PREDICTIONS OF DIFFRACTION MINIMA IN ELASTIC ELECTRON SCATTERING USING FINITE NUCLEAR POTENTIALS*

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Finite nuclear potentials are compared with the conventional harmonic-oscillator potential in studying the diffraction minima in elastic electron scattering from O^{16} and C^{12}. The effects of admixtures of two-particle, two-hole configurations in the ground states on the form factors are considered.

Elastic electron scattering has, in the past, been a rich source of information regarding the ground-state charge distribution $\rho(r)$ of nuclei.[1,2] In particular, theoretical fits to the experimentally measured elastic form factor $F(q)$ have yielded much of our current knowledge about the size and shape of the charge distributions for the nuclei under consideration.[3-6] Moreover, assuming a particular form for the single-particle wave functions of the nucleus has allowed one to choose the correct parameters for the assumed independent-particle potential by comparing the form factor predicted there with the experimental results.[7-9] In this way, for example, the oscillator parameter b, characterizing the often-used harmonic oscillator (h.o.) single-particle wave functions, may be obtained for a given nucleus. In such nuclei as O^{16} and C^{12} reasonable fits to the form factors through the first diffraction minimum ($q \leq 400$ MeV/c) have been possible utilizing h.o. single-particle wave functions.[10] More recent experimental results on light closed-shell nuclei[11,12] have demonstrated that for $q \sim 600$ MeV/c there exists a diffraction minimum not predicted by the h.o. shell model. For the case of He^4, the data exhibit a single diffraction minimum ($q = 625$ MeV/c), while a $(1s_{1/2})^4$ configuration of oscillator states would yield none.

Gibson et al.[7] have employed a basis formed from wave functions in a finite single-particle well which has a hard core, including an appropriate center-of-mass correction, to obtain the diffraction minimum in He^4. Other authors[13,14] have demonstrated that the hard-core–induced correlations can produce additional diffraction minima, although we feel that there has been no really satisfactory treatment of the difficult problem of treating the correlations.

The purpose of this note is to demonstrate that an important and straightforward mechanism for producing the observed second minimum in O^{16} is the utilization of basis wave functions derived from a finite potential well. In addition we predict a second minimum for C^{12} at about the upper limit of the current existing data.[12] We also discuss the effect of configuration mixing in the ground states of C^{12} and O^{16} on the predicted shape of the form factor. The effect is small. The appropriate correction for the center of mass[15] has been utilized and is found to shift the second diffraction minimum ~30-50 MeV/c outward for O^{16} and C^{12}. The important point here is that we have obtained diffraction minima in the region 600-700 MeV/c without introducing correlations.

The basic expression for the elastic form factor of a spin-0 nucleus in the first Born approximation can be written as

$$F(q) = \exp\left[\frac{1}{A}\left(\frac{bq}{2}\right)^2\right]\frac{1}{Z}F(\text{s.n.})G(\text{D.F.})\Big[\sum_{nlj}(2j+1)M_{nl}^{\ nl}(q) - \sum_{\text{h}} B_{\text{h}}^{\ 2} M_{(nl)\text{h}}^{\ (nl)\text{h}}(q) + \sum_{\text{p}} B_{\text{p}}^{\ 2} M_{(nl)\text{p}}^{\ (nl)\text{p}}(q)\Big], \quad (1)$$

where

$$M_{nl}^{\ nl}(q) = (nl|\sin(qr)/qr|nl). \quad (2)$$

The first sum is over all the proton states in the closed shell; the second sum is over the proton-hole states produced when a proton is promoted to a higher shell (the origin of the third sum) in a model including configuration mixing. Z is the charge of the nucleus, the B_{p}^2 represent the probability of finding a proton in the state (nl) outside the closed shell (the B_{h}^2 have a similar interpretation for holes), and q is the three-momentum transfer. The exponential term arises from a needed c.m. correction since we

use shell-model wave functions. The particular choice here is appropriate for harmonic-oscillator basis states. We have also included the single-nucleon form-factor correction[7] F(s.n.), and the Darwin-Foldy term[2,16] G(D.F.). The particle-hole (np-nh) terms are included because we exhibit results showing the change in the form factor when substantial 2p-2h configuration mixing is assumed in the ground state. In obtaining the expression above we have assumed that either the l or j quantum number of the promoted proton is different than for the hole it leaves behind.

The center-of-mass correction appropriate when finite-well basis states are used is considerably more complicated than the simple Gaussian factor used for harmonic oscillators. However, it is in principle known.[15] Theoretical arguments, too lengthy to reproduce here, lead us to the conclusion that the principal effect of the c.m. correction is to alter the location of the predicted diffraction minima from q to $[A/(A-1)]q$.

The elastic electron scattering form factors for O^{16} and C^{12} were calculated using Eq. (1) with harmonic-oscillator and Woods-Saxon single-particle basis states. In Fig. 1 we show the results for O^{16} treated as a closed-shell nucleus. Only the recent data of McCarthy and Sick[12] are shown in Figs. 1 and 3 for clarity; the previous data[3-6] agree within experimental error. An oscillator parameter of $b=(\hbar/M\omega)^{1/2}=1.77$ F was used in the case of h.o. wave functions to place the first diffraction minimum in agreement with experiment.[3,4,12] In the case of the Woods-Saxon well the parameters were also chosen to produce agreement with experiment for the first minimum and otherwise to be representative of such a potential. Subsequently, they were varied to test the sensitivity of the form factor to changes in the parameters. The results in Fig. 1 are given for a Woods-Saxon potential (including a Coulomb barrier) with radius $R=3.25$ F, diffuseness $a=0.5$ F, spin-orbit strength $V_S=6$ MeV, and well depth $V_0=50.6$ MeV, the latter being chosen to give the $1p_{1/2}$ level the proton separation energy of 12.1 MeV. These well parameters were also used for the $1p_{3/2}$(15.6 MeV) and $1s_{1/2}$(28.9 MeV) levels. For the case where 2p-2h configurations in the ground state were considered, involving particles in the $2s$-$1d$ shell, a similar well was used, except that the well depth was taken to be $V_0=58.6$ MeV in order to bind the $1d_{3/2}$ particle by (an arbitrarily chosen) 0.5 MeV, placing the $2s_{1/2}$ and $1d_{5/2}$ levels at 3.3 and 6.6 MeV, respectively. This procedure of course is highly arbitrary; however, Holder and Eisenberg[17] have shown that, when near threshold, continuum wave functions yield results very similar to those for bound states.

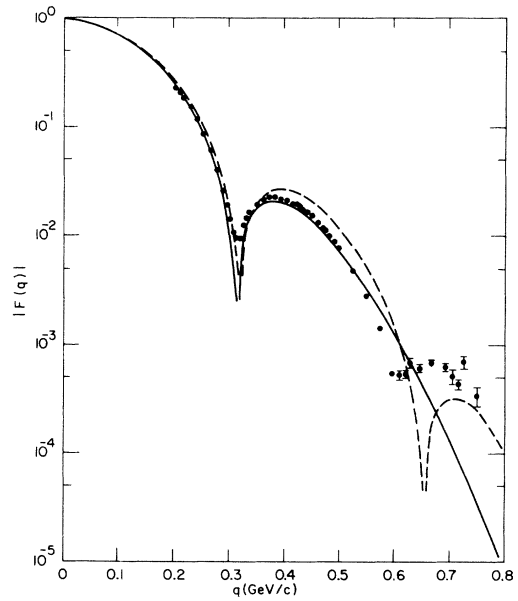

FIG. 1. Electron elastic-scattering form factor for O^{16}. The data are recent results of McCarthy and Sick (Ref. 12) carried to higher momentum transfer than previous data (Refs. 3 and 4) (not shown here for clarity). The solid curve is obtained using harmonic-oscillator wave functions and the dashed curve using Woods-Saxon wave functions treating O^{16} as a closed-shell nucleus with well parameters as described in text.

We see in Fig. 1 that the harmonic-oscillator and finite-well form factors are similar to about $q=600$ MeV/c, after which the latter has a diffraction minimum in qualitative agreement with the diffraction feature found experimentally. The harmonic-oscillator form factor has no other diffraction minimum besides the one near 300 MeV/c, whereas the finite-well form factor continues to undulate with increasing momentum transfer, having diffraction minima spaced at intervals of a few hundred MeV/c.

The form factor was found to be relatively more sensitive to changes in the radius and diffuseness parameters than to changes in the well depths or energy eigenvalue; detailed results will be reported elsewhere.

In Fig. 2 we show the results of h.o. calculations including configuration mixing.[18,19] The effects of configuration mixing are similar when Woods-Saxon wave functions are used. With h.o. wave functions, the inclusion of $2s$-$1d$ states al-

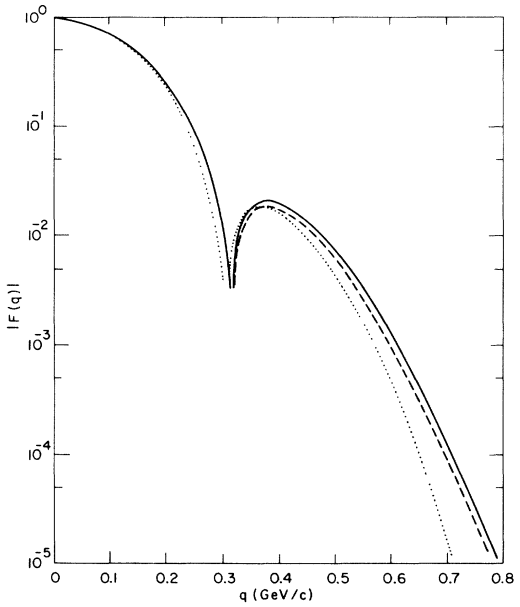

FIG. 2. As for Fig. 1, but showing results using harmonic oscillators for a closed shell (solid curve) and for 2p-2h configuration mixing as employed by Walker (Ref. 18) (dashed curve) and Agassi, Gillet, and Lumbroso (Ref. 19) (dotted curve).

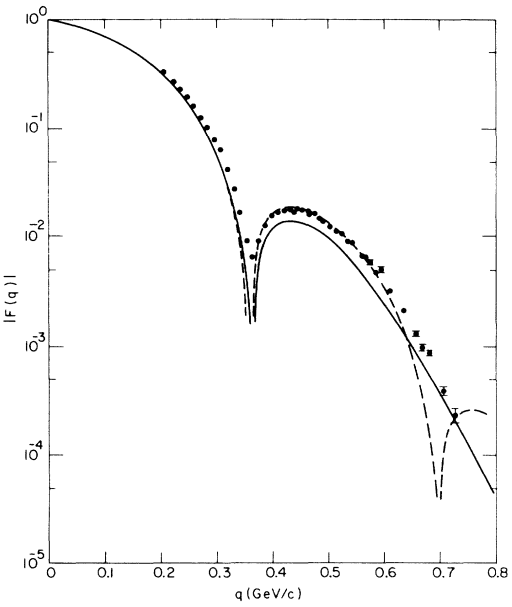

FIG. 3. As for Fig. 1, but for C^{12}, again showing the recent data of McCarthy and Sick (Ref. 12) and omitting the previous data (Refs. 3-6) for clarity. The solid curve is for harmonic-oscillator wave functions and the dashed curve for Woods-Saxon wave functions as described in the text.

lows the form factor to have a second diffraction minimum (beyond 750 MeV with the admixtures used here). However, configuration mixing of the type induced by traditional residual interactions, even in higher order random-phase-approximation calculations, are seen to produce only small changes in the form factors, and in particular, shift the position of the second diffraction minimum in the finite-well case by less than 40 MeV/c.

In Fig. 3 we have plotted the results of calculations for C^{12} treated as a closed $1p_{3/2}$ shell with both h.o. and Woods-Saxon basis states, again comparing with recent experimental data.[12] The oscillator parameter was taken to be $b = 1.64$ F and the Woods-Saxon parameters to be $R = 3$ F, $a = 0.5$ F, $V_S = 6$ MeV, and $V_0 = 55.5$ MeV (giving the $1p_{3/2}$ level the proton separation energy of 16.0 MeV and consequently placing the $1s_{1/2}$ level at 31.2 MeV). As in the case of O^{16} these parameters were chosen to position the first diffraction minimum at its experimental value. The Woods-Saxon calculation predicts a second diffraction minimum at about 700 MeV/c. With the c.m. correction producing a shift to $[A/(A-1)]q$, the minimum would be at about 760 MeV/c or at about the upper limit of the existing data.[12] However, it should be emphasized that another set of well parameters and the possibility of configuration mixing can change these numbers slightly.

The elastic form factor for He^4 has been treated by Gibson, Goldberg, and Weiss[7] using finite nuclear potentials. It appears that for He^4 the use of wave functions in a finite well is insufficient to explain the observed diffraction feature[11] unless unrealistic well parameters are employed.

The agreement obtained in comparing the finite-well calculation for O^{16} and C^{12} with experiment is good for momentum transfers less than 500-600 MeV/c. In the region 600-800 MeV/c, second diffraction minima are predicted, with the one for C^{12} occurring at higher values of q than for O^{16}. The possibility of using different well parameters, of introducing 2p-2h configuration mixing, and of treating the whole problem in higher order than first Born approximation can change the quantitative agreement with experiment. The first two considerations, along with the c.m. correction, can move the diffraction minima and change the magnitude of the form factor, while the last will fill in the zeros obtained in first Born approximation and, at high q, will also change the magnitude. However, it is clear that a powerful and straightforward mechanism for obtaining reasonable agreement

with experiment for O^{16} is simply to use more realistic single-particle basis states, and that perhaps some caution is needed in drawing conclusions about correlations.

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[1]R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).
[2]T. de Forest, Jr., and J. D. Walecka, Advan. Phys. 15, 1 (1966).
[3]H. Crannell, Phys. Rev. 148, 1107 (1966).
[4]H. F. Ehrenberg, R. Hofstadter, U. Meyer-Berkhout, D. G. Ravenhall, and S. E. Sobottka, Phys. Rev. 113, 666 (1959).
[5]H. L. Crannell and T. A. Griffy, Phys. Rev. 136, B1580 (1964).
[6]J. H. Fregeau and R. Hofstadter, Phys. Rev. 99, 1503 (1955).
[7]B. F. Gibson, A. Goldberg, and M. S. Weiss, Nucl. Phys. A111, 321 (1968).
[8]B. F. Gibson and K. J. van Oostrum, Nucl. Phys. A90, 159 (1967).
[9]L. R. B. Elton and A. Swift, Nucl. Phys. A94, 52 (1967).
[10]U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, Ann. Phys. (N.Y.) 8, 119 (1959).
[11]R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearian, Phys. Rev. 160, 874 (1967).
[12]We wish to express our thanks to J. S. McCarthy and I. Sick of the High Energy Physics Laboratory at Stanford for allowing us to use their preliminary data on O^{16} and C^{12} prior to publication.
[13]A. Małecki and P. Picchi, Phys. Rev. Letters 21, 1395 (1968).
[14]C. Ciofi Degli Atti, Nucl. Phys. A106, 215 (1968) and Nuovo Cimento 55B, 570 (1968).
[15]S. Gartenhaus and C. Schwartz, Phys. Rev. 108, 482 (1957).
[16]K. W. McVoy and L. van Hove, Phys. Rev. 125, 1034 (1962).
[17]F. D. Holder and J. M. Eisenberg, Nucl. Phys. A106, 261 (1968).
[18]G. E. Walker, Phys. Rev. 174, 1290 (1968).
[19]D. Agassi, V. Gillet, and A. Lumbroso, Commissariat à l'Energie Atomique Saclay Report, 1968 (unpublished).

THE (^{3}He,p) REACTIONS TO THE ANALOG STATES IN THE REGION OF $N = 28$

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The (^{3}He,p) reactions on ^{46}Ti, ^{48}Ti, ^{52}Cr, ^{54}Fe, ^{56}Fe, ^{58}Ni, and ^{60}Ni leading to the 0^+ analog states were studied and compared with the (t,p) reactions on the same target nuclei and with the predictions of the pairing vibration model.

A systematic study of (^{3}He,p) reactions on even target nuclei with mass number $A = 46$-60 was undertaken with special emphasis upon the $L = 0$ transitions to the analog states. These reactions are made particularly interesting by comparing them with the (t,p) reactions on the same target nuclei.[1] Moreover, the study of $L = 0$ transitions to both analog and antianalog states provides a test of the predictions of the pairing vibration model.[2]

The experiments have been performed using 18-, 17.5-, and 15-MeV ^{3}He-ion beams supplied by the Universität Heidelberg's Model EN tandem Van de Graaff generator. The Ni targets were self-supporting; all the others were made by evaporation onto 1- to 1.5-mg/cm^2 gold foils. The target thicknesses ranged from 100 to 250 μg/cm^2. The protons were analyzed by a broad-range magnetic spectrograph and registered on nuclear emulsion plates. Absolute cross sections were determined with a probable error of less than 30%. Measurements were made in most cases at 5, 10, 20, and 40 deg, which was sufficient to identify the $L = 0$ and many $L = 2$ transitions. The resolution was about 35 keV. Spectra at $\theta_{lab} = 5°$ are shown schmetically in Fig. 1.

The ground-state analogs were identified by using the known reaction Q values[3] and the Coulomb displacement energies given by Sherr, Blair, and Armstrong.[4] Actual agreement with these Coulomb displacement energies was found to better than 20 keV for the analogs of the ground state and for most of the excited states.[5]

The (^{3}He,p) reaction leading to the analog state and the (t,p) reaction leading to the parent state describe an identical process under the following assumptions: All states involved have good isospin and the reaction mechanism is independent of the charge of the mass-3 projectile. In particular, $0^+ \rightarrow 0^+$ transitions involving the transfer of