sik, edited by S. Flügge (Springer-Verlag, Berlin, Germany, 1966), Vol. 18, Pt. II.

[5]In the sense that the field changes in a finite time interval which is much smaller than the spin-wave period. On the other hand, it has to be larger enough to assure the validity of the magnetostatic approximation used to find Eq. (1).

[6]S. M. Rezende and N. Zagury, Phys. Letters 29A, 47 (1969).

[7]The properties of coherent states have been extensively studied by R. J. Glauber, Phys. Rev. 131, 2766 (1963).

MAGNON INTERACTIONS IN HEMATITE (α-Fe_2O_3) *

D. C. Herbert†
Department of Physics (Solid State Theory), Imperial College, London, S.W. 7, England
(Received 20 January 1969)

The main conclusions of a theoretical investigation of magnon-interaction effects in α-Fe_2O_3 are briefly reported. It is shown that the optic modes can have very low energy and it is suggested that these modes may play an important role in the mechanism of the Morin phase transition.

A group-theoretic approach to the problem of calculating magnon self-energies in complicated magnetic structures has recently been developed,[1] and this technique has been combined with the Dyson-Maleev boson mapping in an effort to elucidate the mechanism of the Morin phase transition in hematite. The main conclusions of this work are briefly reported here.

The generally accepted explanation of the Morin phase transition in α-Fe_2O_3 is that of Artman, Murphy, and Foner,[2] who used a molecular-field approach to calculate the temperature dependence of the effective c-axis anisotropy field. They found that small differences in the temperature dependence of the fine structure and dipolar anisotropies could change the sign of the effective anisotropy, thus causing a phase transition. Although their theory predicts a transition temperature which is in excellent agreement with experiment, there are other experimental results which it cannot explain. Foner and Williamson[3] have investigated the temperature dependence of the antiferromagnetic-resonance frequency, and they found that in the low-temperature phase the resonance disappeared abruptly at the transition temperature with a residual-mode energy of 28 kG. The anisotropy theory predicts that the acoustic modes are unstable, since the c-axis anisotropy is coupled to the exchange in the acoustic spin-wave energy, which should therefore tend to zero with the anisotropy.

Symmetry arguments indicate that the phase transition is first order,[4] so that in general metastable states are possible, and the disappearance of a mode energy corresponds to the limit of metastability.[5,6] The residual acoustic-mode energy might therefore be due to the effects of metastability but this seems unlikely, as typical residual mode energies for field-induced spin-flop transitions are of order 1 kG.[7] The curve of spin-flop field against temperature[8-10] also has some unexplained features. Near the Morin transition temperature, spin-flop fields of less than 28 kG have been observed, suggesting that the spin-flop and antiferromagnetic-resonance experiments detect different modes in this temperature region. The spin-flop field curve[8] also has an apparent kink at about 100°K below the transition temperature.

The free spin-wave energies have been calculated exactly at the center of the Brillouin zone ($k=0$) and at certain other symmetry points, in terms of a near-neighbor spin Hamiltonian, and the results at $k=0$ for the low-temperature phase are as follows:

$$(\lambda^{\mu})^2 = 2S^2|K^{\mu}|(j_1+j_2)-S^2[\tfrac{1}{4}(D^{\mu})^2-(K^{\mu})^2],$$

$$(\lambda^{\tau})^2 = 4S^2(j_1-j_3)(j_2-j_3)+2S^2|K^{\mu}|(j_1+j_2-2j_3) -S^2[\tfrac{1}{4}(D^{\tau})^2-(K^{\mu})^2],$$

where λ^{μ} and λ^{τ} are the degenerate acoustic- and optic-mode energies, respectively, $|K^{\mu}|$ is the magnitude of the c-axis anisotropy parameter, and D^{μ} and D^{τ} are antisymmetric spin-interaction parameters;

$$j_1=6J_4, \quad j_2=J_1+3J_3, \quad j_3=3J_2,$$

where J_1, J_2, J_3, and J_4 are the usual exchange parameters ordered according to increasing interatomic distance. Using molecular-field theo-

ry Bertaut[11] has given the following estimates for the exchange parameters:

$$j_1 + j_2 = 310°\text{K}, \quad j_3 = 147°\text{K}.$$

It can be seen from these values that the interaction between parallel spins (j_3) could make the pure exchange contribution to the optic-mode energies vanish, and these modes could then have very low energies. Although α-Fe_2O_3 is antiferromagnetic, parallel spins occur on near-neighbor sites, and the exchange interaction between these spins (j_3) has the wrong sign to favor stability. Since the optic modes at $k = 0$ have spins on parallel sublattices in antiphase, it is clear physically that j_3 should cause these modes to have low energy.

The renormalization technique used successfully for simple systems by Bloch[12] and Low[13] has been extended to handle complicated crystals, and order of magnitude estimates of the magnon self-energy shifts have been obtained at $k = 0$ in the low-temperature phase of α-Fe_2O_3. A number of interesting features have emerged from this theory. With reasonable values of the exchange and anisotropy parameters, any of the magnon modes could be unstable against magnon interaction. The fine structure and dipolar anisotropies could lead to instabilities for the acoustic modes (corresponding to the theory of Artman, Murphy, and Foner), and this is the only possibility if the optic modes have high energy. However, if the optic modes have low energy, the Dzyaloshinski interaction could lead to instabilities of either the acoustic or the optic modes. As the transition is associated with the appearance of a weak ferromagnetic moment, caused by the Dzyaloshinski interaction, this interaction may be expected to play an important part in the mechanism of the transition. If the optic modes were unstable, the renormalization equations would also be extremely sensitive to small changes in the exchange and anisotropy parameters, and this could explain the high sensitivity of the transition temperature and other properties to small impurity concentrations.[14]

It is proposed that the experimental anomalies mentioned above can be understood if the renormalized optic-mode energy tends to zero near the Morin transition temperature. The optic modes do not couple to the applied magnetic field and therefore do not resonate, so that if they are unstable, the resonance experiments of Foner and Williamson can be explained. At low temperatures the magnon-mode energies deduced from antiferromagnetic-resonance and spin-flop experiments agree, and as spin-flop detects the mode of lowest energy, while antiferromagnetic resonance only detects acoustic modes, it follows that the optic-mode energy is higher than the acoustic-mode energy at low temperature. If the optic modes are unstable, then the mode energies must cross at some temperature, and this could explain both the kink in the spin-flop curve and the existence of spin-flop fields lower than 28 kG. To settle the question of mode stability, it will be necessary to investigate the optic-magnon energies experimentally near the transition temperature, possibly using neutron-scattering techniques.

A detailed account of the group theoretic model and the calculations leading to the above conclusions will be given shortly.

The author wishes to thank Dr. S. Doniach for suggesting and supervising this work on hematite. He is also indebted to the Science Research Council for the award of a research studentship.

---

*These results were obtained during the preparation of a Ph. D. thesis recently submitted to the University of London, London, England.

†Science Research Council Postdoctoral Fellow. Present address: Physics Department, University of Bristol, Bristol, England.

[1]D. C. Herbert, thesis, London University, 1968 (unpublished).

[2]J. W. Artman, J. C. Murphy, and S. Foner, Phys. Rev. 138, A912 (1965).

[3]S. Foner and S. J. Williamson, J. Appl. Phys. 36, 1154 (1965).

[4]I. Dzyaloshinski, J. Phys. Chem. Solids 4, 241 (1958).

[5]F. B. Anderson and H. B. Callen, Phys. Rev. 136, A1068 (1964).

[6]J. Feder and E. Pytte, Phys. Rev. 168, 640 (1968).

[7]Y. L. Wang and H. B. Callen, J. Phys. Chem. Solids 25, 1459 (1964).

[8]P. J. Besser, A. H. Morrish, and C. W. Searle, Phys. Rev. 153, 632 (1966).

[9]R. A. Voskanyan, R. Z. Levitin, and V. A. Schurov, Zh. Eksperim. i Teor. Fiz. 53, 459 (1967) [translation: Soviet Phys.—JETP 26, 302 (1968)].

[10]T. Kaneko and S. Abe, J. Phys. Soc. Japan 20, 2001 (1965).

[11]E. F. Bertaut, in Proceedings of the International Conference on Magnetism, Nottingham, England, 1964 (The Institute of Physics and The Physical Society, London, England, 1964), p. 516.

[12]M. Bloch, J. Appl. Phys. 34, 1151 (1963).

[13]G. Low, Proc. Phys. Soc. (London) 82, 992 (1963).

[14]P. J. Flanders and J. P. Remeika, Phil. Mag. 11, 1271 (1965).