high energy for both reactions is then of the form $(s)^{2\alpha(u)-2}$, where $\alpha(u)$ is the real part of the Δ_δ trajectory. This simple model leads to the predictions that $R_\pi = R_\rho$ and $\Delta B_\pi = \Delta B_\rho$.

Our results and these predictions differ by 1.2 standard deviations for the R's and by 2.5 standard deviations for the ΔB's. This discrepancy could be due to the following: (1) An incorrect description of the background. However, the discrepancy remains the same even when we fit the data with more complicated backgrounds. (2) Inadequacies in the simple Regge model outlined above.[7] It should be pointed out that more detailed Regge calculations for backward ρ^- production are in reasonable agreement with our data.[8]

It is a pleasure to acknowledge the generous cooperation and valuable assistance of the AGS staff in the setting up and running of this experiment. We also wish to acknowledge the important contributions made to this experiment by the staffs of the BNL On-Line Data Facility, the BNL Instrumentation Division, and the Physics Design Groups at BNL and Carnegie Mellon University. We are particularly grateful to A. Abrahamson, E. Bihn, R. Rothe, and J. Smith for their invaluable assistance throughout the experiment. We also wish to acknowledge useful discussions with Dr. C. C. Shih and Dr. E. Paschos.

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[1]Some evidence for backward ρ production has been reported in the Aachen-Berlin-Birmingham-Bonn-Hamburg-London (Imperial College)-Munchen Collaboration, Phys. Rev. 138, B897 (1965); Aachen-Berlin-CERN Collaboration, Phys. Letters 12, 356 (1964).

[2]E. W. Anderson, E. J. Bleser, H. R. Blieden, G. B. Collins, D. Garelick, J. Menes, F. Turkot, D. Birnbaum, R. M. Edelstein, N. C. Hien, T. J. McMahon, J. Mucci, and J. Russ, Phys. Rev. Letters 20, 1529 (1968).

[3]We label the background coefficients $a(u)$ with the values of u calculated from the beam energy, the average laboratory angle of the data being fit, and the ρ mass.

[4]Data on Reaction (2) at small u and low energy have been reported by R. Anthony, C. Coffin, E. Meanley, J. Rise, N. Stanton, and K. Terwilliger, in the Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968 (to be published).

[5]In these comparisons we have used our forward π^-p elastic cross section data at 8 GeV/c (to be published) and the forward ρ^+ cross section data at 8 GeV/c of the Aachen-Berlin-CERN Collaboration, to be published.

[6]D. R. O. Morrison, Phys. Letters 22, 528 (1966).

[7]There are ways to modify the model outlined above. For example, it is not necessary to assume a simple exponential behavior of the cross section. Detailed studies by E. Paschos (to be published) of u-channel processes based on Regge-pole model have suggested deviations from the "usual" Regge behavior at small values of u. Also, the possibility of another low-lying $I=\frac{3}{2}$ baryon trajectory contributing to Reaction (1) cannot be ruled out. [Particle Data Group, University of California Radiation Laboratory Report No. UCRL 8030 Revised, 1968 (unpublished)].

[8]C. Shih, following Letter [Phys. Rev. Letters 22, 105 (1969)].

REGGE-POLE ANALYSIS FOR π^- PRODUCTION OF ρ^- MESON NEAR THE BACKWARD DIRECTION*

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ρ^- production near backward direction has been analyzed in a Regge-pole model with the emphasis on kinematic structures and constraint relations. We obtain results which agree well with experimental data and are consistent with backward elastic scattering.

Recently many attempts have been made in Regge-pole models to understand the connections between high-energy backward meson-baryon scattering and baryon trajectories.[1] Theoretically, unequal-mass kinematics,[2] conspiracy,[3] MacDowell symmetry,[4] and Gribov's theorem[5] complicate the problem. With the success for backward elastic scattering, it is natural to investigate whether the same kind of analysis can be achieved for the more complicated production proceses.[6] Because of the data available, and the more complicated spin structure, we shall restrict ourselves here to the vector-meson production only. Through the use of invariant amplitudes, the kinematic structure of the helicity amplitudes and their constraint relations are derived. Special attention is paid to MacDowell symmetry and the pseudothreshold constraints at $u_0=(m_N-m_\rho)^2 \sim 0.03$. As $s=0$ is in the physical region and s_0 is very close to the backward direction, these points are very important in any phenomenological analysis. On the other hand, these relations reduce considerably the arbitrariness of the otherwise unrelated individual Regge-pole residues. A parametrization satisfying all the constraints requires less parameters and therefore makes the comparison with experimental data possible.

Recent data[7] on $\pi^- p \to p\rho^-$ near backward direction are especially suitable for this purpose. Only one known baryon trajectory exchange, Δ_δ, is allowed. Our simplest possible parametrization of the residues for this process uses three parameters. Using the trajectory function $\alpha_\Delta(s^{1/2})$ obtained from backward elastic scattering as the fixed input, we find a best fit which is in excellent agreement with the existing data. Also, the density matrix elements of ρ^- meson thus obtained are sensitive to the input trajectory function $\alpha_\Delta(s^{1/2})$. Thus the measurement of ρ density matrix can be very informative in helping to determine the more detailed structure of the baryon trajectories.

We Reggeize the parity-conserving kinematic-singularity-free helicity amplitudes $\hat{f}_i(w)$ in the usual way. Taking a simple parametrization, we get

$$\hat{f}_i(w,u)\equiv(\hat{f}_{\frac{1}{2}0,\frac{1}{2}1}{}^{\pm},\hat{f}_{\frac{1}{2}0,\frac{1}{2}-1}{}^{\pm},\hat{f}_{\frac{1}{2}0,\frac{1}{2}0}{}^{\pm})=\gamma_i(w)[\alpha^{\pm}(w)+\tfrac{1}{2}][\alpha^{\pm}(w)+\tfrac{3}{2}]\xi_i{}^{\pm}(w)\left(\frac{u}{u_0}\right)^{\alpha^{\pm}(w)-\frac{1}{2}},\quad w=s^{1/2},\qquad(1)$$

where $\gamma_i(w)$ are the reduced residue functions and $\xi^{\pm}(w)$ the signature factors.

MacDowell symmetry[4] takes the form

$$\alpha^+(w)=\alpha^-(-w),\quad \gamma_2(w)=\gamma_1(-w),$$
$$\gamma_{4,6}(w)=-\gamma_{3,5}(-w),\qquad(2)$$

and the constraint relations,

$$\gamma_1=\tfrac{1}{2}w\gamma_3=\mp 2^{-1/2}\gamma_5=0,\quad w=-(m\mp\rho).\qquad(3)$$

The constraint relations at $w=\pm(m-\rho)$ take much simpler form for some linear combinations of the $\gamma_i(w)$. Defining

$$\gamma_2'(w)=\gamma_2(w)-\tfrac{1}{2}w\gamma_4(w),$$
$$\gamma_6'(w)=[-(2)^{1/2}\gamma_6(w)+w(k_0/\rho)\gamma_4(w)]$$
$$\gamma_4'(w)=\gamma_4(w),\quad k_0=\frac{1}{2w}[w^2-m^2+\rho^2],\qquad(4)$$

Eq. (3) reduces to

$$\gamma_2'(w)=\gamma_6'(w)=0,\quad \text{at } w=-(m-\rho),$$
$$\gamma_2'(w)-\gamma_6'(w)=0,\quad \text{at } w=(m-\rho).\qquad(5)$$

If $\gamma_i'(w)$ are slowly varying, it is reasonable to parametrize $\gamma_i'(w)$ as

$$\gamma_4'(w)=A,\quad \gamma_2'(w)=\gamma_6'(w)=B\left(1+\frac{w}{m-\rho}\right).$$

We get finally

$$\gamma_1(-w)=\gamma_2(w)=\tfrac{1}{2}wA+\left(1+\frac{w}{m-\rho}\right)B,$$
$$-\gamma_3(-w)=\gamma_4(w)=A,$$
$$-\gamma_5(-w)=\gamma_6(w)=\frac{1}{\sqrt{2}}\left[\frac{1}{2\rho}[w^2-m^2+\rho^2]A-B\left(1+\frac{w}{m-\rho}\right)\right].\qquad(6)$$

Still another parameter is the scaling factor u_0. For the choice of the trajectory function $\alpha_\Delta(w)$, we take the form given from the analysis of backward elastic scattering. As the same trajectory is exchanged in both cases, $\alpha_\Delta(w)$ should be the same for consistency. Besides, the data for backward production are much less abundant than those for the elastic scattering. A more detailed analysis allowing the variation of $\alpha_\Delta(w)$ can only be done after we have more data.

Using the elastic scattering data at 8 and 16 GeV[1] and the usual parametrization,[1] a reasonable fit to the backward elastic scattering can be obtained with

$$\mathrm{Re}\,\alpha_\Delta(w)=0.049+0.76w^2,$$
$$\gamma(w)=0.36[1+(2.9/m_\Delta)W](1/u_0)^{\alpha(w)-\frac{1}{2}},$$
$$u_0=1.85.\qquad(7)$$

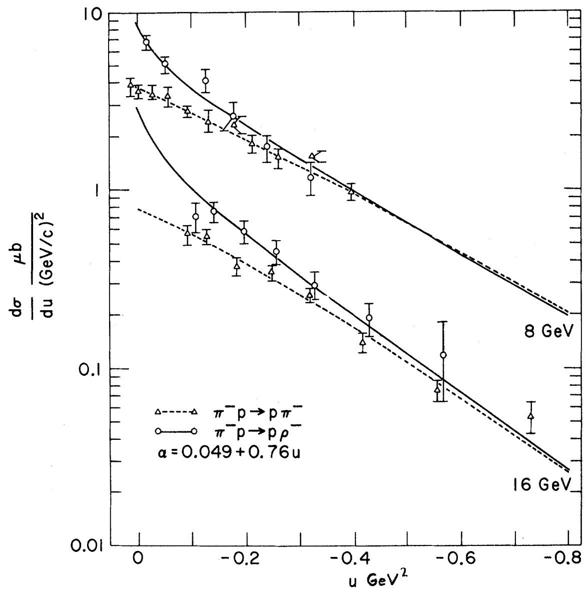

FIG. 1. $d\sigma/du$ vs u for $\pi^- p \to p\pi^-$ and $\pi^- p \to p\rho^-$ at 8 and 16 GeV/c. Experimental data for backward π^- (triangles) and backward ρ^- (circles) are Refs. 1 and 7.

Taking this $\alpha_\Delta(w)$ as fixed input, the best fit for the ρ^- production is obtained with

$$A = 0.27, \quad B = 0.046, \quad u_0 = 3.27. \tag{8}$$

As shown in Fig. 1, both fits agree very well with the experimental data.

A very interesting result is the prediction for the ρ^- density matrix. With different choices of the trajectory functions $\alpha_\Delta(w)$, the best fits all agree very well with the experimental cross-section data, but the predicted density matrices differ considerably. For example, for

$$\alpha^{(1)} = 0.049 + 0.76u, \quad A = 0.27, \quad B = 0.046,$$
$$u_0 = 3.27, \text{ and } \chi^2 = 9.1;$$
$$\alpha^{(2)} = -0.06 + 0.8u, \quad A = 0.67, \quad B = -0.10,$$
$$u_0 = 2.92, \text{ and } \chi^2 = 8.5;$$
$$\alpha^{(3)} = 0.15 + 0.9u, \quad A = 0.26, \quad B = -0.07,$$
$$u_0 = 3.56, \text{ and } \chi^2 = 12.5.$$

As shown in Fig. 2, at 8 GeV, away from the backward direction $\rho^{(1)} \approx 0.75$, ${\rho_{00}}^{(2)} \approx 0.2$, and ${\rho_{00}}^{(3)} \approx 0.1$. We feel therefore that understanding of the ρ density matrix near the backward direction can be very useful in helping to determine the properties of the baryon trajectories.

As for the polarization of the final nucleon, it depends on the phase difference of the ${f_{\lambda\mu}}^{\pm}$ amplitudes and is given asymptotically in our process[6] as

$$\tanh\{\tfrac{1}{2}[\alpha(s^{1/2}) - \alpha((-s)^{1/2})]\}.$$

Similar analysis can be done for the recent π photoproduction data in the backward direction.[10] Because of the different isospin structure and the presence of more trajectories, e.g., N_α and Δ_δ, the results are less unique, and will be mentioned elsewhere. Backward production of higher spin resonances can also be done in a very simi-

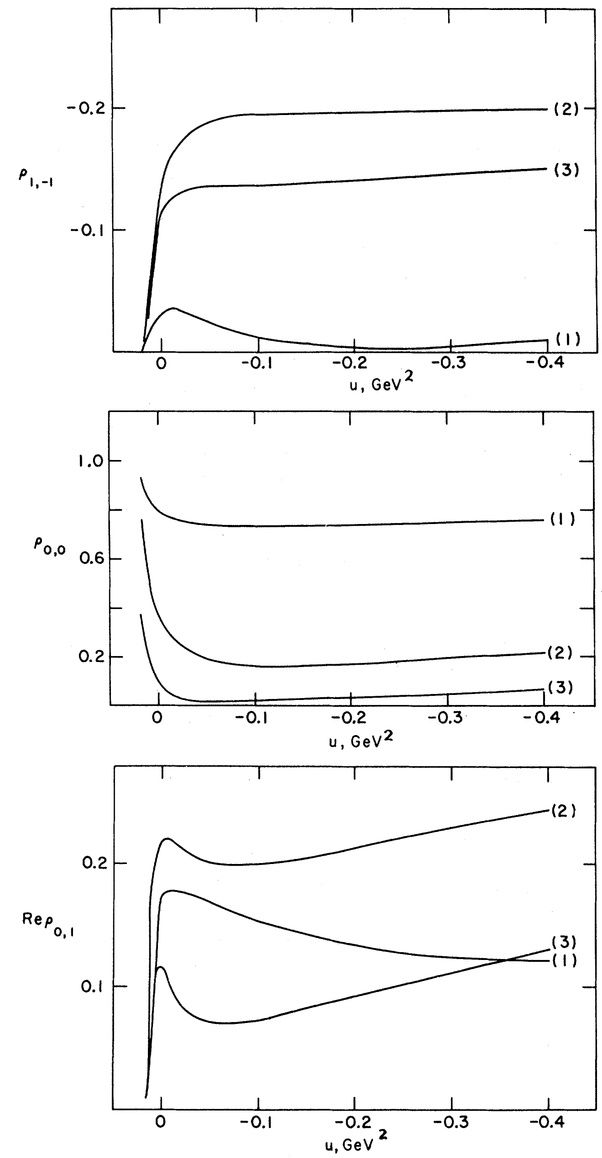

FIG. 2. The predictions of the density matrix elements for backward ρ^- production in the c.m. frame of the direct channel at incident π^--meson momentum of 8 GeV/c.

lar way. It is easier to work directly, and equivalently, in the crossed channel[11] than to introduce invariant amplitudes. Detailed calculations are in progress.

Thanks are due to Dr. H. R. Blieden and Dr. D. Garelick for their very fruitful discussions and to members of the Theory Group at Brookhaven National Laboratory.

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*Work performed under the auspices of U. S. Atomic Energy Commission.

[1]V. Barger and D. Cline, Phys. Rev. Letters 21, 392 (1968) and many other references cited therein; E. W. Anderson, E. J. Bleser, H. R. Blieden, G. B. Collins, D. Garelick, J. Menes, F. Turkot, D. Birnbaum, R. M. Edelstein, N. C. Hien, T. J. McMahon, J. Mucci, and J. Ross, ibid. 20, 1529 (1968); K. Igi, S. Matsuda, Y. Oyanagi, and H. Sato, ibid. 21, 580 (1968).

[2]For example, D. Z. Freedman and J. M. Wang, Phys. Rev. 153, 1596 (1967); E. A. Paschos, ibid. 171, 1620 (1968).

[3]C. C. Shih, Brookhaven National Laboratory Report No. BNL 13038 (to be published); J. D. Jackson and G. E. Hite, Phys. Rev. 169, 1248 (1968); T. L. Trueman, Phys. Rev. 173, 1684 (1968). The other three $\gamma_i(s)$ satisfy similar constraint relations. They can be obtained by taking the MacDowell reflection from Eq. (3).

[4]S. W. MacDowell, Phys. Rev. 116, 774 (1959).

[5]V. N. Gribov, Zh. Eksperim. i Teor. Fiz. 43, 1529 (1962) [translation: Soviet Phys.—JETP 16, 1080 (1962)].

[6]G. Zweig, Nuovo Cimento 32, 679 (1964); M. P. Rekalo, Zh. Eksperim. i Teor. Fiz. 46, 612 (1964) [translation: Soviet Phys.—JETP 19, 416 (1964)]; V. G. Gorskhov, M. P. Rekalo, and G. V. Frolov, Zh. Eksperim. i Teor. Fiz. 46, 2132 (1964) [translation: Soviet Phys.—JETP 19, 1439 (1964)]; E. A. Paschos, Stanford Linear Accelerator Center Report No. SLAC-PUB-448, July, 1968 (to be published).

[7]E. W. Anderson, E. J. Bleser, H. R. Blieden, G. B. Collins, D. Garelick, J. Menes, F. Turkot, D. Birnbaum, R. M. Edelstein, N. C. Hien, T. J. McMahon, J. Mucci, and J. Russ, preceding Letter [Phys. Rev. Letters 22, 102 (1969)].

[8]The values of the residue functions can be compared with the elementary pole calculations. Because of the long extrapolation in the w plane and the uncertainty in the $(\rho N\Delta)$ coupling, the comparison is less meaningful than the case of elastic scattering.

[9]This is in the c.m. frame of the (direct) physical u channel.

[10]R. Anderson, D. Gustavson, J. Johnson, D. Ritson, R. Weinstein, W. G. Jones, and D. Kreinick, Phys. Rev. Letters 21, 479 (1968).

[11]Working directly with the crossed-channel helicity amplitudes we get

$$\frac{d\sigma}{ds}=\frac{1}{uq_u^{\,2}}\sum_i |f_{\lambda_f,\lambda_i}{}^s|^2.$$

Although the daughter mechanism is assumed to use the asymptotic expression for $\hat{f}_i^{\,s}$, the remaining half-angles are unchanged, and they are quite rapidly varying, e.g., $\cos\theta_s=-1$ at $s=s_m(\cos\theta_u=1)$, $\cos\theta_s=1$ at $s=0$, and $\cos\theta_s\simeq -u/2q_sk_s$ at $s\sim -|\text{a few}||s_m|$. Thus because of the unequal-mass kinematics, at the backward direction those amplitudes involving $(1+\cos\theta_s)$ vanish exactly. At the same time, $1/s\sim u$. We expect, therefore, that $(\sin\frac{1}{2}\theta_s)/(s)^{1/2}\sim(u)^{1/2}$ and $\cos\frac{1}{2}\theta_s$ remains small in backward core including $(s)^{1/2}=0$. Away from the extreme backward direction all half angles may be replaced by their asymptotic expression. This has also been emphasized by Paschos (Ref. 6).

COMMENTS ON YUTA-OKUBO η ASYMMETRY MECHANISM*

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The asymmetry generated by the interference between the η and nonresonant background 3π amplitudes has been calculated for previously reported data in the decay $\eta\to\pi^+\pi^-\pi^0$. An upper limit of 0.23 % has been found for the effect in this sample of data.

Recently Yuta and Okubo[1] proposed a mechanism that could produce a charge asymmetry of ~2 % in experimental samples of the decay

$$\eta\to\pi^+\pi^-\pi^0.$$

This proposal did not invoke C nonconservation. The mechanism involved an asymmetry generated by the interference between the η and nonresonant background 3π amplitudes. In this Letter, we report that after further study of our previously reported data,[2] we find that only a small part of the asymmetry of 1.5 % can be explained by this mechanism. We find an upper limit of 0.23 % as the largest possible asymmetry in our data due to this kind of interference.[3]

In our calculation of the maximum asymmetry of this kind, we have taken into account the dependence of the 3π background amplitude on the Dalitz x coordinate. The Dalitz coordinates, x and y, are

$$x=\sqrt{3}(T_+-T_-)/Q,$$
$$y=3T_0/Q-1,$$