RESONANCE CONES IN THE FIELD PATTERN OF A SHORT ANTENNA IN AN ANISOTROPIC PLASMA*

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We report experimental observation of resonance cones in the angular distribution of the radio-frequency electric field of a short antenna in a plasma in a static magnetic field. The cone angle is observed to vary with incident frequency, cyclotron frequency, and plasma frequency in agreement with simple plasma dielectric theory. We discuss the relationship of these cones to the limiting phase- and group-velocity cones which appear in the theory of plane wave propagation.

The electromagnetic fields and radiation of a short dipole antenna in an anisotropic plasma have been the subject of many theoretical studies,[1-4] and have become of practical interest in connection with investigations employing rocket and satellite vehicles. The analysis of an oscillating point dipole shows that the fields should become infinite along a cone whose axis is parallel to the static magnetic field and whose opening angle is determined by the plasma density, magnetic field strength, and incident frequency. It has also been shown that the Poynting vector is singular along these cones, yielding an infinite radiation resistance for the point dipole antenna, a result which has stirred considerable controversy.[5] Effects such as electron collisions, electron thermal velocities, and antennas of nonzero dimensions would be expected to cause the fields to remain finite along the cones. We report here experimental verification of the existence of resonance cones along which the observed fields become very large.

Consider an antenna oscillating at frequency ω in an infinite cold plasma with an applied static magnetic field $\vec{B}_0 = B_0\hat{z}$. The near-zone fields ($r \ll c/\omega$) may be derived using the quasistatic approximation $\vec{E} = -\nabla\varphi$. We must solve Poisson's equation $\nabla\cdot\vec{D} = \rho_{ext}$, where $\vec{D} = \epsilon_0 K\cdot\vec{E}$ and K is a tensor with $K_{xx} = K_{yy} = K_\perp$, $K_{xy} = -K_{yx} = K_H$, and $K_{zz} = K_\|$ as its nonzero components. Neglecting terms of order m_e/m_i, we have $K_\perp = 1 - \omega_p^2/(\omega^2 - \omega_c^2)$ and $K_\| = 1 - \omega_p^2/\omega^2$. Using Fourier transform methods we may solve for the potential of an oscillating monopole $\rho_{ext} = qe^{-i\omega t}\delta(\vec{r})$, yielding

$$\varphi(\rho, z) = qe^{-i\omega t}(4\pi\epsilon_0)^{-1}(K_\perp{}^2 K_\|)^{-\frac{1}{2}} \times(\rho^2/K_\perp + z^2/K_\|)^{-\frac{1}{2}} \quad (1)$$

in cylindrical coordinates. To solve for the potential and fields of an oscillating dipole we merely take the appropriate derivatives. All spatial derivatives of Eq. (1) will also have a singularity along the cone defined by the vanishing of the denominator,

$$K_\| \sin^2\theta + K_\perp \cos^2\theta = 0, \quad (2)$$

where $\theta = \tan^{-1}\rho/z$ is the polar angle in spherical coordinates. Thus the cones exist only in the frequency regions where either $K_\|$ or $K_\perp$ becomes negative, but not both. In a magnetoplasma these regions are ω less than both ω_p and ω_c, hereafter called the "lower branch," and ω greater than both ω_p and ω_c but less than the upper hybrid frequency $(\omega_p^2 + \omega_c^2)^{1/2}$, hereafter called the "upper branch." From Eq. (2) the resonance cone angle is given by

$$\sin^2\theta = \omega^2(\omega_p^2 + \omega_c^2 - \omega^2)/\omega_c^2\omega_p^2. \quad (3)$$

Kuehl[3] and others predict that the fields also become singular at these same cone angles in the far-zone region ($r \gg c/\omega$).

Using a steady-state argon rf discharge (pressure $\simeq 1\mu$) in the apparatus illustrated in Fig. 1(a), an experimental search for these cones was undertaken and proved successful. The source, or transmitting antenna, was fixed in the center of the plasma column. A second antenna was used to probe the fields set up by the transmitting antenna. This receiving antenna was constructed to rotate in a circular arc whose center is the transmitting antenna. The received signal was fed into a heterodyne receiver whose output drove an X-Y recorder. A typical trace showing the power received by the rotating probe versus the angle the rotating probe makes with the magnetic field direction is also included in Fig. 1(b). When the cone angle exceeded about 60°, multiple re-

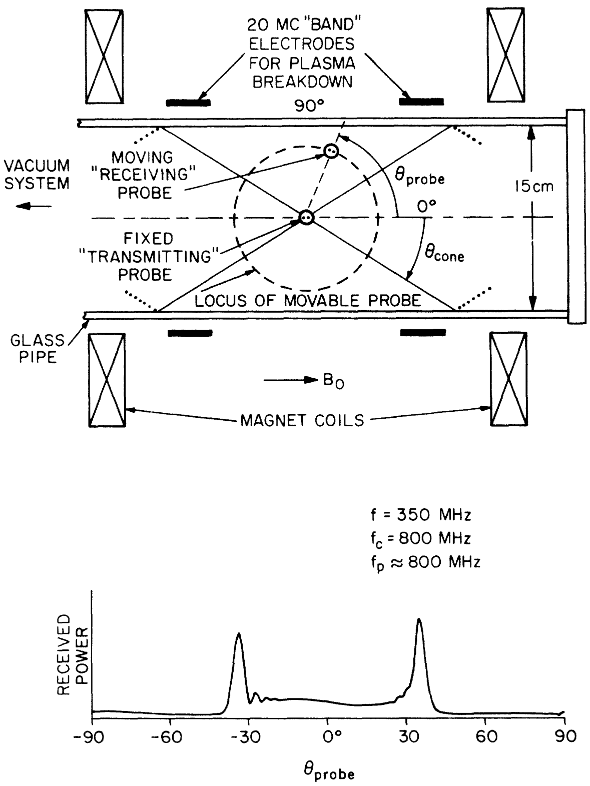

FIG. 1. (a) Schematic of experiment to measure resonance cone angles. (b) Portion of a typical trace showing the power received by the moving probe versus the angle the receiving probe makes with the magnetic field direction.

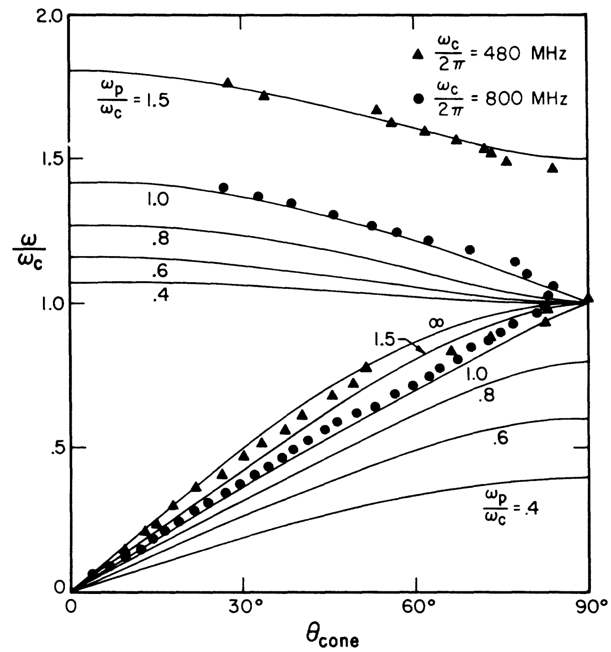

FIG. 2. Graph showing the experimentally observed location of the resonance cone angle versus the ratio ω/ω_c. Solid curves are those predicted by Eq. (3) for labeled values of ω_p/ω_c.

flections from the glass walls [dashed lines in Fig. 1(a)] complicated the traces.

The plasma density and magnetic field were held constant and traces were taken at various incident frequencies. Figure 2 is a plot of the experimentally observed cone angle versus the ratio ω/ω_c for two different values of ω_c. On the same graph the cone-angle–frequency relationship predicted by Eq. (3) is plotted for various values of the parameter ω_p/ω_c. The two sets of experimental data indicate a density roughly corresponding to $\omega_p/\omega_c \simeq 1$ and 1.5 (or $n_e \simeq 8\times10^9$ cm^{-3} and 6.5×10^9 cm^{-3}, respectively). An independent measurement using a 10.5-GHz microwave interferometer (not shown in Fig. 1) confirms these electron densities to within 20%. Data taken at the lower magnetic field show the upper and lower branches clearly separated by a frequency band in which, as predicted, no cones were present.

The cones studied in this experiment, which we shall refer to as group-velocity cones since they describe directions in which the group velocity vanishes, occur at the complementary angle of the phase-velocity cones which arise in the theory of plane-wave propagation.[6] The latter are sometimes referred to as resonance cones because the index of refraction tends to infinity (and the phase velocity tends to zero) as the angle ψ between the wave vector and the static magnetic field tends to the (phase-velocity) cone angle ψ_c. The latter are just the limiting cones in the Clemmow-Mullaly-Allis diagrams[6] which indicate the allowed direction (with respect to the magnetic field) and magnitude of the phase-velocity vectors. For the lower branch the angle ψ must be less than ψ_c, and for the upper branch the angle ψ must be greater than ψ_c. Kuehl,[3] in discussing the fields of an oscillating dipole in an uniaxial plasma ($B_0\rightarrow\infty$, for which there is no upper branch), shows that Poynting vector is nonzero only inside the group-velocity cone. Now in an anisotropic plasma the directions of the phase velocity and group velocity are generally different. Indeed, for the uniaxial case where cones exist ($\omega^2 < \omega_p^2$), we have shown that as the angle ψ of the phase-velocity vector increases from 0 to ψ_c, the angle θ of the group-velocity vector increases from 0 to $\theta_c = \frac{1}{2}\pi - \psi_c$, the complementary angle. Although the group velocity tends to zero as $\theta \to \theta_C$, the Poynting vector and energy density tend to infinity.[3] We believe that the necessity for examining allowed directions of the group-velocity vector rather than the allowed directions of the phase-velocity vector, when determining whether transmission between two antennas in a plasma in a magnetic field is possible, has not been sufficiently emphasized. While our experiment was not performed in the far-field region, the observed cone angles are those predicted by considerations of the group velocity and the fields in the far zone.

Measurement of the resonant cone angle can also be used as a diagnostic measure of the plasma density in a plasma in a magnetic field. The upper branch especially is very dependent on the electron density. This experiment was originally undertaken in part as an experimental check on what might be observed when a satellite equipped with a transmitting and a receiving antenna is sent aloft into the ionosphere. Since the ionosphere is an anisotropic plasma in the presence of the earth's magnetic field, then as the satellite tumbles in space, the signal propagating between the two antennas might be expected to exhibit peaks when the line between the antennas happens to lie along the surface of one of the cones set up by the transmitting antenna. Note that inclusion of the ion terms in the expressions for the plasma dielectric tensor would predict similar cones at frequencies near the ion cyclotron frequency.

In conclusion, we have experimentally observed the widely discussed resonance cones. We noted that similar cones might be expected to appear near the ion cyclotron frequency. We attempted to clarify the apparent discrepancy between the angle at which these cones appear and the limiting phase-velocity cones which occur in the theory of plane-wave propagation. Further studies into the nature of these cones is being undertaken.

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