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†On leave of absence from University of Torino, Torino, Italy.

‡On leave of absence from University of Louvain, Louvain, Belgium.

§Alfred P. Sloan Foundation Fellow.

[1]K. Igi, Phys. Rev. Letters 9, 76 (1962); A. Logunov, L. D. Soloviev, and A. A. Tavkhelidze, Phys. Letters 24B, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters 18, 625 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[2]We generalize the concept of "exchange degeneracy" to include degenerate trajectories of different SU(3) quantum numbers.

[3]J. L. Rosner, Phys. Rev. Letters 21, 950 (1968); H. J. Lipkin, to be published; M. Kugler, Phys. Rev. (to be published); R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

[4]M. Gell-Mann, Phys. Letters 8, 214 (1964); G. Zweig, CERN Reports Nos. Th 401 and Th 412, 1964 (unpublished), and in Proceedings of the International School of Physics "Ettore Majorana," Erice, Italy, 1964, edited by N. Zichichi (Academic Press, Inc., New York, 1965).

[5]P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, Phys. Rev. Letters 20, 1395 (1968).

[6]Singlets of mesons or baryons give no additional information.

[7]We used the crossing matrices of C. Rebbi and R. Slansky, to be published.

[8]R. H. Capps, Phys. Rev. Letters 22, 215 (1969). His result differs from ours because he considers only one channel.

[9]See, e.g., H. Harari, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968).

ASYMPTOTIC BEHAVIOR OF ELECTROPRODUCTION STRUCTURE FUNCTION*

Richard A. Brandt
Center for Theoretical Physics, Department of Physics and Astronomy,
University of Maryland, College Park, Maryland 20742
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The electroproduction function $F_2(-\nu/q^2)$, defined as the limit of the structure function $\nu W_2(q^2,\nu)$ for $\nu\to\infty$, ν/q^2 fixed, is experimentally observed to approach a constant for $-\nu/q^2\to\infty$. We derive this result from an integral representation of the scattering amplitude and the assumption of Regge behavior for the limit $\nu\to\infty$, q^2 fixed.

Bjorken[1] has recently shown that the electroproduction structure functions[2] $W_i(\kappa,\nu)$, $i=1, 2$, defined by[3]

$$\frac{p_0}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J_\mu(x),J_\nu(0)]|p\rangle=\left(p_\mu-\frac{\nu q_\mu}{\kappa}\right)\left(p_\nu-\frac{\nu q_\nu}{\kappa}\right)W_2-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{\kappa}\right)W_1, \tag{1}$$

are expected to have simple behavior in the limit[4] $\kappa\to\infty$, $\rho\equiv-\nu/\kappa$ fixed. Bjorken derived

$$\lim_A \nu W_2(\kappa,\nu)=F_2(\rho),\quad \lim_A W_1(\kappa,\nu)=F_1(\rho), \tag{2}$$

where the limits satisfy $0\leq F_i(\rho)<\infty$. Present experiments[5] are in agreement with (2) and, furthermore, indicate the a priori rather surprising property

$$F_2(\rho)\xrightarrow[\rho\to\infty]{}\text{const}\neq 0. \tag{3}$$

In this note we shall present a derivation of (3) and estimate the value of the constant. The essential idea is to relate the A limit to the Pomeranchuk-dominated Regge limit[6] $\nu\to\infty$, κ fixed.[7] The derivation is not rigorous, but is valid in the absence of pathologies.

Current Regge-pole theory[8] implies

$$W_2\xrightarrow[R]{} w_2(\kappa)\nu^{\alpha-2},\quad W_1\xrightarrow[R]{} w_1(\kappa)\nu^{\alpha},$$

where α is the $t=0$ intercept of the leading appropriate Regge trajectory. Assuming that the Pomeranchuk trajectory with $\alpha=1$ dominates, we have

$$W_2\xrightarrow[R]{} w_2(\kappa)\nu^{-1}, \tag{4}$$

$$W_1\xrightarrow[R]{} w_1(\kappa)\nu, \tag{5}$$

where $w_i\neq 0$.

Our essential assumption is that T_2 satisfies the Deser-Gilbert-Sudarshan (DGS) representation[9]

$$W_2(\kappa,\nu)=\int_0^\infty da\int_{-1}^{1}db\,\kappa\sigma_2(a,b)\times\delta(\kappa+2b\nu+b^2-a)\epsilon(\nu+b). \tag{6}$$

We assume that the spectral function $\sigma_2(a, b)$ vanishes rapidly for large a but that it may be singular at $b=0$. We shall always take $\nu>1$ so that we can put $\epsilon(\nu+b)=1$ in (6).

In the A limit we find

$$\nu W_2 \underset{A}{\rightarrow} \frac{\kappa}{2}\int da\, \sigma_2\left(a, \frac{1}{2\rho}\right).$$

Thus, in order that the limit be finite, we must have

$$\int da\, \sigma_2(a, 1/2\rho)=0. \tag{7}$$

Then (6) gives

$$\nu W_2 \underset{A}{\rightarrow} -\frac{1}{4\rho}\int da\, \sigma_2'\left(a, \frac{1}{2\rho}\right)a \equiv F_2(\rho), \tag{8}$$

where

$$\sigma_2'(a, b)=\partial\sigma_2(a, b)/\partial b.$$

If σ_2' were finite at $b=0$, we would have

$$F_2(\rho) \xrightarrow[\rho\to\infty]{?} -\frac{1}{4\rho}\int da\, \sigma_2'(a, 0)a.$$

To see if this is possible, we calculate the R limit of (6):

$$W_2 \underset{R}{\rightarrow} \frac{\kappa}{2\nu}\int da\, \sigma_2\left(a, \frac{a-\kappa}{2\nu}\right). \tag{9}$$

We now take $\nu \gg -\kappa \gg 1$ so that, since σ_2 is assumed to vanish rapidly for large a, we can assume that $-a/\kappa \ll 1$ inside the integral. Then, using (7), (9) becomes

$$W_2 \underset{R}{\rightarrow} \frac{\kappa}{4\nu^2}\int da\, \sigma_2'\left(a, -\frac{\kappa}{2\nu}\right)a, \qquad \nu \gg -\kappa \gg 1. \tag{10}$$

Thus, ignoring possible difficulties at $b=0$, we find

$$W_2 \underset{R}{\overset{?}{\rightarrow}} \frac{\kappa}{4\nu^2}\int da\, \sigma_2'(a, 0)a. \tag{11}$$

Since this violates (4), we must conclude that σ_2' is singular at $b=0$. It must, in fact, diverge linearly to account for the extra power of ν^{-1} in (11). Therefore we can write

$$\sigma_2'(a, b)=\sigma(a)b^{-1}+\tau(a, b), \tag{12}$$

where $\tau(a, b)$ is less singular at $b\sim 0$ than b^{-1}. It then follows from (7) [which implies $\int da\, \sigma_2'(a, b)=0$] that both

$$\int da\, \sigma(a)=0 \tag{13}$$

and $\int da\, \tau(a, b)=0$.

Insertion of (12) into (10) gives

$$W_2 \underset{R}{\rightarrow} -\frac{1}{2\nu}\int da\, \sigma(a)a \equiv w_2\nu^{-1}, \qquad \nu \gg -\kappa \gg 1. \tag{14}$$

Thus we are now consistent with (4) and find that $w_2(\kappa)\sim w_2$ is independent of κ for large κ. We can find $w_2(\kappa)$ for general κ by integrating (12) to obtain

$$\sigma_2(a, b)=\sigma(a)\ln b+\eta(a, b), \tag{15}$$

where η is less singular at $b\sim 0$ than $\ln b$. Substitution in (9) gives, in view of (13),

$$W_2 \underset{R}{\rightarrow} \frac{\kappa}{2\nu}\int da\, \sigma(a)\ln(a-\kappa) \equiv w_2(\kappa)\nu^{-1}. \tag{16}$$

This reduces to (14) for large κ. For use below we define here the photon amplitude

$$\hat{W}_2(\nu)=-\lim_{\kappa\to 0}\kappa^{-1}W_2(\kappa, \nu)$$

and find from (16) that

$$\hat{W}_2(\nu) \underset{R}{\rightarrow} -\frac{1}{2\nu}\int da\, \sigma(a)\ln a \equiv \hat{w}_2\nu^{-1}.$$

We now return to (8) and use (12) to find

$$F_2(\rho) \xrightarrow[\rho\to\infty]{} -\tfrac{1}{2}\int da\, \sigma(a)a = w_2,$$

which is the desired result (3). Although it is conceivable that $w_2=0$, we shall indicate below that this is not the case. Thus (3) will be satisfied in any theory obeying Bjorken behavior (2), Regge behavior (4), and the representation (6) with a rapidly decreasing spectral function.

We can obtain a rough estimate of w_2 as follows. We assume that the a integrations are approximately saturated near some effective squared mass a_0. Then

$$\int da\, \sigma(a)\ln a \cong \int da\, \sigma(a)\left[\ln a_0 + \frac{a-a_0}{a_0}+\cdots\right] \cong a_0^{-1}\int da\, \sigma(a)a,$$

using (13). Thus we have

$$\hat{w}_2 \cong w_2 a_0^{-1}.$$

Now the total γp cross section $\sigma_\gamma(\nu)$ satisfies

$$\sigma_\gamma(\nu) \xrightarrow[\nu\to\infty]{} 4\pi^2\alpha\hat{w}_2 \cong 4\pi^2\alpha w_2 a_0^{-1}.$$

Taking $a_0^{1/2}$ to be the ρ mass and using $\sigma_\gamma(\infty) \sim 120\ \mu\text{b}$,[10] this gives $w_2\sim 0.6$, in rough agreement with the experimental[5] value ~0.35 in view of the uncertainties in our choice of a_0 and $\sigma_\gamma(\infty)$.

The above procedures can be applied to W_1 in exactly the same way. The representation is[11]

$$W_1(\kappa, \nu) = \int da db [\nu \sigma_1(a, b) + \bar{\sigma}_1(a, b)] \delta(\kappa + 2b\nu + b^2 - a),$$

and we find

$$W_1 \underset{R}{\rightarrow} w_1(\kappa)\nu, \quad w_1(\kappa) \sim w_1 \kappa^{-1}, \tag{17}$$

$$F_1(\rho) = \tfrac{1}{2}\int da\, \sigma_1\left(a, \frac{1}{2p}\right) \xrightarrow[\rho \to \infty]{} -w_1\rho. \tag{18}$$

Estimating as above, we find $-w_1 \cong w_2$.

The form (15) of the spectral function can also be used to determine the asymptotic properties of the complete amplitude $T_2(\kappa, \nu)$, where $W_2 = \pi^{-1} \operatorname{Im} T_2$. In fact, this form can be derived directly from the DGS representation of T_2 by considerations similar to those given above.

Although we have made extensive use of the representation (6), our conclusions do not depend on its validity. We can obtain the same results directly from the representation (1) by using (4) and (5) to determine the behavior of the commutator near a lightlike surface. This information is sufficient to determine the A limit of (1) and the result is in agreement with (3). This derivation will be given elsewhere.

Let us now compare our analysis with that of Abarbanel, Goldberger, and Treiman.[7] These authors assumed that the residue function $w_2(\kappa)$ is such that $0 < F_2(\rho) < \infty$ and that the A limit can be obtained by first taking the R limit and then taking $\kappa \to -\nu/\rho$. Our use of the DGS representation enables us (i) to relate the large-κ case to the $\kappa = 0$ case and thus roughly indicate that $F_2(\rho) \neq 0$ and (ii) to take the A limit directly, without assuming that the above double limit is equivalent to the A limit.

Although our analysis is far from rigorous, we conclude that the property (3) is not so surprising but is likely to be a consequence of the generally accepted behavior (4) and the representation (6) or (1). Real optimism, however, must await further experimental tests of (3) and (18).

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[1]J. D. Bjorken, Phys. Rev. (to be published).

[2]For recent investigations of these functions, see J. D. Bjorken, Phys. Rev. 163, 1769 (1967); C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969), and 21, 311 (1968).

[3]In Eq. (1), $|p\rangle$ is a one-nucleon state of momentum p^μ (we take the nucleon mass to be 1 so that $p^2 = 1$), $\nu = q \cdot p$, $\kappa = q^2$, and an average over nucleon spins is understood.

[4]We refer to this limit as the A limit.

[5]The experimental results are summarized in the talk by W. K. H. Panofsky, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 23.

[6]We refer to this limit as the R limit.

[7]H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969), have also recently derived (3) in this way. We shall compare our work with theirs below.

[8]See, for example, A. H. Mueller and T. L. Trueman, Phys. Rev. 160, 1296, 1306 (1967).

[9]For references and a discussion of the validity of (6) and the analogous representation of W_1, see J. M. Cornwall and R. E. Norton, Phys. Rev. 173, 1637 (1968). We emphasize the presence of the κ factor in (6), which is necessary to enforce the kinematical vanishing of W_2 at $\kappa = 0$. Without this factor our calculation would be much simpler—but incorrect.

[10]Experimental limits: H. B. Hilbert et al. [Phys. Letters 27B, 474 (1968)] give $\sigma_\gamma = 116 \pm 17\ \mu$b at 3.5 to 5.4 GeV, and J. Ballam et al. [Phys. Rev. Letters 21, 1544 (1968)] give $\sigma_\gamma = 126 \pm 17\ \mu$b at 7.5 GeV.

[11]The $\nu\sigma_1$ spectral function is present because the ν dispersion relation for W_1 requires a subtraction. See Ref. 9.