PECULIARITIES OF THE PION-NUCLEAR INTERACTION*

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We argue that the observed attraction in π-nuclear interactions for $l \neq 0$ should become a repulsion for large nuclear radius. The critical radius depends on angular momentum.

The nonabsorptive π-nuclear interaction at low energy has two dominant features. (a) The interaction for s-wave pions is repulsive as exemplified by the negative $1s$ level shifts of π-mesonic atoms.[1,2] Since the coherent sum of the s-wave scattering lengths over the nucleons is close to zero for nuclei of zero isospin as required by soft-pion theorems, this repulsion arises mostly from the fluctuation scattering by the (polarized) nuclear medium, i.e., from the discrete nature of nuclear matter.[3] (b) For higher angular momenta the interaction is observed to be attractive, since the measured level shifts of $2p$, $3d$, and $4f$ states of π-mesonic atoms are positive. For a p-wave pion this is due to the elementary p-wave πN interaction: A coherent sum of the p-wave amplitudes gives attraction. The attractive p-wave interaction dominates contributions from higher πN waves also when the π-nuclear angular momentum $l > 1$: Even in very light elements the coherent sum of πN amplitudes with $l > 1$ is negligible compared with contributions in which the pion is in a p state with respect to individual nucleons.[4]

It is the purpose of the present Letter to point out that the observed attractive interaction for states with $l \neq 0$ may reverse into repulsion for heavier elements. The origin of the effect is basically the same as the one which makes the p-wave πN interaction dominate higher angular momentum states in the π-nuclear system. With increasing nuclear size the net π-nuclear interaction for p-wave (d-wave, etc.) pions is more and more dominated by the elementary s-wave πN interaction which is repulsive. The nuclear size for which the elementary p-wave attraction exactly balances the elementary s-wave repulsion (so that no net interaction occurs but absorption) varies as a function of the angular momentum state of the pion with respect to the nucleus. An estimate of the critical nuclear size is obtained in the following way:

The π-nuclear interaction strength is approximately given by the expectation value of πN scattering amplitudes on the individual nucleons in the nucleus. In practice only s- and p-wave πN scattering is of importance, as mentioned above. For a single nucleon and a plane wave

$$\exp\{i\vec{k}\cdot\vec{x}\} = 1 + \vec{k}\cdot\vec{x} + \cdots,$$

the interaction strength is simply

$$I_1 \simeq a_s + 3a_p k^2 + \cdots,$$

where a_s and a_p are πN s- and p-wave scattering lengths. For the bound wave function $\varphi(\vec{r})$ we have to take proper s- and p-state contributions for the individual nucleons. In the neighborhood of a point $\vec{r}$ this is immediately obtained by comparison of the plane-wave expansion above with the Taylor expansion of the wave function,

$$\varphi(\vec{r}+\vec{x}) \simeq \varphi(\vec{r}) + \vec{x}\cdot(\nabla\varphi)_{\vec{r}}.$$

With a nuclear density $\rho(\vec{r})$ and the average scattering lengths $\bar{a}_s$ and $\bar{a}_p$ the A-particle interaction strength is then

$$I_A \simeq \int[\bar{a}_s \varphi^2(\vec{r}) + 3\bar{a}_p(\nabla\varphi)^2]\rho(\vec{r})d\vec{r}.$$

The parameters $\bar{a}_s$ and $\bar{a}_p$ are to be interpreted as the effective πN scattering lengths in the nucleus: Fluctuation scattering by the discrete nuclear medium produces an effective field at each scatterer which causes $\bar{a}_s$ and $\bar{a}_p$ to differ from a simple coherent average of πN scattering lengths. Non-negligible contributions are also expected from virtual-pion absorption. (Real-pion absorption has no significant effect on the phenomenon we discuss here.)[2] Trivial kinematic binding effects in amplitudes and momenta must be included. It is obvious from the expression for the interaction strength I_A that the net interaction strength may be repulsive <u>or</u> attractive depending on the relative size of the integrals containing φ^2 and $(\nabla\varphi)^2$, since $\bar{a}_s$ and $\bar{a}_p$ have opposite signs. These integrals depend strongly on angular momentum and nuclear radius. A qualitative insight into this dependence is simply obtained by taking the pion wave function to be dominated by the centrifugal barrier over nuclear dimensions. Thus $\varphi(\vec{r}) \propto r^l Y_{lm}$ in this region. The interaction strength is then

$$I_A \propto \bar{a}_s \langle r^{2l} \rangle + 3\bar{a}_p l(2l+1)\langle r^{2(l-1)} \rangle.$$

This expression can be explicitly evaluated for a uniform nuclear-matter distribution with radius R. The critical radius R_l for pion angular momentum l for which the interaction changes sign is then given by

$$R_l^2 = -(3\bar{a}_p/\bar{a}_s)l(2l+3).$$

It is quite obvious that the higher the angular momentum, the larger the critical radius. In practice we expect the critical radius for $l > 2$ to occur for nonstable superheavy elements and this may be the case even for $l = 2$. The effective values for $\bar{a}_s$ and $\bar{a}_p$ which describe experiments are[2]

$$\bar{a}_s = -\{0.034 + [(N-Z)/A]0.092\}\mu^{-1},$$

$$\bar{a}_p = 0.06\mu^{-3}.$$

With these values and a neutron excess of 10% we have

$$R_l = 2.9[l(2l+3)]^{1/2} \text{ fm}.$$

We then find $R_1 = 6$ fm, $R_2 = 11$ fm, and $R_3 = 15$ fm. The neutron excess is not of negligible importance; it increases $|a_s|$ and therefore decreases the critical radius. These values for the critical radius are only approximate since both the pion wave function and the nuclear density distribution were oversimplified.

A detailed analysis of all the experimental energy shifts and absorption widths in pionic atoms[2] has been completed by two of us (M.K. and T.E.). We have used a nonlocal optical potential of the form suggested by multiple-scattering theory. The experimental data are in good agreement with the best-fit calculated values. With the parameters of the best fit we can predict the shifts in nuclei where no measurement yet exists. The calculated shift for the $2p$ state of $_{38}Sr^{88}$ is indeed repulsive. The ratio to the calculated width is -0.07.

In Fig. 1 we have plotted the ratio of nuclear-interaction shift to the nuclear-absorption broadening as deduced from experiments. This ratio provides a natural frame for the effect since most of the intrinsic Z dependence of the interaction shift is eliminated. The curve is consistent with the expected change from attractive to repulsive interaction. The crossover point should be around $Z = 36$ with an uncertainty of a few units in Z.

We emphasize that an interaction which changes from attraction to repulsion as a function of nuclear size is unusual in nuclear physics. Since the amplitude changes sign even in the Born approximation, the nature of the effect is completely different from the well-known change of sign of neutron scattering lengths at a nuclear size resonance (anomalous dispersion). A direct experimental confirmation of the phenomenon would therefore be very interesting since it would clearly display the nonlocal interaction structure, and since it would permit an accurate determination of the relative interaction components.

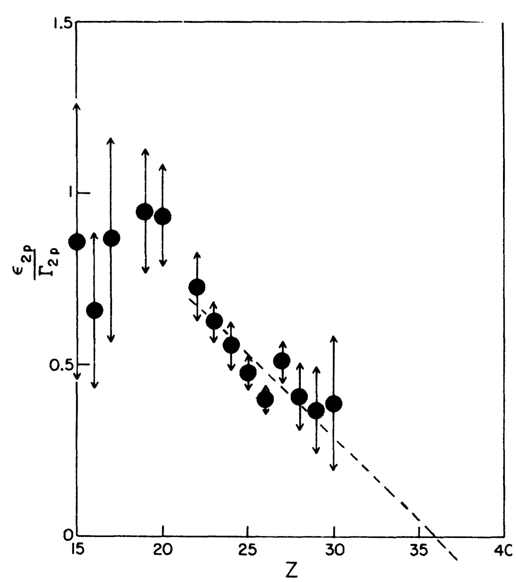

FIG. 1. Ratios $\epsilon_{2p}/\Gamma_{2p}$ of energy shifts to widths for $2p$ states versus charge number Z. Experimental energies and widths are from G. Poelz, H. Schmitt, L. Tauscher, G. Backenstoss, S. Charalambus, H. Daniel, and H. Koch, Phys. Letters <u>26B</u>, 331 (1968). The energy shift is the deviation of the $3d$-$2p$ transition energy from its electromagnetic point-nuclear value (including vacuum polarization).

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THREE-QUASIPARTICLE MULTIPLET IN Pr^{139}

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Nd^{139m} in its ϵ decay preferentially populates a multiplet of six high-lying, high-spin, odd-parity states in Pr^{139}. We interpret this as $(\pi d_{5/2})^2(\nu d_{3/2})^{-2}(\nu h_{11/2})^{-1} \rightarrow (\pi d_{5/2})(\nu d_{3/2})^{-1} \times (\nu h_{11/2})^{-1}$, with the Pr^{139} states being three-quasiparticle states.

Well-characterized three-particle states in nuclei are comparitively rare, and recognizing them most often has depended on the isomeric properties of a few high-spin states. Consequently, the excitation of a multiplet of such states in one nucleus, each state decaying to a number of lower lying states, has many interesting theoretical implications. Here we report our identification of the relatively unique population of a multiplet of six high-spin, negative-parity levels in Pr^{139} by the electron-capture decay of Nd^{139m}. We interpret this as the configuration of Nd^{139m} being peculiarly suited for populating three-quasiparticle states.

The two isomers of Nd^{139} follow the trend of $N = 79$ isomers, there being a 30-min $\frac{3}{2}^+$ ground state and a 5.5-h $\frac{11}{2}^-$ metastable state. Because the spacing between them is fairly small (231.2 keV) and the energy available for electron-capture decay is large (≈3 MeV), only 12.7% of the decay of the $\frac{11}{2}^-$ state proceeds via the $M4$ isomeric transition, and the two isomers decay almost independently. We prepared the isomers by the $Pr^{141}(p, 3n)Nd^{139m+g}$ reaction, using a 29-MeV proton beam from the Michigan State University sector-focused cyclotron, and have studied the γ-ray spectra following their decay with Ge(Li) and NaI(Tl) detectors in various singles, coincidence, and anticoincidence configurations. The resulting decay scheme of Nd^{139m} is given in Fig. 1; the details on how this was constructed and the decay scheme of Nd^{139g} will be presented in another publication.[1]

The decay of Nd^{139g} is more or less straightforward and much like the decay of many similar nuclei in this region,[2] which have most of their decay going directly to the ground state of the daughter nucleus. However, it can easily be seen

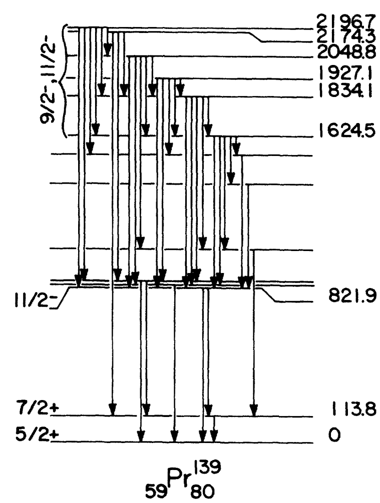

FIG. 1. Simplified level scheme of Pr^{139} showing the states that are populated in the decay of Nd^{139m} and the γ-ray transitions that de-excite them. Energies and spin-parity assignments are listed only for states pertinent to the present discussion. For more details about this scheme and its construction and for the Nd^{139g} decay scheme, see Ref. 1.