[5]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); I. S. Gerstein and H. J. Schnitzer, Phys. Rev. 170, 1638 (1968). See also S. Brown and G. West, Phys. Rev. Letters 19, 812 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters 19, 1085 (1967).

[6]C. J. Goebel, J. L. Blackmon, and K. C. Wali, to be published.

[7]S. Weinberg, Phys. Rev. Letters 17, 168 1966.

[8]M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters 22, 83 (1969); S. Weinberg, to be published.

[9]It is easy to find models for $V_3(\alpha(s), \alpha(t))$ which have the correct high-energy behavior and satisfy $V_3(\alpha(0),\alpha(0)) = 0$ to $O(m_\pi^2/m_\rho^2)$. Although not necessary, it is a good working hypothesis. In any case, the effects of V_3 are negligible for $p_4^2 \ll m_A^2$ and s, t, u not all of order m_π^2. By contrast, the simplest models for V_1 which satisfy $V_1(\alpha(0), \alpha(0)) = 0$ also have a ghost at $\alpha = 0$.

[10]F. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968). See also S. Weinberg, to be published.

[11]J. Ballam et al., Phys. Rev. Letters 21, 934 (1968).

[12]J. Shapiro and J. Yellin, to be published.

[13]This is reminiscent of the difference between the soft- and hard-pion calculations of $A_1 \to \rho + \pi$ and $\rho \to \pi + \pi$. See Ref. 5.

FIELD-CURRENT IDENTITY, PARTIALLY CONSERVED CURRENTS, AND THE VENEZIANO MODEL*

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Hard-meson partial conservation of axial-vector current and partial conservation of vector current are imposed upon the Veneziano amplitudes. It is seen that these determine inelastic amplitudes (e.g., $\pi\pi \to \pi A_1$, etc.) in terms of corresponding elastic amplitudes (e.g., $\pi\pi \to \pi\pi$). Mass formulas for the κ and δ meson, equality of many trajectory slopes of either normality, and πK scattering lengths are obtained, in good agreement with experiment.

Several authors[1] have recently combined Veneziano's original formula for scattering ampltudes[2] with the Adler soft-pion condition[3] to obtain several interesting new results. These include a number of mass formulas and the equality of slopes of Regge trajectories of opposite normality. In this Letter we wish to make use of the field-current identities[4] to obtain hard-meson partial conservation of axial-vector current (PCAC) constraints on the Veneziano amplitudes. We find the Veneziano amplitudes in general consistent with these continuous PCAC constraints. In this way we obtain several new mass formulas (e.g., for the δ and κ mesons) and equality of slopes of trajectories which do not necessarily have opposite normality, all in good agreement with experiment. In addition, the hard-meson technique gives a complete determination of decay amplitudes such as $A_1 \to 3\pi$, $K_A \to 2\pi + K$, etc., and affords a natural explanation as to why inelastic forward amplitudes for processes such as $\pi\pi \to \pi A_1$ vanish asymptotically.[5] A number of coupling constant relations are also obtained.

As a prototype of our method we consider the π-A_1 system. The field-current identity for the axial-vector current $A_a^\mu(x)$ reads[6]

$$A_a^{\ \mu}(x) = g_A a_a^{\ \mu}(x) + F_\pi \partial^\mu \pi_a(x), \tag{1a}$$

which yields the PCAC condition

$$(-\Box^2 + m_\pi^{\ 2})\pi_a(x) = g_A (F_\pi)^{-1} \partial_\mu a_a^{\ \mu}(x). \tag{1b}$$

The Feynman amplitude for the process $\pi_a(p_1) + A_{1b}(p_2) \to \pi_c(p_3) + \pi_d(p_4)$ for an off-shell A_1 particle (all other particles on the mass shell) can be decomposed into its invariant amplitudes as[7]

$$T_{(abcd)}^{\mu}=[P^{\mu}F_{1(abcd)}+p^{\mu}F_{2(abcd)}+p_2^{\mu}F_{3(abcd)}], \tag{2}$$

where $\epsilon_\mu(p_2)T^\mu$ is the corresponding on-shell amplitude for $p_2^2=-m_A^2$. Equation (1b) now yields

$$[2F_\pi m_A^2 g_A^{-1}]T^{(I)}(s,t,u;p_2^2)=(t+u-2m_\pi^2)F_1^{(I)}-(t-u)F_2^{(I)}+2p_2^2[F_1^{(I)}+F_3^{(I)}], \tag{3}$$

where $T^{(I)}(s,t,u;p_2^2)$ is the corresponding off-shell $\pi\pi\to\pi\pi$ amplitude. The Lovelace-Veneziano $\pi\pi\to\pi\pi$ amplitude for $I=2$ is[1]

$$T^{(2)}(s,t,u;p_2^2)=\beta_\pi(p_2^2)\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(u))}{\Gamma(1-\alpha_\rho(t)-\alpha_\rho(u))}. \tag{4}$$

The absence of doubly-charged mesons and satellite terms in the Veneziano formula implies that the $t\leftrightarrow u$ antisymmetric amplitude $F_2^{(2)}$ vanish. Equation (3) then yields

$$F_i^{(2)}(s,t,u;p_2^2)=\beta_i(p_2^2)\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(u))}{\Gamma(2-\alpha_\rho(t)-\alpha_\rho(u))},\quad i=1,3, \tag{5}$$

where

$$\beta_1(p_2^2)=-\beta_3(p_2^2)=[2m_A^2F_\pi g_A^{-1}]\alpha_\rho'\beta_\pi(p_2^2);\quad \alpha_\rho'=(2m_\rho^2-2m_\pi^2)^{-1}. \tag{6}$$

The slope condition is, of course, the same as found by soft-pion methods.[1] In addition, however, the hard-pion method has completely determined the $\pi A_1\to\pi\pi$ amplitude in terms of the $\pi\pi\to\pi\pi$ amplitude.[8] We note that one of the two independent amplitudes that arises in a general mass-shell analysis[9] of $\epsilon_\mu T^\mu$ has been evaluated to be zero. The strictness of the Veneziano form is further indicated by the fact that while in the general Ward's-identity analysis[6] one can only determine the $\pi\pi\to\pi\pi$ amplitude in terms of the divergence of the $\pi A_1\to\pi\pi$ amplitude, here one may invert the relation and solve for the full $\pi A_1\to\pi\pi$ amplitude.

One may straightforwardly calculate the $A_1\rho\pi$ couplings in terms of the $\rho\pi\pi$ coupling from the above. Defining the vertices (with all particles on the mass shell) by

$$\mathcal{L}[A_{1\,a}^{\mu}(p+q)\to\rho_b^{\nu}(p)+\pi_c(q)]=i\epsilon_{abc}[\eta^{\mu\nu}G_{\pi\rho A}+(2F_\pi)^{-1}(1-\lambda_A)q^\mu q^\nu], \tag{7a}$$

$$\mathcal{L}[\rho_a^{\mu}(k_1+k_2)\to\pi_b(k_1)+\pi_c(k_2)]=\epsilon_{abc}(k_1-k_2)^\mu G_{\rho\pi\pi}, \tag{7b}$$

one finds $\lambda_A=1$ and $G_{\pi\rho A}=2m_A^2F_\pi(g_A)^{-1}G_{\rho\pi\pi}$, where λ_A is the A_1 anomalous moment.[10] From Eq. (3) we also see that F_i goes asymptotically as $u^{\alpha_\rho(t)-1}$ (rather than the maximally allowed behavior[11] of $u^{\alpha_\rho(t)}$) and hence automatically vanishes in the forward direction.

We consider next the $\pi K\to\pi K$ amplitude and its connection to the $\pi K_A\to\pi K$ and $\pi K\to A_1K$ amplitudes through the partial conservation of the strange and nonstrange axial currents. If we let $T^{(I)}$ be the isotopic amplitudes for πK elastic scattering, then the absence of satellites and of $I=\frac{3}{2}$ poles leads to[12,13]

$$T^{(1/2)}=\gamma[3V_{st}-V_{ut}], \tag{8a}$$

$$T^{(3/2)}=2\gamma V_{ut}, \tag{8b}$$

where

$$V_{st}=\frac{\Gamma(1-\alpha_{K^*}(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_{K^*}(s)-\alpha_\rho(t))}. \tag{9}$$

The off-shell $\pi K \to A_1 K$ and $\pi K_A \to \pi K$ amplitudes may be written as[7]

$$T_\mu^{(A_1)} = q_\mu T_1 + Q_\mu T_2 + p_{3\mu} T_3, \tag{10a}$$

$$T_\mu^{(KA)} = k_\mu G_1 + K_\mu G_2 + p_{2\mu} G_3, \tag{10b}$$

and the two PCAC conditions relate these to the πK elastic amplitude according to

$$T = g_A [F_\pi m_A^2]^{-1} p_{3\mu} T^{\mu(A_1)} = g_K [F_K m_{KA}^2]^{-1} p_{2\mu} T^{\mu(KA)}. \tag{11}$$

Equations (8)-(11) then determine the inelastic amplitudes in terms of the elastic ones. Thus the $I_t = 0$ amplitudes are

$$T_1^{(0)} = c_1(B_{st} + B_{ut}), \quad T_2^{(0)} = c_2(B_{st} - B_{ut}), \quad T_3^{(0)} = 0, \tag{12a}$$

$$G_1^{(0)} = g_1(B_{st} + B_{ut}), \quad G_2^{(0)} = g_2(B_{st} - B_{ut}), \quad G_3^{(0)} = 0, \tag{12b}$$

where $V_{st} \equiv [1 - \alpha_{K^*}(s) - \alpha_\rho(t)] B_{st}$ and

$$c_1 = c_2 = (6\alpha_\rho')^{1/2}(F_\pi m_A^2 g_A^{-1})\gamma, \tag{13a}$$

$$g_1 = -g_2 = -(6\alpha_\rho')^{1/2}(F_K m_{KA}^2 g_{KA}^{-1})\gamma. \tag{13b}$$

In addition, the K^* and ρ trajectories must obey $\alpha_\rho(m_\pi^2) + \alpha_{K^*}(m_K^2) = 1 = \alpha_\rho(m_K^2) + \alpha_{K^*}(m_\pi^2)$ which implies

$$\alpha_{K^*}' = \alpha_\rho'; \quad \alpha_{K^*}(m_K^2) = \alpha_\rho(m_\pi^2) = \tfrac{1}{2}. \tag{14}$$

Equation (14) predicts $m_{K^*} \simeq 905$ MeV in excellent agreement with the experimental value[14] of 891 MeV. The slope relation $\alpha_{K^*}' = \alpha_\rho'$ is further verified by noting that it predicts a $J^P = 2^+$ "Veneziano recurrence" at 1392 MeV, in good agreement with the $K_N(1420)$ mass. If one normalizes γ of Eq. (8) to give a K^* width of 50 MeV, one obtains for the S-wave πK scattering lengths the values $a_{\frac{1}{2}} = 0.25 m_\pi^{-1}$ and $a_{\frac{3}{2}} = -0.098 m_\pi^{-1}$, in good agreement with the soft-pion current-algebra values[15] of $a_{\frac{1}{2}} = 0.26 m_\pi^{-1}$ and $a_{\frac{3}{2}} = -0.13 m_\pi^{-1}$, and consistent with the present data.[16] With this normalization, one also finds $\Gamma(K_N(1420) \to K\pi) = 50$ MeV in excellent agreement with experiment[14] ($\simeq 49$ MeV). Equations (12) and (13) determine the complete $K_A \to 2\pi + K$ and $\pi + K \to A_1 + K$ amplitudes. In particular, one may relate the residue of the K^* pole to that of the ρ pole to yield the relation $(G_{K^*K\pi})^2 = (\frac{3}{4}) G_{\rho\pi\pi} G_{\rho K\bar{K}}$, a result that also follows from SU(3) invariance, and agrees with experiment to within about 10%.

We consider next $\pi\kappa$ elastic scattering (where κ is the $I = \frac{1}{2}$, 0^+ meson). If in addition to pion PCAC one also assumes partial conservation of vector current (PCVC), i.e., that the divergence of the strangeness-changing vector current is proportional to the κ field, then one may determine the $A_1\kappa \to \pi\kappa$ and $\pi\kappa \to \pi K^*$ inelastic amplitudes in terms of the $\pi\kappa \to \pi\kappa$ amplitude. We assume, as usual, that there are no satellites and no $I = \frac{3}{2}$ resonances and use the $K_A(1320)$ trajectory in the s and u channels and ρ trajectory in the t channel. The two partial-conservation laws then imply that $\alpha_{KA}' = \alpha_\rho'$ and the mass formula $m_\kappa^2 = m_{KA}^2 - m_\rho^2 + m_\pi^2$. The latter yields $m_\kappa = 1095$ MeV in agreement with the present experimental value[16] of 1100 to 1200 MeV.

Pion PCAC allows one to relate $\pi\eta \to A_1\eta$ amplitudes to elastic $\pi\eta$ scattering. Assuming the $\delta(962)$ trajectory in the s and u channels and the σ trajectory in the t channel,[17] one finds (in addition to determining the inelastic amplitude) the results $\alpha_\sigma(m_\pi^2) = \frac{1}{2} = \alpha_\delta(m_\eta^2)$ and $\alpha_\delta' = \alpha_\sigma'$. (The last relation follows only from the hard-meson PCAC and cannot be obtained by soft-pion techniques.) This leads to the mass formula $m_\delta^2 = m_\rho^2 - m_\pi^2 + m_\eta^2$ yielding $m_\delta = 935$ MeV, to be compared with the experimental value[14] of 962 MeV. One may also calculate the S-wave scattering length for the $\pi\eta$ channel. Assuming[18] $\Gamma_\delta \simeq 60$ MeV we find $a_0 \simeq 0.011 m_\pi^{-1}$. (The soft-pion value for a_0 is zero since η is an isoscalar.)

Lastly, one may check on the self-consistency of our model. Thus the $\pi\pi \to \pi\pi$ amplitude requires certain constraints among the trajectory and residue parameters which affect the $\pi\sigma \to \pi\sigma$ amplitude where some of the same vertices occur. These consistency requirements are satisfied. Thus the Veneziano amplitude for $\pi\sigma \to \pi\sigma$

reads

$$T_{\pi\sigma\to\pi\sigma}=g_\sigma[V(\alpha_\pi(s),\alpha_\sigma(t))+V(\alpha_\pi(u),\alpha_\sigma(t))+V(\alpha_\pi(s),\alpha_\pi(u))],\quad(15)$$

where for convenience we have normalized $\alpha_\pi(m_\pi^2)=1$. One may now determine the $\pi\sigma\to A_1\sigma$ amplitude by PCAC [which also implies $\alpha_\pi'=\alpha_\sigma'=(2m_\sigma^2-2m_\pi^2)^{-1}$]. From this one may calculate the $A_1\pi\sigma$ coupling in terms of the $\pi\pi\sigma$ coupling which turns out to be identical to the relation obtained from the $\pi\pi\to A_1\pi$ amplitude of Eqs. (5) and (6). (We also find $G_{\pi\pi\sigma}=G_{\sigma\sigma\sigma}$.)

In conclusion, it appears that the partial-conservation laws for the π, K, and κ particles give results for mass formulas and trajectory slopes in good agreement with experiment. We should like to stress, however, the power of the PCAC conditions when combined with the Veneziano forms as functional equations for determining inelastic amplitudes. The simple models we have considered here with no satellites present appear to be approximations containing various amounts of SU(3) and chiral symmetries. For the true physical amplitudes, the f and A_2 and the σ and ρ are not degenerate; or, more drastically, $\Gamma_\sigma\approx150$-300 MeV and not $\frac{9}{2}\Gamma_\rho$ as predicted; λ_A is not 1; SU(3) couplings are not exactly valid, etc. (In addition, for KK, $\eta\eta$, and $\pi\eta$ amplitudes, the simultaneous imposition of PCAC constraints on all external legs proves impossible with only a single trajectory per channel and no satellites.) However, SU(3) and chiral invariance are in fact a reasonable zeroth approximation, which suggests that it should be possible to obtain more realistic amplitudes with only small modifications of the ones considered here (and thus maintain the good results obtained from the simple models presented above). A discussion of some of the modifications needed to obtain more realistic amplitudes will be presented elsewhere.

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[1]C. Lovelace, Phys. Letters 28B, 264 (1968); M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters 22, 83 (1969).

[2]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[3]S. Adler, Phys. Rev. 137, B1022 (1965).

[4]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[5]This result is to be compared with the assumption made by S. Weinberg, Phys. Rev. (to be published), and in Massachusetts Institute of Technology Report No. CTP 58, 1968 (to be published), cf. footnote 8.

[6]For the hard-pion chiral current-algebra analysis of vertex functions based on these relations see, e.g., H. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters 19, 1085 (1967); S. G. Brown and G. W. West, Phys. Rev. Letters 19, 812 (1967). Our normalization of g_A and F_π follows that of the second paper. $a_a^\mu(x)$, $\pi_a(x)$, etc., are the A_1, π, etc., fields.

[7]$\epsilon_\mu(p_2)$ is the polarization vector of the A_1 particle. $P=p_3+p_4$, $p=p_3-p_4$; $q=p_2-p_4$, $Q=p_2+p_4$; $k=p_1-p_3$, $K=p_1+p_3$; $s=-(p_1+p_2)^2$, $t=-(p_1-p_3)^2$, $u=-(p_1-p_4)^2$. Subscripts a, b, c, and d are isospin indices. One may decompose $F_{i(abcd)}$ into its isospin amplitudes $F_i^{(I)}$, $I=0,1,2$.

[8]The solution of Eqs. (5) and (6) also automatically satisfies Eq. (3) for $I=0,1$ as well as the analogous PCAC conditions when one continues the p_1, p_3, or p_4 legs off-shell.

[9]C. J. Goebel, M. L. Blackmon, and K. C. Wali, Argonne National Laboratory Report No. ANL/HEP 690, 1969 (to be published).

[10]This is to be contrasted to the hard-pion current-algebra result (Ref. 6) for $\lambda_A=1$: $G_{\pi\rho A}=(\frac{2}{3})m_A^2F_\pi\times(g_A)^{-1}G_{\rho\pi\pi}$. The difference arises from the fact that in Eq. (7) we have assumed that $\beta_\pi(p_2^2)$ is a constant. If one gives $\beta_\pi(p_2^2)$ the same off-shell p_2^2 dependence required by the current algebra, the two results then agree.

[11]The general amplitude constructed in Ref. 9 does indeed go asymptotically as $u^{\alpha_\rho(t)}$. Their amplitude reduces to ours if one sets their parameter λ_1 to zero (which corresponds to choosing $\lambda_A=1$).

[12]This result has also been obtained by K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Letters 28B, 432 (1968), where an application of Lovelace's arguments to the πK and KK amplitudes is made.

[13]We note that a V_{su} term in Eqs. (8) can in principle be present, but only if $I=\frac{3}{2}$ poles exist.

[14]All particle data are taken from the compilation of the Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

[15]S. Weinberg, Phys. Rev. Letters 17, 616 (1966); R. Griffith, Phys. Rev. 176, 1705 (1969).

[16]T. G. Trippe et al., Phys. Letters 28B, 203 (1968). The signs of the scattering lengths appear to be in agreement with the theoretical predictions. The experimental $K^+\pi^-$ threshold cross section is 6.6 ± 3.3 mb, while the Veneziano scattering lengths above yield 2.2 mb at threshold.

[17]The experimental situation regarding the A_2 is at present unclear (see Ref. 14). However, the A_2 branching ratio into $\pi\eta$ appears to be small and so we have ignored the A_2 trajectory in this process. For a theoretical estimate see H. Goldberg and Y. Srivastava, Phys. Rev. Letters 22, 749 (1969).

[18]B. French, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 91.