall show in the following that it is not quite right.

Since we are working with the duality among poles alone, we are interested only in three-body vertices. Effective couplings relevant to our process are written in tensor notation as

$$\sum_{i,j} G_w^{(ij)}(\bar{B}_i\lambda_6 B_j)+\text{H.c.},\tag{1}$$

$$\sum_i G_w^{(i)}(M_i\lambda_6 M_0)+\text{H.c.},\tag{2}$$

$$\sum_{i,j} G_s^{(i)}(\bar{B}_i M_0 B_j)+\text{H.c.},\tag{3}$$

where B_i and M_i stand for the baryon and meson multiplets, respectively, M_0 being the 0^- octet containing the π. It is so understood that the tensor products in the parentheses are contracted appropriately, and there are two distinct couplings (symmetric and antisymmetric) for each of the octet-octet baryon couplings. The first and second couplings are with the weak spurion while the last one is the strong Yukawa coupling. All space-time dependences are suppressed.

On the other hand, the medium-strong SU(3)-breaking interactions responsible for the Gell-Mann–Okubo mass splittings give rise likewise to similar couplings among which the following are relevant:

$$\sum_{i,j} G_{m-s}^{(ij)}(\bar{B}_i\lambda_8 B_j)+\text{H.c.},\tag{4}$$

$$\sum_i G_{m-s}^{(i)}(M_i\lambda_8 M_0)+\text{H.c.},\tag{5}$$

where $G_w \ll G_{m-s} < G_s$ is satsified. By rotating all the SU(3) multiplets around the λ_7 axis by very small angles of the order of G_w/G_{m-s},[3]

$$M_0\rightarrow M_0'=M_0+i\theta[F_7,M_0],\tag{6}$$

$$M_i\rightarrow M_i'=M_i+i\theta_i[F_7,M_i]\quad(i\neq 0),\tag{7}$$

$$B_i\rightarrow B_i'=B_i+i\theta[F_7,B_i],\tag{8}$$

where F_7 is the seventh generator of SU(3), one can transform away the coupling (2) into the coupling (4). Thus one succeeds in eliminating entirely the weak pole transitions between the mesons without inducing any new effective weak coupling relevant to our processes. Note that the rotation angle θ_i is different for each meson multiplet, but M_0 and all the baryon multiplets are rotated by the same angle θ. It is crucially important to keep in mind that under this rotation all the SU(3) multiplets given above remain the same; in other words the exotic channels are still exotic after this transformation because M_0 and B_i are rotated by the same angle.

The weak pole transitions between the baryons, however, do not get transformed away simultaneously unless the weak spurion falls in the same multiplet as the medium-strong spurion so that $G_w^{(ij)}/G_{m-s}^{(ij)}$ becomes universal not only for all the baryon multiplets but also the mesons and the baryons.[4] We have, therefore, found the suitable reference frame in the SU(3) space in which there is no meson exchange in the t channel but only the baryon exchange in the s and/or the u channel.

We are now in a position to derive a selection rule. Consider first $\Sigma^- \to n\pi^-$, or $S+\Sigma^- \to n+\pi^-$. There is no resonance either in the s channel or, owing to the argument given above, in the t channel. Then, whatever u-channel poles may be allowed by the quantum numbers, the duality implies that they do not couple with the $(\bar{S}n)$ system, thus forbidding this process. In other words, it is dynamically impossible that u-channel poles exist alone in the absence of poles in neither of the other channels. We therefore conclude that $P(\Sigma_-^-)=0$.

The selection rule applies to the $\Omega^- \to \Xi\pi$ decays, too, where there is no resonance either in the u channel or in the t channel. We thus predict that the p-wave amplitude of the $\Omega^- \to \Xi\pi$ decays should be vanishing. All the other p-wave decays are allowed since there are baryon resonances both in the s and u channels.

The present argument may be extended to the parity-nonconserving decay amplitudes. But the weak-spurion–meson couplings are not eliminated since there is no parity-nonconserving medium-strong SU(3) breaking into which they are to be transformed away by a rotation. Although the diagonal transitions between the ground-state octet baryons are SU(3) forbidden, there are a lot of nondiagonal transitions left. Therefore the duality derives no selection rule for the parity-nonconserving decays. It is not the duality selection rule that leads us to $S(\Sigma_+^+)=0$. It should be explained from different standpoints among which the soft-pion calculation looks most reasonable.

In summary, we have derived from the duality principle a selection rule for the p-wave amplitudes of the nonleptonic decays. The rule predicts $P(\Sigma_-^-)=0$ and $P(\Omega^- \to \Xi\pi)=0$. The former is in accord with experiment, and the latter is a solid prediction which will be tested with experiment in the not-too-distant future. A similar discussion was once made in the static Chew-Low model.[5]

I am thankful to H. Sugawara and N. Tokuda for stimulating conversations.

[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2]C. Lovelace, Phys. Letters 28B, 265 (1968).

[3]We are familiar with this kind of transformation in the p-wave nonleptonic decays. See B. W. Lee, Phys. Rev. 140, B152 (1965).

[4]The weak spurion originates in, for instance, the current ×current interactions like $V_\mu V_\mu + A_\mu A_\mu$ whether a contact interaction or an interaction mediated by an intermediate boson. In order that the weak and medium-strong spurions may fall in the same multiplet, both must have the same structure in origin. If this were the case, there would exist no p-wave decay whatsoever. See S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964). Experimentally the F/D ratios are rather close to each other for the diagonal transitions of the $\frac{1}{2}^+$ ground-state octet baryons, $F/D=-3.1$ for λ_8 and -2.3 for λ_6. Enhancement theory predicts in the impulse approximation to the perturbations that they must be equal apart from higher order symmetry breaking. However, the discrepancy is appreciable and seems to be larger than higher order effects. We know far less about the universality between the mesons and the baryons.

[5]See for example N. Tokuda, Progr. Theoret. Phys. (Kyoto) 41, 153 (1969).

MULTI-REGGE BARYON EXCHANGE AND CENTRAL INTERACTIONS*

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We find that the double differential cross section for pp inelastic collisions at high energy can be well reproduced over a range of two decades by a simple version of the multi-Regge model, in which baryon exchange is responsible for large momentum transfers to the final proton. The model is an asymptotic one, and we show predictions at 70 and 200 GeV/c.

The study of inelastic processes at high energy presents a twofold interest: On a purely phenomenological ground, these processes amount to roughly three-fourths of the total cross sections; in addition, it has been suggested for a long time that their contribution to the unitarity sum is essential for building the asymptotic behavior of two-body processes.[1] Recently, considerable attention has been given to the multi-Regge-model (MRM) which generates, through