INELASTIC ELECTRON-NUCLEON SCATTERING AND THE POMERANCHUK SINGULARITY*

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(Received 14 January 1969)

It is suggested that the amplitude for virtual Compton scattering at large spacelike photon masses is completely dominated at all energies by the Pomeranchukon exchange or diffraction mechanism and that all the contributions of s-channel N^* resonances and "ordinary" t-channel exchanges decrease rapidly with q^2. Experimental and theoretical implications of this conjecture are discussed.

Inelastic electron-nucleon scattering experiments are interesting in many ways. Their relevance to various aspects of the nucleon structure is widely recognized and their connection to specific commutators of current densities has been recently studied in great detail.[1] To lowest order in α, such experiments determine the total photoabsorption cross sections on nucleons. Consequently, we may indirectly use them to measure the imaginary part of the forward Compton scattering amplitude $f(\gamma N \rightarrow \gamma N)$ as a function of ν, the laboratory energy of the incoming (virtual) photon, and q^2, the square of its mass.

In this paper we study the amplitude for virtual Compton scattering assuming that, in many respects (and to the lowest order in α), it follows the usual behavior of elastic hadronic amplitudes. In particular, we suggest that the diffractive part (or the Pomeranchukon exchange contribution) of the Compton amplitude may exhibit a q^2 dependence which is completely different from that of the contributions of all the "ordinary" Regge trajectories, such as the P' or the A_2. We further propose that the residue functions of these "ordinary" trajectories have a q^2 dependence similar to that of the excitation form factors of the low-lying N^* resonances. These assumptions are consistent with (and partly motivated by) the preliminary analysis of partial results of a Stanford Linear Accelerator–Massachusetts Institute of Technology (SLAC-MIT) experiment[2] currently in progress at SLAC. They lead to specific testable predictions for future experiments and to some interesting consequences related to the Adler sum rule and the proton-neutron mass difference.

Our starting point is the possibility that the imaginary part of the amplitude for virtual Compton scattering on hadronic targets can be described in terms of the following currently accepted ideas of strong interaction dynamics: (i) At low energies the amplitude is dominated by s-channel N^* resonances. (ii) At high energies, a few t-channel Regge trajectories give an adequate approximate description of the forward amplitude. (iii) In the sense of finite energy sum rules, "ordinary" trajectories (P', A_2, etc.) are "built" mainly from s-channel resonances. (iv) The contribution of the Pomeranchuk singularity (otherwise known as the diffractive part of the amplitude) is mainly "built" from nonresonating background in the s channel.[3]

All these features of the imaginary part of the amplitude should be reflected by the total photoabsorption cross sections. On the other hand, the real part of the Compton amplitude probably involves terms which are absent in pure hadronic processes, such as the various proposed fixed poles.[4] Such terms are relevant to most of the electron-scattering sum rules, but they do not appear directly in the measured total γN cross sections.

The SLAC-MIT experiment[2] represents the first measurements of electron inelastic-scattering cross sections in the energy region above the masses of the first few N^* resonances. One dramatic feature revealed by this experiment for the first time is the observation that the q^2 variation of the total photoabsorption cross section is much more moderate at high energies than in the region of the lowest N^* states. The excitation form factors of the 1236-, 1520-, and 1688-MeV resonances behave roughly like the nucleon form factors and "disappear" very rapidly at large q^2. On the other hand, the high-energy photoabsorption cross sections ($\nu = 2$-8 BeV, $q^2 = 1$-2 BeV^2) change very slowly with q^2, perhaps slower than $1/q^2$.

This remarkable experimental fact is somewhat puzzling. Consider fixed-q^2 finite-energy sum rules in ν for the inelastic electron-proton form factor[5] $W_2(q^2, \nu)$:

$$\int_0^N \nu W_2(q^2, \nu) d\nu = \sum_i \beta_i (q^2) N^{\alpha_i(0)}. \tag{1}$$

Here $\alpha_i(0)$ is the $t = 0$ intercept of the trajectory

α_i and $\beta_i(q^2)$ is the $t=0$ value of its residue function.[6] Some of the α_i's may represent fixed poles. The SLAC-MIT data indicate that[2]: (i) A reasonable value for N can be selected somewhere around the energy of the 1920-MeV N^* resonance (at least for $|q^2| \leq 2$ BeV^2). (ii) There is no clear indication for the existence of a prominent non-Regge term (such as Bjorken's quasielastic peak[1]) above $\nu = N$. Equation (1) is therefore expected to hold, with a small number of terms on the right-hand side. (iii) For the above N values, the left-hand side of Eq. (1) decreases very rapidly with q^2. (iv) At least some of the residue functions $\beta_i(q^2)$ of the leading poles on the right-hand side of Eq. (1) change very slowly with q^2, since W_2 is almost q^2 independent at large ν.

We are therefore faced with an apparent inconsistency between the q^2 variation of the two sides of Eq. (1). How can the equation remain valid over a wide range of q^2 values?

Our answer to this question is based on the distinction between the Pomeranchuk singularity and the "ordinary" trajectories.[3] We propose the following set of conjectures:

(A) The contributions of the "ordinary" trajectories $(P', A_2, \cdots)$ are "built" from the N^* resonances. The q^2 dependence of their residue functions should therefore be similar to that of the (squared) excitation form factors of the resonances: $\beta(q^2)$ will fall off like a high power of q^2 ($1/q^8$, if the dipole formula holds, but certainly much faster than $1/q^2$).

(B) The (diffractive) Pomeranchukon exchange contribution, which is built mainly by the nonresonating background, does not vary strongly with q^2. At $\nu \gg N$ this is the dominant term in the amplitude and it is responsible for the slow q^2 variation. On the other hand, at $\nu = N$ the P' and A_2 are still very important and the q^2 variation is faster.

(C) The entire physical Compton amplitude can be approximately accounted for by adding the s-channel resonances or the "ordinary" trajectories to the Pomeranchukon contribution with no extra terms.[7]

These assumptions are consistent with the present SLAC-MIT data[2]; they "explain" how the fixed-q^2 finite-energy sum rules can be valid for a wide range of q^2 values; they preserve the duality between the ordinary trajectories and s-channel resonances, which has led to interesting results in hadron physics; and, finally, they can be unambiguously tested by experiments in the near future.

What are the experimental consequences of our conjectures?

(1) We predict that at large q^2 values, the photoabsorption cross section is purely diffractive at all energies. In other words, the energy dependence of $\sigma(\gamma N)$[5] for large fixed q^2 should start off at threshold with some (q^2-dependent) threshold factor, will smoothly rise to a constant value (the Pomeranchuk contribution), and will remain constant as $\nu \to \infty$. It is clear that we cannot define rigorously what we mean by "large q^2," "threshold factor," etc. The explicit experimental meaning of our prediction is therefore: As q^2 increases, we should observe a flatter and flatter ν dependence of $\sigma(\gamma N)$.

(2) The difference between $\sigma(\gamma p)$ and $\sigma(\gamma n)$ at a fixed c.m. energy should decrease rapidly with q^2, since it is due to $I=1$ t-channel exchanges. In other words, as q^2 increases, the measured proton and neutron inelastic form factors should become equal beginning at lower and lower values of c.m. energy. The decrease of $[\sigma(\gamma p) - \sigma(\gamma n)]$ with q^2 should be much faster than that of $\sigma(\gamma p)$ itself. Experimentally, it may be easier to test the analogous prediction for nuclei by performing experiments on H^3 and He^3 targets rather than extract the neutron data from deutron experiments.

(3) An overall detailed fit of future ep and ed inelastic data at all q^2 and high ν in terms of the P, P', and A_2 poles should exhibit the different q^2 behavior of the P coupling on one hand and the P' and A_2 couplings on the other hand.

Our conjecture, if true, has interesting implications with respect to various sum rules. We conclude with a few theoretical remarks related to these matters:

(a) The Adler sum rule[8] can be written as[9]

$$M_N^{-1}\int_0^\infty W_2^-(\nu, q^2)d\nu = 1, \tag{2}$$

where $W_2^-(q^2, \nu)$ is the form factor for inelastic electron scattering having $I=1$ and $C=-1$ quantum numbers in the t channel for $\gamma N \to \gamma N$.[10] The high-ν part of the integral in Eq. (2) is presumably given by the ρ trajectory. According to our assumptions the ρ, as well as any other "ordinary" trajectory, should contribute very little at large q^2. The same statement presumably holds for the resonances which dominate the low-ν part of the integral. If this is the case, the only way to save the sum rule is to assume that the fixed $J=1$ pole on the right-hand side of Eq. (2) is somehow "built" by a nonresonating, non-Regge, nondiffractive background which remains constant with q^2. We find it hard to accept this[11] since at $q^2=0$ the fixed pole is completely accounted for by the nucleon Born term. We are therefore led to the "suspicion" that if our conjecture is experimentally verified, the Adler sum rule may have to include a q^2-dependent term on its right-hand side. This may occur if in addition to the fixed pole in the A amplitude in Compton scattering there is another $J=1$ fixed pole in the B amplitude. A and B are defined by the usual expression for the Compton amplitude:

$$M_{\mu\nu}=A(q^2,\nu)P_\mu P_\nu+B(q^2,\nu)(q_\mu P_\nu+q_\nu P_\mu)+C(q^2,\nu)q_\mu q_\nu+D(q^2,\nu)\delta_{\mu\nu}. \tag{3}$$

A fixed $J=1$ pole in B will prevent it from vanishing at $\nu\to\infty$. We will have

$$B(q^2,\nu)\xrightarrow[\nu\to\infty]{}K(q^2),$$

where $K(q^2)$ is the residue of the fixed pole. The modified Adler sum rule will then read

$$M_N^{-1}\int_0^\infty W_2^-(q^2,\nu)d\nu=1+q^2K(q^2), \tag{4}$$

and $K(q^2)$ will have to decrease like $1/q^2$ at large q^2 in order to ensure a strong q^2 dependence of both sides of Eq. (4).

(b) The Cottingham formula[12] for the p-n mass difference, with a subtraction in the dispersion relation caused by the A_2 trajectory,[13] involves a q^2 integration of the inelastic form factors. If the A_2 contribution follows our assumption and falls off very rapidly at large q^2, the q^2 integration converges and the calculation results in a finite mass difference[14] (except for the possible divergence pointed out by Bjorken[15]). If, however, the q^2 dependence of the A_2 contribution is the same as the one observed[2] for[5] $\sigma_T(\gamma p)+\sigma_S(\gamma p)$ (namely, it does not decrease faster than $1/q^2$), the q^2 integration in the subtracted version of the Cottingham formula will diverge, introducing a most embarrassing new unknown divergent factor.

(c) We are not able to predict the q^2 dependence of the diffractive part of $\sigma(\gamma p)$. This can be guessed, however, if we accept Bjorken's argument[15] that at large q^2, $\nu W_2(q^2,\nu)$ is a function of q^2/ν only. Since the diffractive part of νW_2 is constant in ν, it should also be constant in q^2, leading to $1/q^2$ dependence for[5] $\sigma_T+\sigma_S$.

Needless to say, there is no compelling reason to accept our speculations as a probable truth. We would like to emphasize, however, that the simplicity of the physical picture which we have described, its consistency with the present data, and its controversial implications for the Adler sum rule and the p-n mass difference make it worthwhile to pursue the straightforward experimental tests suggested here.

The author would like to thank his colleagues in the theoretical groups at SLAC and the Weizmann Institute for many illuminating discussions.

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*Research supported in part by the Air Force Office of Scientific Research through the European Office of Aerospace Research, U. S. Air Force under Contract No. F-61052-68-C-0070.

[1]J. D. Bjorken, Phys. Rev. Letters 16, 408 (1966), and in Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967 (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968), and Phys. Rev. 163, 1767 (1967), and in Current Algebra at Small Distances, Proceedings of the International School of Physics, "Enrico Fermi," Course IXL, edited by J. Steinberger (Academic Press, Inc., N. Y., 1968), p. 55. See also J. D. Bjorken, Phys. Rev. Letters 148, 1467 (1966), and Phys. Rev. 179, 1547 (1969); J. Cornwall and R. Norton, Phys. Rev. (to be published); J. D. Bjorken and M. Bander, Phys. Rev. 174, 1704 (1968); C. Callan and D. Gross, Phys. Rev. Letters 21, 311 (1968); K. Gottfried, Phys. Rev. Letters 18, 1174 (1967); J. D. Walecka, in Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967 (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968).

[2]E. D. Bloom et al., in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (unpublished); W. K. H. Panofsky, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968).

[3]H. Harari, Phys. Rev. Letters 20, 1395 (1968). See also P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); F. J. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters 21, 323 (1968).

[4]At least two kinds of fixed poles are relevant here: (i) the $I=J=1$ pole found in the Adler-Fubini sum rule by J. Bronzan, I. S. Gerstein, B. W. Low, and F. E. Low [Phys. Rev. Letters 18, 32 (1967)] and by V. Singh [Phys. Rev. Letters 18, 36 (1967)]; (ii) the fixed pole suggested by H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz [Phys. Rev. 160, 1329 (1967)] in order to reinstate the Pomeranchukon coupling to the photon. The particular mechanism which resurrects the Pomeranchukon in the Reg-

ge formulation of Compton scattering does not really matter here. The only relevant information we need is that at large ν there is some diffractive contribution which we call Pomeranchukon exchange. This is guaranteed if $\sigma(\gamma p)$ remains constant in energy, as it appears to do experimentally.

[5]We use the definitions and notations of F. J. Gilman, Phys. Rev. 167, 1365 (1968); $\nu = q_0$; at large ν, $W_2 \sim \nu^{\alpha-2}$; $\sigma(\gamma p)$ represents σ_T and/or σ_S for virtual γp scattering; and W_2 is related to the combination $\sigma_T + \sigma_S$.

[6]$\beta_i(q^2)$ includes all the necessary factors of α, $\alpha-1$, $\Gamma(\alpha)$, etc. In particular $\beta_p(q^2) \neq 0$ because of the mechanism which reinstates the Pomeranchukon contribution (see Ref. 4 above).

[7]For the Compton scattering of neutral photons we do not expect any other contribution to the physical amplitude. In the case of the Adler sum rule [see Eq. (2) and the following discussion] "charged" photons are involved and we do have an extra fixed pole contributing to the real part.

[8]S. L. Adler, Phys. Rev. 143, 1144 (1966).

[9]$W_2^-/M_N = \beta^{(-)} - \beta^{(+)}$. Notation as in Ref. 5.

[10]Note that no combination of neutral photon amplitudes can contribute to W_2^-. Only neutrino experiments can, in principle, test Eq. (2) in a model independent way.

[11]Bjorken (Ref. 1) has suggested that many high-spin resonances at intermediate energies may "build" the fixed pole in Eq. (2). If there is no "quasielastic peak" (and the data do not support such a peak), the high-spin resonances will be properly averaged by the ordinary trajectories and we do not have to consider them in addition.

[12]W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[13]H. Harari, Phys. Rev. Letters 17, 1303 (1966).

[14]The electromagnetic self-mass will diverge in this case because of the slow q^2 variation of the Pomeranchukon contribution. Only the mass differences will converge.

[15]J. D. Bjorken, Phys. Rev. Letters 148, 1467 (1966).

[16]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

EMPIRICAL N/D POTENTIAL AND CASTILLEJO-DALITZ-DYSON POLES IN THE PION-NUCLEON P_{11} STATE*

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(Received 16 January 1969; revised manuscript received 9 April 1969)

The CERN phase-shift analysis is used to evaluate the left-cut term of the P_{11} partial wave of pion-nucleon scattering. The inelastic N/D equation is solved with this term and the elasticity η as input. The output and input phase shifts agree below 2 BeV if and only if at least two Castillejo-Dalitz-Dyson poles are included in the canonical D function. This result rules out single-channel bootstrap models and furnishes a constraint on many-channel models.

We report model-independent calculations that furnish a constraint which any model of pion-nucleon P_{11} scattering must satisfy. The Chew[1] theory of 1962 fails to meet this empirical constraint. So do all published models that we know of in which the nucleon appears as a composite state of one or several channels.[2] A search for more suitable models is indicated if one believes in a bootstrap theory with a composite nucleon.

We have used a recent phase-shift analysis[3] which provides the real phase shift δ and elasticity η of the P_{11} state to compute the N/D potential B directly from experiment. With this potential and η as input, solution of the N/D integral equation[4] yields an output phase shift $\hat{\delta}$ which agrees with δ below 2 BeV if and only if we include at least two Castillejo-Dalitz-Dyson (CDD) poles[4,5] in a "canonical" single-channel D function defined in the following. This is the constraint mentioned above: A single-channel N/D treatment of the P_{11} state must entail at least two CDD poles of the canonical denominator function. The result turns out to be independent of the unknown high-energy behavior of δ and η.

We work with the following partial-wave amplitude:

$$f(s) = \frac{\eta \exp(2i\delta) - 1}{2iq^3} = \frac{N(s)}{D(s)}. \tag{1}$$

Here s and q are the squared energy and momentum, respectively, in the center-of-mass frame, and the D function is defined such that $D(s+i0) = |D(s+i0)| \exp(-i\delta)$ for s greater than the threshold energy s_0. The D function has the representa-