strated that Regge trajectories derived from all these models are always bounded from below by $-a\sqrt{-t}$, $a > 0$, as $t \to -\infty$. It is interesting to note that a common characteristic of these models is the incorporation of unitarity. In constrast, models based on linear trajectories constantly are having difficulties with unitarity.[8]

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[1] G. F. Chew, M. L. Goldberger, and F. Low, Phys. Rev. Letters 22, 208 (1969); L. Caneschi and A. Pignotti, Phys. Rev. (to be published); G. F. Chew and C. DeTar, Phys. Rev. (to be published); and I. G. Halliday and L. M. Saunders, Imperial College Report No. ICTP/68/2 (to be published).

[2] M. L. Goldberger, C.-I. Tan, and J. M. Wang, "A Simplified Integral Equation for the Multiperipheral Bootstrap" (to be published). Although our equation is derived from the multiperipheral bootstrap model, our technique can be used to include the case with Mandelstam "crossed-diagram" (S. Mandelstam, to be published).

[3] G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

[4] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento 26, 896 (1962); S. Mandelstam, Nuovo Cimento 30, 1148 (1963).

[5] H. Rothe, Phys. Rev. 159, 1471 (1967).

[6] In a multichannel problem, the output trajectory will, in general, acquire additional singularities due to trajectory crossings. However, this will not affect our argument on the asymptotic growth of $\alpha(t)$.

[7] S. Mandelstam, in Second Japanese Summer Institute in Theoretical Physics, Oiso, Japan, 1966, edited by G. Takeda and A. Fujii (W. A. Benjamin, Inc., New York, 1967), Pt. II.

[8] R. Roskies, Phys. Rev. Letters 21, 1851 (1968).

VENEZIANO MODEL AND CURRENT ALGEBRA: THE HYPOTHESIS OF PARTIAL CONSERVATION OF AXIAL-VECTOR CURRENT.

Howard J. Schnitzer*
Brandeis University, Waltham, Massachusetts 02154
(Received 27 January 1969)

A systematic procedure for applying the hypothesis of the partial conservation of the axial-vector current to the Regge-pole models of the Veneziano type is illustrated by a discussion of the matrix element of the axial-vector current for the weak process $A_\mu + \pi \to \pi + \pi$. Additional information is obtained which relates the reactions $\pi + \pi \to \pi + \pi$ and $\pi + \pi \to \pi + A_1$.

The discovery of a model by Veneziano[1] explicitly exhibiting the property of duality is a promising advance in Regge-pole theory. In this connection, an important idea has been advanced by Lovelace,[2] who suggested that these models may provide reasonable continuations off mass shell of the external particles. With this in mind, he applies the Adler consistency[3] condition for soft pions to models of the Veneziano type,[1,4] and obtains some interesting results. In light of this, we believe that an important question to be asked is whether local current algebra is compatible with duality of the strong interactions. If they are, then one should be able to combine Regge-pole models which exhibit duality with current algebra in a systematic fashion so as to provide a reasonable definition of hadron currents, which satisfy current-algebra commutation relations. Conversely, the current algebra should then provide useful constraints on strong-interaction models. In particular, if models of the Veneziano type permit a natural continuation off mass shell, then one should be able to obtain a parametrization of current algebra (not necessarily unique), satisfying the duality principle, even when the pions are not soft. We suggest that an appropriate framework for this kind of investigation is the hard-pion method[5] of current algebra, where Ward identities connecting time-ordered products of currents are re-expressed in terms of similar Ward identities involving the off-shell continuations of hadron amplitudes.

The first step in such a program is the definition of the matrix elements of a single hadron current. In the hard-pion method this requires a knowledge of hadron amplitudes with a single particle off shell. According to Lovelace's suggestion, a reasonable amplitude is provided by the off-shell continuation of amplitudes of the Veneziano type. We illustrate our point of view with a simple example, the application of the hypothesis of the partial conservation of the axial-vector current (PCAC) to $\pi\pi$ scattering. In this case we will be dealing with a simple divergence condition; however, these ideas certainly apply to more complicated examples involving more than one current. Consider the application of PCAC to the processes $\pi+\pi\to\pi+A_\mu$, where A_μ is the hadron axial-vector current. This weak process can be related to the off-shell continuation of the hadron amplitudes for $\pi\pi$ scattering and the pion production of an axial-vector meson ($\pi+\pi\to\pi+A_I$) as follows[5]:

$$\langle\pi^c(p_3)|A_\mu^{\ d}(0)|\pi^a(p_1)\pi^b(p_2)\rangle=\frac{-ip_{4\mu}}{m_\pi^{\ 2}-p_4^{\ 2}}F_\pi M(s,t,u;p_4^{\ 2})_{ab,cd}$$
$$-\frac{ig_A}{m_A^{\ 2}-p_4^{\ 2}}(g_{\mu\nu}-p_{4\mu}p_{4\nu}/m_A^{\ 2})M^\nu(s,t,u;p_4^{\ 2})_{ab,cd}. \qquad (1)$$

We have defined the pion field by the divergence of the axial-vector current, $\partial^\mu A_\mu(x)=m_\pi^{\ 2}F_\pi\varphi(x)$, and made the further assumption that the axial-vector current is dominated by the pion and a single axial-vector meson, at least for $p_4^{\ 2}$ in the interval $0\le p_4^{\ 2}\lesssim m_A^{\ 2}$. The amplitude M (M^ν) is the off-shell continuation for the process $\pi^a(p_1)+\pi^b(p_2)\to\pi^c(p_3)+\pi^d(p_4)$ $[\pi^c(p_3)+A_1^{\ d}(p_4)]$; the particle with four-momentum p_4 and isospin d being off shell, and all particles incoming so that $p_1+p_2+p_3+p_4=0$ $[s=(p_1+p_2)^2,\ t=(p_2+p_3)^2$, and $u=(p_3+p_1)^2]$. As usual F_π is the pion decay rate and g_A is the vacuum axial-vector-meson matrix element of the axial-vector current. The divergence of Eq. (1) together with our definition of the pion field gives the condition

$$M(s,t,u;p_4^{\ 2})=g_Am_A^{\ -2}F_\pi^{\ -1}p_{4\mu}M^\mu(s,t,u;p_4^{\ 2}). \qquad (2)$$

Hence, if one can construct a reasonable model for $\pi+\pi\to\pi+A_1$, with the axial-vector meson continued off shell, then Eq. (2) enables us to construct a model for $\pi+\pi\to\pi+\pi$, with a single pion off shell, which automatically satisfies the Adler consistency condition. (Of course, the converse is not true in that a model for $\pi+\pi\to\pi+\pi$ does not determine $\pi+\pi\to\pi+A_1$.)

Let us assume that M_μ may be represented by models of the Veneziano type. That is, let[6]

$$M_\mu(s,t,u;p_4^{\ 2})_{ab,cd}=\lambda_1(p_4^{\ 2})\{I_{abc}[p_{1\mu}V_1(\alpha(s),\alpha(t))+p_{3\mu}V_1(\alpha(t),\alpha(s))]+\text{cyclic terms}\}$$
$$+\lambda_2(p_4^{\ 2})\{I_{abc}p_{2\mu}V_2(\alpha(s),\alpha(t))+\text{cyclic terms}\}$$
$$+\lambda_3(p_4^{\ 2})p_{4\mu}\{I_{abc}V_3(\alpha(s),\alpha(t))+\text{cyclic terms}\}, \qquad (3)$$

where $\alpha(s)$ is the leading Regge trajectory connecting two pions (the Pomeranchuk trajectory aside). This representation is sufficiently general to encompass any of a number of models which exhibit duality, containing a single ρ family. The isospin factor

$$I_{abc}=\delta_{ab}\delta_{cd}+\delta_{bc}\delta_{ad}-\delta_{bd}\delta_{ac}$$

excludes the exchange of $I=2$ trajectories. We assume that Eq. (3) provides an off-shell continuation for $\pi+\pi\to\pi+A_1$ (with s and t fixed, say) both through the implicit relation $s+t+u=3m_\pi^{\ 2}+p_4^{\ 2}$ and the explicit dependence of the coupling parameters λ_1, λ_2, and λ_3 on $p_4^{\ 2}$. The high-energy behavior of $M_\mu\epsilon^\mu$ on the A_1 mass shell requires[6]

$$V_1(\alpha(s),\alpha(t))\underset{\substack{s\to\infty\\ t\text{ fixed}}}{\longrightarrow}s^{\alpha(t)},\quad V_1(\alpha(t),\alpha(s))\to(s)^{\alpha(t)-1}, \qquad (4)$$

and

$$V_2(\alpha(s),\alpha(t))=V_2(\alpha(t),\alpha(s))\to s^{\alpha(t)-1};$$

however the high-energy behavior of V_3 cannot be similarly determined, since it does not contribute to the physical A_1 amplitude. However, it is consistent with the divergence condition for it to have pure

Regge behavior, in which case, for $s\to\infty$, t fixed,

$$V_3(\alpha(s),\alpha(t))=V_3(\alpha(t),\alpha(s))\to s^{\alpha(t)}$$

(although it could also have non-Regge terms). The divergence of Eq. (3) thus defines a $\pi\pi$ scattering amplitude with a single pion off shell (not restricted to be soft). Notice that the Adler condition for the off-shell pion is automatically satisfied (without imposing conditions on the Regge trajectories).

We turn our attention to other constraints. Recall that the divergence condition which defines the pion field must be supplemented by the hypothesis of PCAC, which is an assumption about the continuation for $p_4^2=0$ to m_π^2. Since the continuation of the Veneziano amplitudes vary slowly with p_4^2 in this range, in this context PCAC implies that the coupling parameters satisfy $\lambda(m_\pi^2)=\lambda(0)+O(m_\pi^2/m_\rho^2)$, and in addition $\lambda_3(0)=O(1)$. The scattering lengths are easily determined from Eqs. (2) and (3), while an overall scale can be fixed from the current-algebra condition[7,8]

$$\left.\frac{\partial M(s,0,-s)}{\partial s}\right|_{s=0}=-\tfrac{1}{2}F_\pi^{-2}(\delta_{ab}\delta_{cd}-\delta_{ad}\delta_{cb}),$$

in which case

$$a_0=\frac{g_Am_\pi}{4\pi m_A^{\,2}F_\pi}[-2\lambda_1(0)V_1(\alpha(0),\alpha(0))+7\lambda_2(0)V_2(\alpha(0),\alpha(0))-5\lambda_3(0)V_3(\alpha(0),\alpha(0))],$$

$$a_2=\frac{2g_Am_\pi}{4\pi m_A^{\,2}F_\pi}[2\lambda_1(0)V_1(\alpha(0),\alpha(0))-\lambda_2(0)V_2(\alpha(0),\alpha(0))-2\lambda_3(0)V_3(\alpha(0),\alpha(0))],\tag{5}$$

and

$$\lambda_1(0)V_1(\alpha(0),\alpha(0))-\lambda_2(0)V_2(\alpha(0),\alpha(0))=-\tfrac{1}{2}F_\pi^{-1}g_A^{-1}m_A^{\,2},$$

where terms of $O(m_\pi^2/m_\rho^2)$ have been neglected. If one further assumes, with Weinberg,[7] that the pion and σ form a chiral quadruplet, then $a_0/a_2=-\frac{7}{2}$, in which case

$$\lambda_1(0)V_1(\alpha(0),\alpha(0))-\lambda_3(0)V_3(\alpha(0),\alpha(0))=0.$$

We believe a plausible physical assumption is that $V_3(\alpha(0),\alpha(0))$ is $O(m_\pi^2/m_\rho^2)$ and $\lambda_1(0)=0$ to this order.[9] That is, though these are not necessary conditions, they lead to a parametrization of the current with several attractive features. Additional information is obtained from the resonance structure of the model. For example, if we require the lowest lying resonances of $\pi+\pi\to\pi+\pi$ to be degenerate S and P states (σ and ρ), for $\alpha(s)\to 1$, then the residues at $s=m_\rho^2$ of $V_1(\alpha(s),\alpha(t))$ and $V_2(\alpha(s),\alpha(t))$ are independent of t, while the residues of $V_1(\alpha(t),\alpha(s))$ and $V_3(\alpha(s),\alpha(t))$ are at most linear in t. They are easily related to the widths $\Gamma_{\sigma\pi\pi}$ and $\Gamma_{\rho\pi\pi}$. Gilman and Harari[10] saturate current-algebra and superconvergence sum rules to obtain $\Gamma_\sigma/\Gamma_\rho=\frac{9}{2}$. In our language, this occurs when $\lambda_1(0)=0$ or

$$\operatorname{Res}V_1(\alpha(-m_\rho^{\,2}),\alpha(s))=\operatorname{Res}V_1(\alpha(s),\alpha(0)),\quad s=m_\rho^{\,2}.$$

A third possibility, in which V_1 has no residue at $s=m_\rho^2$, is ruled out by the experimental spin dependence of $A_1\to\rho+\pi$.[11]

A much stronger hypothesis is to parametrize the process $\pi+\pi\to\pi+A_1$ by a Veneziano model with leading terms only, in which case[6]

$$V_2(\alpha(s),\alpha(t))=\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))},$$

$$V_1(\alpha(s),\alpha(t))=[1-\alpha(s)]V_2(\alpha(s),\alpha(t)),\tag{6}$$

and

$$V_3(\alpha(s),\alpha(t))=[1-\alpha(s)-\alpha(t)]V_2(\alpha(s),\alpha(t)).$$

An interesting additional assumption is to require the $\pi\pi$ scattering at $p_4^2=0$ to be given by a Veneziano model with leading term only:[2,8,12]

$$M(s,t,u;\,0)_{ab,cd}=\beta(0)\left[I_{abc}\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}+\text{cyclic terms}\right]. \tag{7}$$

The constraint

$$\lambda_1(0)[2m_\pi^2-u-2t\alpha(s)-2s\alpha(t)]+\lambda_1(0)m_\pi^2[\alpha(s)+\alpha(t)]+\lambda_2(0)(u-m_\pi^2)$$
$$=2F_\pi m_A^2 g_A^{-1}\beta(0)[1-\alpha(s)-\alpha(t)] \tag{8}$$

obtained from (2), (6), and (7) is satisfied for linear trajectories and arbitrary s and t if and only if

$$\alpha_\rho(m_\pi^2)=\tfrac{1}{2},\quad \lambda_1(0)=0, \tag{9}$$

and

$$\lambda_2(0)=2m_A^2 g_A^{-1}F_\pi\alpha'\beta(0),$$

where α' is the slope of the ρ trajectory. It is tempting to assume that λ_1 and λ_2 are independent of p_4^2; however, we argue that if these models are to be taken at all seriously, this cannot be the case. One obtains the matrix element for the decay $A_1\to\rho+\pi$ from the residue of the ρ pole in $M_\mu\epsilon^\mu$, Eq. (3), i.e.,

$$M_{\mu\nu}=-g_{\rho\pi\pi}^{-1}[\alpha'^{-1}\lambda_2(m_A^2)g_{\mu\nu}+2\lambda_1(m_A^2)p_{\rho\mu}p_{A\nu}]\epsilon_A^{\ \mu}\epsilon_\rho^{\ \nu}. \tag{10}$$

If $\lambda_1(p_4^2)=0$ and $\lambda_2(p_4^2)=\text{const}$, then the predicted ratio of widths Γ_{A_1}/Γ_ρ would be an order of magnitude larger than experiment.[13] Further, $M_{\mu\nu}\sim g_{\mu\nu}$ predicts a spin correlation for $A_1\to\rho+\pi$ which is not in agreement with preliminary experiments.[11] Thus, both λ_1 and λ_2 depend on p_4^2. A rough description of the experimental spin correlation is given by $\lambda_2(m_A^2)\simeq 0$. If we assume that λ_1 and λ_2 are at most quadratic functions of p_4^2 (at least for $p_4^2\leq m_A^2$),[11] then the above properties are summarized by

$$\lambda_1(p_4^2)\simeq p_4^2 m_A^{-2}\lambda_4, \tag{11}$$

and

$$\lambda_2(p_4^2)\simeq 2g_A^{-1}F_\pi\alpha'\beta(0)(m_A^2-p_4^2).$$

We close with some comments.

(1) Although these models are not uniquely determined by current-algebra conditions, it is clear that duality is compatible with PCAC. The simple requirement $\lambda_1(0)=0$ and $V_3(\alpha(0),\alpha(0))\approx 0$ in Eq. (3) leads to a particularly attractive class of models, consistent with several current-algebra constraints.

(2) The condition on the ρ Regge trajectory, $\alpha_\rho(m_\pi^2)=\frac{1}{2}$,[2,8] is not a necessary one. The same remark applies to the general quantization conditions of Ademollo, Veneziano, and Weinberg.[8] Still, the empirical data indicate the dominance by Veneziano models, <u>with leading terms only</u>, is an extremely attractive conjecture because of the success of this quantization condition. This conjecture leads to very specific models for weak amplitudes of type $A_\mu+B\to C+D$, which are expected to be valid for a wide range of energies, momentum transfers, and p_4^2. This may be testable for $A_\mu+N\to N+\pi$, etc.

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[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2]C. Lovelace, Phys. Letters 28B, 204 (1968); M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters 22, 83 (1969); S. Weinberg, to be published.

[3]S. L. Adler, Phys. Rev. 137, B1022 (1965).

[4]S. Mandelstam, Phys. Rev. Letters 21, 1724 (1968); J. Mandula (unpublished); S. Fubini, to be published.

[5]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); I. S. Gerstein and H. J. Schnitzer, Phys. Rev. 170, 1638 (1968). See also S. Brown and G. West, Phys. Rev. Letters 19, 812 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters 19, 1085 (1967).

[6]C. J. Goebel, J. L. Blackmon, and K. C. Wali, to be published.

[7]S. Weinberg, Phys. Rev. Letters 17, 168 1966.

[8]M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters 22, 83 (1969); S. Weinberg, to be published.

[9]It is easy to find models for $V_3(\alpha(s), \alpha(t))$ which have the correct high-energy behavior and satisfy $V_3(\alpha(0), \alpha(0)) = 0$ to $O(m_\pi^2/m_\rho^2)$. Although not necessary, it is a good working hypothesis. In any case, the effects of V_3 are negligible for $p_4^2 \ll m_A^2$ and s, t, u not all of order m_π^2. By contrast, the simplest models for V_1 which satisfy $V_1(\alpha(0), \alpha(0)) = 0$ also have a ghost at $\alpha = 0$.

[10]F. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968). See also S. Weinberg, to be published.

[11]J. Ballam et al., Phys. Rev. Letters 21, 934 (1968).

[12]J. Shapiro and J. Yellin, to be published.

[13]This is reminiscent of the difference between the soft- and hard-pion calculations of $A_1 \to \rho + \pi$ and $\rho \to \pi + \pi$. See Ref. 5.

FIELD-CURRENT IDENTITY, PARTIALLY CONSERVED CURRENTS, AND THE VENEZIANO MODEL*

R. Arnowitt, P. Nath, and Y. Srivastava
Department of Physics, Northeastern University, Boston, Massachusetts 02115

and

M. H. Friedman†
Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 17 March 1969)

Hard-meson partial conservation of axial-vector current and partial conservation of vector current are imposed upon the Veneziano amplitudes. It is seen that these determine inelastic amplitudes (e.g., $\pi\pi \to \pi A_1$, etc.) in terms of corresponding elastic amplitudes (e.g., $\pi\pi \to \pi\pi$). Mass formulas for the κ and δ meson, equality of many trajectory slopes of either normality, and πK scattering lengths are obtained, in good agreement with experiment.

Several authors[1] have recently combined Veneziano's original formula for scattering ampltudes[2] with the Adler soft-pion condition[3] to obtain several interesting new results. These include a number of mass formulas and the equality of slopes of Regge trajectories of opposite normality. In this Letter we wish to make use of the field-current identities[4] to obtain hard-meson partial conservation of axial-vector current (PCAC) constraints on the Veneziano amplitudes. We find the Veneziano amplitudes in general consistent with these continuous PCAC constraints. In this way we obtain several new mass formulas (e.g., for the δ and κ mesons) and equality of slopes of trajectories which do not necessarily have opposite normality, all in good agreement with experiment. In addition, the hard-meson technique gives a complete determination of decay amplitudes such as $A_1 \to 3\pi$, $K_A \to 2\pi + K$, etc., and affords a natural explanation as to why inelastic forward amplitudes for processes such as $\pi\pi \to \pi A_1$ vanish asymptotically.[5] A number of coupling constant relations are also obtained.

As a prototype of our method we consider the π-A_1 system. The field-current identity for the axial-vector current $A_a{}^\mu(x)$ reads[6]

$$A_a{}^\mu(x) = g_A a_a{}^\mu(x) + F_\pi \partial^\mu \pi_a(x), \tag{1a}$$

which yields the PCAC condition

$$(-\Box^2 + m_\pi{}^2)\pi_a(x) = g_A (F_\pi)^{-1} \partial_\mu a_a{}^\mu(x). \tag{1b}$$

The Feynman amplitude for the process $\pi_a(p_1) + A_{1b}(p_2) \to \pi_c(p_3) + \pi_d(p_4)$ for an off-shell A_1 particle