wave function were purely

$$|[[f_{7/2}\nu f_{7/2}\nu]^{j=0} d_{3/2}{}^{-1}\pi]^{J=\frac{3}{2}}\rangle,$$

the spectroscopic factor of this 5^- state would be 0.46. In the case of the 3.95-MeV 4^- state, the spectroscopic factor is 0.47 and the calculated spectroscopic factor is 0.04. The 0.04 is due almost entirely to the

$$|[[f_{7/2}\nu f_{7/2}\nu]^{j=2} d_{3/2}{}^{-1}\pi]^{J=\frac{3}{2}}\rangle$$

component of the ^{41}K wave function. Similarly, the spectroscopic factor for the 4.44-MeV 2^- state is 0.36, and the corresponding calculated spectroscopic factor is essentially zero.

The small values calculated for the ^{42}Ca spectroscopic factors of the 4^- and 2^- states can be directly attributed to the weak contribution of the ^{40}Ca 4^- and 2^- RPA core states. Thus, the strength of the lowest two odd-parity core states in the RPA is overestimated to the exclusion of the other core states. This overestimation is a reflection of a corresponding overestimation of the ground-state correlations in ^{40}Ca. This exclusion is an RPA effect; that is, when shell-model core states are employed, many core states are mixed in the low-lying levels of ^{42}Ca. Unfortunately, there is no theory which simultaneously accounts for the transition rates and the spectroscopic factors.

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SHELL MODEL FOR $N=82$ NUCLEI*

B. H. Wildenthal†

Texas A & M University College Station, Texas 77843

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Characteristics of the level structure of $N=82$ nuclei are studied with a shell model.

There is extensive evidence that, in the language of the nuclear shell model, 50 and 82 constitute good "magic numbers." It thus seems reasonable to suppose that nuclei with 82 neutrons and $Z > 50$ protons can be well described by a shell model in which the $Z=50$ and $N=82$ shells are closed while the remaining $(Z-50)$ protons are distributed over the next higher major shell. In the usually accepted ordering of the single-particle orbits above $Z=50$, the $1g_{7/2}$ and $2d_{5/2}$ orbits come lowest in energy, being almost degenerate with each other, while the $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$ orbits lie about an MeV or so higher. Kisslinger and Sorenson[1] initiated theoretical examination of these nuclei with their approximate pairing–shell-model calculations, and this same type of approach has been followed in later studies.[2]

Until recently, conventional shell-model calculations for the $N=82$ isotones were not feasible because of the necessity of including both the $1g_{7/2}$ and $2d_{5/2}$ shells on an essentially equal footing, since the relatively high spins of these orbits result in quite large dimensions for the shell-model matrices to be handled. However the development of sophisticated computer codes[3,4] now allows straightforward calculation of shell-model structure in such large spaces. At the same time, the recent explosion of experimental information[5-7] on the properties of these nuclei has stimulated effort to produce a comprehensive theoretical explanation of the observed phenomena. We describe here some initial results from a shell-model calculation which seems remarkably successful in accounting for salient features of the $N=82$ nuclei.

In forming our shell-model vector space we allow the n active protons outside the $Z=50$, $N=82$ core to form any configuration involving $1g_{7/2}$ and $2d_{5/2}$ orbits only. In addition, we allow all configurations in which $n-1$ protons occupy the "$\frac{7}{2}$-$\frac{5}{2}$" space, while one particle is in either the $3s_{1/2}$ or $2d_{3/2}$ orbit. We have not as yet included the $1h_{11/2}$ orbit in the calculation and this means that we do not consider negative-parity levels. No difficulty in extending the calculations to include this feature is anticipated. With our model, eight-particle states (which would correspond to states in ^{140}Ce) have from between 100 and 300 components in their wave functions.

We parametrize the two-body part of the shell-model Hamiltonian in terms of the modified surface delta interaction (MSDI).[8] Thus the Hamiltonian is determined by six adjustable parameters: a strength A of the surface delta interaction, a strength B for the extra (modifying) monopole term, and the single-particle energies for the $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, and $3s_{1/2}$ orbits. The strength A and the three single-particle-energy splittings (the only parameters which affect the energy-level spectra) were adjusted to give a least-squares fit to 40 excitation energies of levels of known J^π in $N=82$ nuclei from $A=136$ to $A=145$. The strength B and the absolute value of the $g_{7/2}$ single-particle energy were adjusted to give a best fit to the known binding energies of $N=82$ ground states. The values of the Hamiltonian parameters thus determined were $A=0.383$ MeV, $B=0.597$ MeV, $E_{7/2}=-10.14$ MeV, $E_{5/2}=-9.62$ MeV, $E_{3/2}=-7.02$ MeV, and $E_{1/2}=-7.19$ MeV. With this model Hamiltonian we have calculated level energies and wave functions for ten of the $N=82$ nuclei. We report here some model results for the ground and first excited states of the odd-mass nuclei and for the excitation spectrum of the ^{140}Ce nucleus.

The ground states of ^{137}Cs and ^{139}La have $J^\pi=\frac{7}{2}^+$, and their first excited states have $J^\pi=\frac{5}{2}^+$. For ^{141}Pr, ^{143}Pm, and ^{145}Eu this sequence is reversed, the ground states having $J^\pi=\frac{5}{2}^+$. In all cases the $\frac{5}{2}^+$ and $\frac{7}{2}^+$ states are separated from higher excited states by an energy gap of about 1 MeV. The energy gaps and $\frac{7}{2}^+$-$\frac{5}{2}^+$ orderings are given quite accurately by our model. Figure 1 shows the behavior of the $\frac{7}{2}^+$ and $\frac{5}{2}^+$ states as a function of the mass number. The shell-model wave functions for these states can be characterized as describing a $g_{7/2}$ or $d_{5/2}$ proton coupled to the 0^+ ground state of the $A-1$ even-mass nucleus. There is good agreement between calculated and measured spectroscopic factors for single-

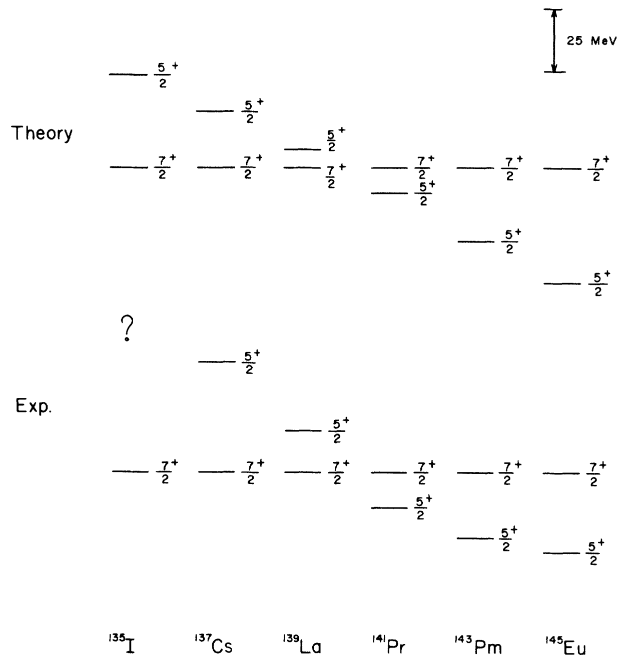

FIG. 1. The spins of the ground and first excited states of $N=82$ nuclei as a function of mass.

nucleon transfer to these states, which indicates that the model wave functions have realistic $g_{7/2}$-$d_{5/2}$ mixtures. (It should be noted that the experimental $\frac{7}{2}^+$-$\frac{5}{2}^+$ trends are also correctly predicted by the earlier pairing calculations.[1,2])

The ^{140}Ce nucleus provides the most thoroughly documented experimental energy-level spectrum[7] against which to match the detailed energy-level results of the model. Experiment and theory for ^{140}Ce are compared in Fig. 2. The main features of the model spectrum are as follows: a 2^+ first-excited state followed by a gap; a $0^+, 4^+, 6^+$ triad at about 1.9-MeV excitation followed by another gap; an octuplet of levels at about 2.4 MeV, including spins of 6, 5, 4, 3, 2, and 1; then still another gap. All of these shell-model features are matched by the experimental data. Of the first 13 model states, only the 5^+ level lacks a probable experimentally observed counterpart, and all of the experimentally observed even-parity states below 3-MeV excitation have shell-model counterparts.

We emphasize that the theoretical-experimental agreement for ^{140}Ce is typical of the overall results from this $N=82$ calculation, and that the same Hamiltonian was used for all of the ten nuclei studied. We think that the ease with which our few-parameter calculation reproduces the available experimental data indicates that our

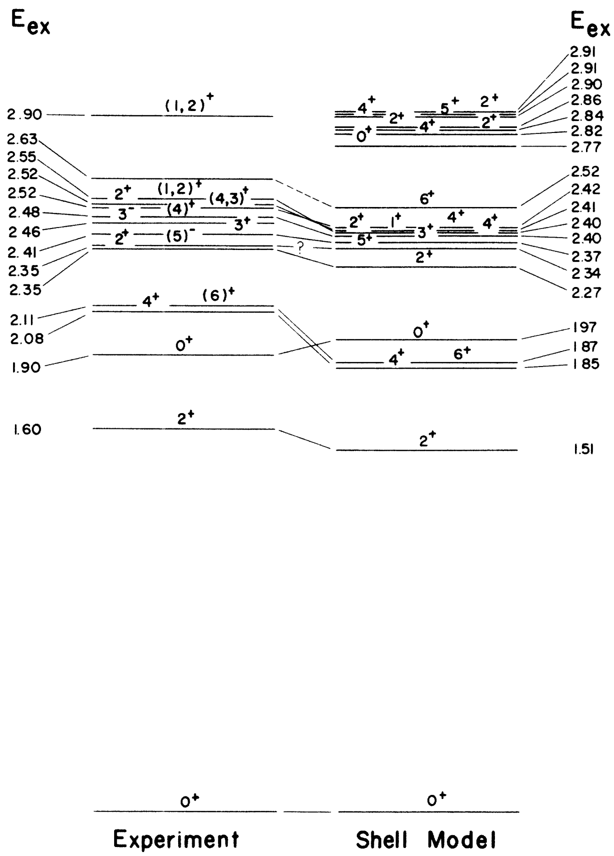

FIG. 2. Experimental and theoretical energy-level spectra of ^{140}Ce.

model vector space includes all the configurations that are of prime importance for describing levels of excitation $\lesssim$2 MeV for odd-mass nuclei and $\lesssim$3 MeV for even-mass nuclei. The model, on present evidence, can account for the main features of each of the first twenty or so energy levels of these nuclei.

The most desirable expansion of the basis space would be to include two-particle excitations into the $\frac{3}{2}^+$, $\frac{1}{2}^+$, and $\frac{11}{2}^-$ orbits, so as to be able to account for the traces of admixtures of these configurations which are indicated by data from pickup reactions. Even without this refinement, however, the present model yields results which indicate that the $N=82$ region is an especially suitable region for shell-model study.

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