hand, no cutoff is necessary, double spectral functions can be largely ignored, and the role of the Pomeranchukon[5] is clearer.

(d) Crossing is given first attention and unitarity is numerically imposed as well as possible, in contrast to partial-wave dispersion-relation approaches where unitarity is explicit and crossing symmetry is sought by computation.

Equations (1) and (2) can be written for more general processes and in more general forms[3,14] for handling more complicated problems. Applications extend in at least two directions, dynamical and phenomenological. Firstly, one may examine the questions raised by Collins, Johnson, and Squires[7] and see whether a parameter-free bootstrap appears possible, and secondly one may use the approach to help in constructing sets of low-energy phase shifts (π-N, K-N, etc.) consistent with high-energy models. We hope to consider some of these questions in the near future.

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REGGE TRAJECTORIES AND PARTICLE SIZE*

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Two models of rising trajectories are contrasted. The first, based on single-channel rotational excitations, leads to a mean square particle radius which increases linearly with angular momentum for excitations on a trajectory. The second model, which makes use of a conjectured weakly violated selection rule, gives a particle radius which is constant along a trajectory. The approximate selection rule explains the narrowness of the higher resonances.

In this article we consider two mechanisms leading to linear rising Regge trajectories. Both models are nonrelativistic but we believe that similar mechanisms can exist in a relativistic setting. The two models lead to radically different behavior of form factors for particles lying on a trajectory. The measurement of various form factors of higher Regge excitations should distinguish the two models.

(A) Harmonic oscillator model. – The model of mesons as quark-antiquark bound states, bound in a three-dimensional harmonic potential, has enjoyed some popularity because the spectrum of states is described by a linear trajectory together with underlying even daughter trajectories. If this model makes physical sense then we would expect the mean square radius of the excited states to increase linearly with increasing angular momentum. Although for unstable particles the concept of a radius and charge distribution are necessarily somewhat ambiguous, we assume that in the narrow-resonance approximation they are approximately defined. In that case the oscillator model would predict increasing charge radius along a trajectory.

Note that if we identify the average particle radius through Regge's formula[1]

$$\langle r^{-2}\rangle^{-1} \sim \frac{2l+1}{2m}\frac{dl}{dE} \qquad (1)$$

then a linear trajectory predicts a linearly increasing radius squared. Equation (1) of course merely represents the centrifugal effects of rotation on the size of a bound system.

In order to guess the relativistic generalization of Eq. (1) we employ an analogy between relativistic physics at infinite momentum and Galilean two-dimensional physics discussed recently by the authors.[2] The result is

$$(r^{-2})^{-1} \sim 2l\, dl/dm^2, \qquad (2)$$

where m^2 is the square of the mass of a bound system and l is the longitudinal component of angular momentum. Both the harmonic oscillator wave functions and Eq. (2) suggest a linear increase of squared radius for a linear trajectory.

(B) Coupled-channel system. – We next present a second nonrelativistic model of linear trajectories which does not lead to increasing size. The model consists of an infinite set of coupled channels with higher and higher spin angular momenta. The states high on a trajectory are approximately bound states in high spin channels and therefore do not suffer from the centrifugal effects of high orbital angular momentum.

Consider a set of particles with the spectrum of states of the three-dimensional oscillator. A basis of particle states is provided by the states

$$a_i^\dagger \cdots a_k^\dagger |0\rangle = |i, \cdots, k\rangle, \qquad (3)$$

where $a_i^\dagger$ is a creation operator for excitation in the ith direction. The subscript i goes over x, y, and z, and the states $|i, \cdots, k\rangle$ with n entries transform under rotation as a symmetric nth-rank tensor. The square of the mass of such a state is given by $n = n_x + n_y + n_z$.

We now postulate that each particle of total excitation n is a bound state in the scattering of particles of excitation m and $n-m$ so that a selection rule in n exists. We assume spin-independent forces, which bind a single s-wave bound state lying on an ordinary trajectory which rises, passes through[3] $l=0$, and then, somewhere above threshold, turns over as in ordinary potential theory. The coupling scheme between the particle $|i_l \cdots i_n\rangle$ and its constituents $|j_l \cdots j_m\rangle$, $|j_{m+1} \cdots j_n\rangle$ can be taken to be $\delta_{i_1 j_1} \delta_{i_2 j_2} \cdots \delta_{i_n, j_n}$ + symmetrizing terms. In this model the particles of high excitation and high spin derive their large angular momenta from the spins of their constituents and not orbital angular momentum.

In Fig. 1 we show the leading (maximum J) trajectories for different total excitation superimposed on one another. If a relatively weak perturbation which couples different values of n is introduced, we encounter a typical level-crossing repulsion phenomenon where the trajectories cross. The effect will be to produce a single new trajectory somewhat above the old trajectories (see Fig. 2). The new trajectory will be somewhat straightened out and along it we will find the perturbed states of the uncoupled system. The graph shows the real part of the trajectory. Qualitatively we may express this situation by saying that as we proceed along a trajectory, the states couple most strongly to closed channels of higher and higher spin. The decay process on any state is forbidden in the limit of n conservation which would suggest that the widths of the

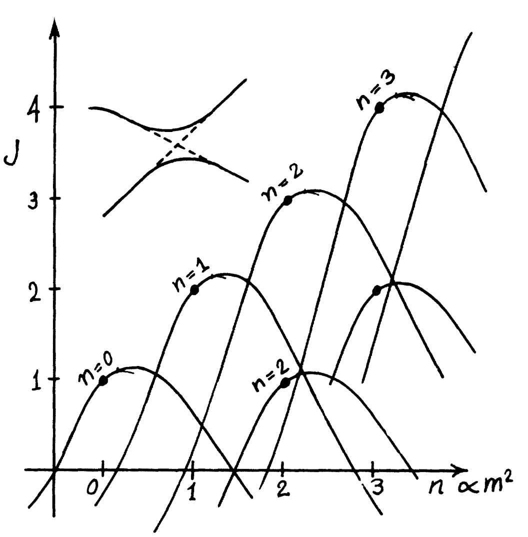

FIG. 1. Regge trajectories of the many-coupled-channel model in the approximation in which the channels are uncoupled. The channels are defined by a quantum number $n = n_x + n_y + n_z$ and are decoupled when n is conserved. The insert illustrates level repulsion when there is a weak interchannel coupling.

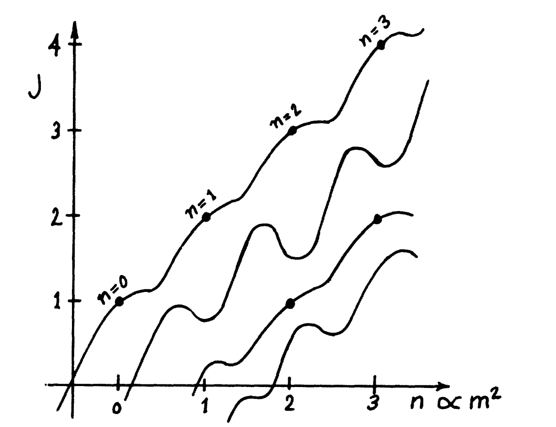

FIG. 2. Regge trajectories of the many-coupled-channel model when level repulsion is invoked at each point where trajectories of the decoupled channels intersect. Stronger interchannel coupling tends to flatten the leading trajectory into a straight line.

unstable particles tend to stay small as we increase their mass.

We emphasize that these models should be distinguishable by measurements of electromagnetic form factors or diffraction scattering[4] involving higher Regge excitations. The charge radius in the oscillator model would increase as $J^{1/2}$ along a trajectory while the lack of centrifugal effects in the coupled-channel model suggest a universal size for a single trajectory.

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DUALITY AND THE HADRON SPECTRUM*

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A form of exchange degeneracy for mesons and baryons is derived from duality and the absence of resonances in exotic channels. The implications of this structure for the hadron spectrum are discussed.

From crossing, SU(3) symmetry, no resonances in exotic channels, and a weak form of duality,[1] we deduce a form of exchange degeneracy which implies specific patterns in the hadron mass spectrum. Nothing in our approach is in conflict with unitarity. In particular, pole residues factor and trajectories need not be linear. We find exchange-degenerate nonets for mesons, while the "natural" solution for baryons requires the degeneracy of a $\underline{1}\oplus\underline{8}$ of trajectories with one signature and an $\underline{8}\oplus\underline{10}$ of the other.[2] For mesons, we have all representations in $(\underline{3}^*\otimes\underline{3})\oplus(\underline{3}\otimes\underline{3}^*)$, while for baryons all representations in $\underline{3}\otimes\underline{3}\otimes\underline{3}$ appear. The result for mesons is essentially well known[3] and consistent with the quark model ($\underline{36}$, all L).[4] Our result for baryons is new and suggests the following quark model structure for the baryon states: $\underline{56}\oplus\underline{70}$, even L; $\underline{20}\oplus\underline{70}$, odd L. For mesons, we then deal with all representations in $(\underline{6}^*\otimes\underline{6})\oplus(\underline{6}\otimes\underline{6}^*)$; for baryons, all representations in $\underline{6}\otimes\underline{6}\otimes\underline{6}$ are included.

The scattering amplitude for a process is divided into two parts, a resonant amplitude and the background.[5] The background is assumed to be primarily diffractive while the remaining resonant piece may be expressed at high energies in terms of Regge trajectories. In "exotic" channels where we expect no resonances to appear, we assume that the imaginary part of the resonant amplitude is zero. This weak form of duality is satisfied by requiring cancellation of the imaginary parts of the contributing Regge trajectories.[3] This cancellation, together with factorization, can be accomplished only with definite patterns of exchange-degenerate trajectories. We work in the SU(3)-symmetric limit where we assume the nonexotic channels to be $\underline{1}$ and $\underline{8}$ for mesons and $\underline{1}$, $\underline{8}$, and $\underline{10}$ for baryons.

The reactions considered are

$$\begin{aligned} &M+M' \to M''+M''', \\ &M+B \to M'+B', \\ &M+B \to M'+\Delta, \\ &M+\Delta \to M'+\Delta'. \end{aligned} \tag{1}$$

M stands for an octet of mesons, and B and Δ correspond to an octet and decuplet of baryons.[6]

To derive exchange-degeneracy constraints on s-channel trajectories and residues, consider a definite t- or u-channel SU(3) representation a. The imaginary part of the nondiffractive piece of the scattering amplitude for large t or u is given by

$$\begin{aligned} &\mathrm{Im}A_{(t)}{}^a = \textstyle\sum_b (X_{ts})^{ab}\,\mathrm{Im}R_{(s)}{}^b, \\ &\mathrm{Im}A_{(u)}{}^a = \textstyle\sum_b (X_{us})^{ab}\,\mathrm{Im}R_{(s)}{}^b, \end{aligned} \tag{2}$$

where X_{ts} is the crossing matrix from the s to