This band model does not resolve the dilemma of the positive thermopower versus the negative Hall coefficient observed in covalent alloy glasses,[3,14,20,21] nor does it explain the magnitude of the density of band states near the mobility edges which is needed to explain the magnitude of the factor σ_0 in the conductivity.[12,22,23]

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QUANTUM STATISTICS OF ONE-PHOTON INTERACTION OF LIGHT WITH MATTER

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Quantum statistics of one-photon interaction of light with matter is investigated. It is found that in general the density operator changes. For a chaotic field, however, the density operator changes only in a trivial way.

Quantum statistics of nonlinear interactions of light with matter has recently been investigated.[1,2] In nonabsorbing media, one-photon interactions obviously do not change the photon statistics. Shen[1] has shown that if the incident radiation resonates with the atoms of the medium, the density operator changes in a trivial fashion and the nature of the radiation remains the same. In this paper, we report the results of our investigations of change in density matrix in one-photon interactions. The evaluation of the density matrix is simple and can easily be extended to two- or more-photon interactions. We find that in general the density matrix changes in a nontrivial way. Coherent light does not remain coherent and the photon fluctuations increase. The chaotic light, as a special case, however, remains chaotic.

Lambropoulos,[2] while discussing statistics of the two-photon amplifier, found the density matrix by considering the matrix element in occupation number space for the field, and solving the differencedifferential equations in the matrix elements. This method has limited applicability as has been observed by Lambropoulos.[2] It is, however, easy to find[3] the matrix in the P representation.[4]

In the P representation,[4] the density matrix of radiation $\rho(t)$ is written as[5]

$$\rho(t)=\int d^2v\,P(v,t)|v\rangle\langle v|, \tag{1}$$

where $|v\rangle$ is the normalized eigenstate of the annihilation operator a with the complex eigenvalue v. Equation (1) can be used to find $P(v,t)$; it gives[3]

$$P(v,t)=\pi^{-2}\int d^2\alpha\exp[|v|^2+|\alpha|^2+\alpha^*v-\alpha v^*]\langle-\alpha|\rho(t)|\alpha\rangle. \tag{2}$$

We consider the model commonly used[6] for studying laser theory, viz., interaction of light with an atom having effectively two levels only. For one-photon interactions, we need consider the Hamiltonian $H=H_0+H_I$, where

$$H_0=\omega a^\dagger a+\omega_0\sigma^\dagger\sigma,\quad H_I=\xi\sigma^\dagger a+\xi^*\sigma a^\dagger. \tag{3}$$

Here $\sigma^\dagger$ and σ are, respectively, the exciting and de-exciting operators of the molecule whose natural frequency is ω_0, and $\xi=L^{-\frac{3}{2}}(2\omega)^{-\frac{1}{2}}\langle u|e^{i\vec{k}\cdot\vec{x}}\vec{p}\cdot\vec{\epsilon}|l\rangle$, L^3 being the volume of the box used for normalization, $|u\rangle$ and $|l\rangle$ being the upper and lower states of the atom, and $\vec{\epsilon}$ being a unit vector along the direction of polarization of light. The equation of motion of $\rho(t)$, viz., $d\rho(t)/dt=i[\rho(t),H]$, has the solution

$$\rho(t+\tau)=u(\tau)\rho(t)u^\dagger(\tau), \tag{4}$$

where the time-evolution operator $u(\tau)$ is given by

$$u(\tau)=u^{(0)}(\tau)+\sum_{n=1}^{\infty}(-i)^n\int_0^\tau d\tau_1\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{n-1}}d\tau_n u^{(0)}(\tau-\tau_1)H_Iu^{(0)}(\tau_1-\tau_2)H_I\cdots u^{(0)}(\tau_{n-1}-\tau_n)$$
$$\times H_Iu^{(0)}(\tau_n),\quad u^{(0)}(\tau)\equiv e^{-iH_0\tau}. \tag{5}$$

If the interaction starts at time t_0, $\rho(t_0)=\rho^{\mathbf{A}}(t_0)\rho^{\mathbf{R}}(t_0)$, where superscripts A and R refer to the atom and radiation, respectively. At time $t_0+\tau$, we have

$$P(v,t_0+\tau)=\pi^{-2}\int d^2\alpha\exp[|v|^2+|\alpha|^2+\alpha^*v-\alpha v^*]\mathrm{Tr}_{\mathbf{A}}\langle-\alpha|\rho(t_0+\tau)|\alpha\rangle. \tag{6}$$

Lengthy but direct calculations for one-photon emission and absorption lead to

$$P(v,t_0+\tau)=P(V,t_0)+|\xi|^2[N_u\{H(\partial^2/\partial V\partial V^*)-H-G^*V(\partial/\partial V)-GV^*(\partial/\partial V^*)\}$$
$$+N_l\{H+GV(\partial/\partial V)+G^*V^*(\partial/\partial V^*)\}]P(V,t_0), \tag{7}$$

where $V=ve^{i\omega\tau}$, $N_u\equiv\rho_{uu}{}^A(t_0)$, $N_l\equiv\rho_{ll}{}^A(t_0)$, $G=(\omega_0-\omega)^{-2}\{\exp[i(\omega_0-\omega)\tau]-1\}$, and $H=-(G+G^*)$, if we take $|\omega_0-\omega|^{-1}\ll\tau$. In Eq. (7), the term $HN_u|\xi|^2(\partial^2/\partial V\partial V^*)P(V,t_0)$ represents the effect of spontaneous emission. The other terms involving N_u and N_l represent the effects of induced emission and absorption, respectively.

For a coherent field, $P(v,t_0)=\delta^2(v-v_0)$, $P(v,t_0+\tau)$ contains, besides the delta function, its derivatives also. The field therefore does not remain coherent after interaction. For a chaotic field, $P(v,t_0)=(\pi\langle n\rangle)^{-1}\exp[-|v|^2/\langle n\rangle]$, Eq. (7) leads to

$$P(v,t_0+\tau)=(\pi\langle n\rangle)^{-1}\exp[-|v|^2/\langle n\rangle][1-H|\xi|^2\{N_u(\langle n\rangle^{-1}-|v|^2\langle n\rangle^{-2})+(N_u-N_l)(1-|v|^2\langle n\rangle^{-1})\}]. \tag{8}$$

This can be written as $P(v,t_0+\tau)=(\pi\langle n'\rangle)^{-1}\exp[-|v|^2/\langle n'\rangle]$, where

$$\langle n'\rangle=\langle n\rangle+H|\xi|^2[N_u+(N_u-N_l)|v|^2]. \tag{9}$$

From Eq. (8), it is seen that $\langle\rho(t_0+\tau)a^\dagger a\rangle=\langle n'\rangle$ given by Eq. (9). This clearly shows that the chaotic field does not change its nature; only the total number of photons changes.

For initially coherent and chaotic fields, the probabilities for detection of m photoelectrons[7] in the interval T at time $t_0+\tau$ are

$$p^{\mathrm{coh}}(m,t_0+\tau,T)=P_m^{\mathrm{coh}}+HKT|\xi|^2[N_u\{\epsilon P_{m-2}^{\mathrm{coh}}+(1-2\epsilon)P_{m-1}^{\mathrm{coh}}-(1-\epsilon)P_m^{\mathrm{coh}}\}$$
$$+(N_u-N_l)|v_0|^2(P_{m-1}^{\mathrm{coh}}-P_m^{\mathrm{coh}})],\quad \epsilon=KT|v_0|^2, \tag{10}$$

$$p^{\mathrm{cha}}(m,t_0+\tau,T)=P_m^{\mathrm{cha}}+(H/KT)|\xi|^2[P_m^{\mathrm{cha}}-(m+1)P_{m+1}^{\mathrm{cha}}][N_u+(N_u-N_l)\langle n\rangle], \tag{11}$$

where $P_m^{\mathrm{coh}}=(m!)^{-1}\epsilon^m e^{-\epsilon}$ and $P_m^{\mathrm{cha}}=(KT)^m\langle n'\rangle^{-1}[KT+\langle n'\rangle^{-1}]^{-(m+1)}$ are, respectively, the probabilities for detection of m photoelectrons in time interval T before interaction with matter of the coherent and chaotic fields and K is the photocathode efficiency. A little calculation shows that Eq. (11) can be written as $p^{\mathrm{cha}}(m,t_0+\tau,T)=(KT)^m\langle n'\rangle^{-1}[KT+\langle n'\rangle^{-1}]^{-(m+1)}$.

The variance of photoelectrons is[8] $\Delta_m^2=\langle m\rangle+(KT)^2(\Delta_n^2-\langle n\rangle)$, where $\Delta_m^2=\langle m^2\rangle-\langle m\rangle^2$, and $\Delta_n^2=\langle n^2\rangle-\langle n\rangle^2$, m and n being the number of photoelectrons and of photons, respectively. It can easily be shown with the help of Eq. (7) that $\Delta_n^2-\langle n\rangle$ is >0 for both initially coherent and initially chaotic fields and thus these fields do not exhibit anticorrelation[9] at any time.

As a measure of fluctuations in the number of photons let us consider $\mathcal{E}_n^2=\Delta_n^2/\langle n\rangle^2$. For initially coherent fields,

$$\mathcal{E}_n^{\mathrm{coh}\,2}(t_0+\tau)=|v_0|^{-2}[1+H|\xi|^2(N_u+N_l-N_u|v_0|^2)]. \tag{12}$$

As $|v_0|^2\gg1$, this shows that fluctuations always increase. For initially chaotic field,

$$\mathcal{E}_n^{\mathrm{cha}\,2}(t_0+\tau)=1+\langle n\rangle^{-1}+H|\xi|^2\langle n\rangle^{-2}[N_l\langle n\rangle-N_u(1+\langle n\rangle)]. \tag{13}$$

This equation shows that if $N_u>N_l$, fluctuations decrease after interaction. From this, Lambropoulos[2] has wrongly concluded that the Planck distribution tends towards something intermediate between the Planck and Poisson distributions. Equation (13) can, however, be written as $\mathcal{E}_n^{\mathrm{cha}\,2}(t_0+\tau)=1+\langle n'\rangle^{-1}$ which shows that the field retains its nature, the only change being in $\langle n\rangle$.

Our results, viz., change in the nature of a coherent field and retention of the nature of a chaotic field in one-photon interactions with matter, can easily be verified experimentally. It is well known that for a coherent beam, $\Delta_m^2=\langle m\rangle+\langle m^2\rangle$. From Eq. (7), we find that for laser light which has traveled through an absorbing medium,

$$\Delta_m^{\mathrm{coh}\,2}(t_0+\tau)=\langle m(t_0+\tau)\rangle+2(KT)^2H|\xi|^2N_+|v_0|^2$$
$$\neq\langle m(t_0+\tau)\rangle, \tag{14}$$

if $N_+\neq0$. For chaotic light, however, one will observe that

$$\Delta_m^{\mathrm{cha}\,2}(t_0+\tau)=\langle m(t_0+\tau)\rangle+\langle m^2(t_0+\tau)\rangle. \tag{15}$$

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MIXMASTER UNIVERSE*

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The generic, nonrotating, homogeneous cosmological model for a closed space (Bianchi type IX) has a very complex singularity which can, however, be described in detail. It appears that only the exceptional (previously studied) cases will have particle horizons. Thus these models may lead to some insight into how the broad-scale homogeneity of the universe may have been produced at very early times.

Particle horizons[1] in cosmological models are limits on the possibilities of causal interactions between different parts of the universe in the time available since the initial singularity. In the standard metric $ds^2=\eta^2\{-d\eta^2+dx^2+dy^2+dz^2\}$ for the radiation-dominated early phase of a Robertson-Walker (RW) cosmological model, it is clear that the coordinate time $\Delta\eta$ required for a light signal ($ds^2=0$) to connect two regions of spatial-coordinate separation Δx is $\Delta\eta=|\Delta x|$. Thus at a fixed epoch $\eta_0>0$, no causal interactions subsequent to the singularity at $\eta=0$ have occurred between regions of coordinate separation $|\Delta x|>\eta_0$. In observational terms this effect says, for example, that if the 3°K background radiation[2] were last scattered at a redshift $z=7$, then the radiation coming to us from two directions in the sky separated by more than about 30° was last scattered by regions of plasma whose prior histories had no causal relationship. These Robertson-Walker models therefore give no insight into why the observed microwave radiation from widely different angles in the sky has[2] very precisely ($\lesssim 0.2\,\%$) the same temperature.

We will describe a model of a closed (type-IX)[2] universe which has a very different singularity behavior than the RW models, but which could evolve into the closed RW model at the present epoch. Several aspects of the description parallel the much simpler behavior of a type-I universe with metric

$$ds_{\mathrm{I}}^2=-dt^2+\sum_k(l_k)^2dx_k^2 \tag{1}$$

which is closed artificially by assuming that each space coordinate x_k is periodic with, say, period 4π. Near the singularity the matter or radiation density terms in the Einstein equations can be neglected, and one finds the Kasner[3] solutions $l_k=t^{2p_k}$ with $\sum(p_k)^2=1=\sum p_k$. The model with $p_k=\delta_k{}^1$ then has $ds^2=-dt^2+t^2dx^2+dy^2+dz=e^{2\eta}\times(-d\eta^2+dx^2)+dy^2+dz^2$, where $\eta=\ln t$. Evidently light rays ($ds=0$) can completely circle the universe in the x direction ($\Delta x=4\pi$) in a coordinate-time interval $\Delta\eta=4\pi$ for this metric. Since the singularity is at $\eta=-\infty$ here, this much coordinate time has preceded every nonsingular epoch in this model, and there exist no horizons for causal propagation in the x direction.[4] To compare later with the type-IX model, note that this interval $\Delta\eta=4\pi$ corresponds to a volume expansion ratio of $\Delta\ln(l_1l_2l_3)=4\pi$. In the course of our description of the type-IX model, we will see that it closely approximates this model during periods involving large expansion ratios, but does this infinitely many times with different directions having the open channels of communication each time. On this basis we expect that the absence of horizons in one direction only in this particular Kasner metric corresponds to a total absence of horizons in the generic nonrotating, type-IX metric.

The Bianchi type-IX metric is

$$ds_{\mathrm{IX}}^2=-dt^2+\sum_k(l_k)^2\sigma_k^2, \tag{2}$$

where $\sigma_z=-(d\psi+\cos\theta\,d\varphi)$, $\sigma_x=\sin\psi\,d\theta-\cos\psi\sin\theta\times d\varphi$, and $\sigma_y=\cos\psi\,d\theta+\sin\psi\sin\theta\,d\varphi$ satisfy $d\sigma_i=\frac{1}{2}\epsilon_{ijk}\sigma_j\wedge\sigma_k$ and are differential forms on the three-sphere (covering group of the rotation group) parametrized by Euler angles $\psi\theta\varphi$ with $0\leqslant\psi\leqslant4\pi$, $0\leqslant\theta\leqslant\pi$, and $0\leqslant\varphi\leqslant2\pi$. To distinguish between expansion (volume change) and anisotropy (shape change) we write $l_k=R\exp\beta_k$, where

$$R\equiv e^{-\Omega}=(l_1l_2l_3)^{\frac{1}{3}} \tag{3}$$