vide a completely independent confirmation of this result. In Kawasaki's approach mentioned above, Eq. (3) is an almost direct consequence of the assumption that energy and concentration are the only macroscopic variables whose Fourier components must be included in the expansion of the microscopic fluxes. Then Eq. (3) follows quite simply from the time-reversal property of these variables.[6,11]. If we accept the validity of the expansion procedure, our results seem to indicate that other variables beside concentration and energy must be introduced. In this respect the recent work by Swift[8] and Kadanoff and Swift[13] are very interesting. There are two main differences between Kadanoff and Swift's and Kawasaki's methods. First, they estimate correlation functions with the aid of the "scaling-law" idea. As we have discussed above, this does not represent a substantial improvement of the theory. In our opinion the most important difference between the two approaches is that Kadanoff and Swift in their perturbation theory take into account the entire set of densities of the conserved additive quantities. This method has been directly applied only to the liquid-gas critical point. The relevant results for mixtures have been obtained by Swift[8] simply by establishing an analogy between transport process in binary mixtures and in simple fluids. According to this analogy diffusion corresponds to heat conduction in simple fluids and therefore α corresponds to the thermal conductivity λ. As far as the shear viscosity is concerned, his conclusions are that it has at most a logarithmic divergence. This prediction is in agreement with our experimental results.

Finally, we must mention that in a series of recent papers[14] Kawasaki has suggested that near the critical point certain modes of motion that usually have been considered to be microscopic begin to slow down enormously. As a consequence, new dynamical variables beside the original macroscopic variables must be introduced to account for all the anomalies of the transport coefficients. However no prediction on this basis has been made for binary mixtures.

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PLASMA DENSITY MEASUREMENT BY ION-BEAM PROBING*

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The collisional dissociation of a beam of energetic H_2^+ ions provides a localized and rapid measurement of plasma density and perturbation of plasma density, yet leaves the plasma undisturbed.

In some fields important quantities are hard to measure, and plasma physics is such a field. The physicist who wants to measure plasma density, for instance, finds himself caught on the horns of a dilemma: Either he can make a measurement with low spatial resolution on an undisturbed plasma by using microwaves, interferometers, or similar techniques which measure a line integral of the density, or else he can make a high-resolution measurement using Langmuir probes, but on a plasma disturbed to an unknown degree by the presence of the probe. If the plasma has high density, he can use laser holography or Thomson scattering to get a high-resolution measurement, but then the measurement is not continuous in time. We have developed an ion-beam probe for measuring plasma density which permits an escape from this dilemma: This energetic ion-beam probe has the good temporal and spatial resolution of a Langmuir probe (0.3 μsec, 1 mm^3), is continuous in time, and yet does not disturb the plasma.[1,2] Since an ion beam can be swept across the plasma electronically instead of mechanically, it can completely map a plane through the plasma in a very short time: Indeed it could, in principle, monitor simultaneously every point in a plane. Using a beam probe system, we have mapped the density of an arc plasma rapidly and in great detail and have also made maps of the perturbation in density associated with an azimuthal wave.

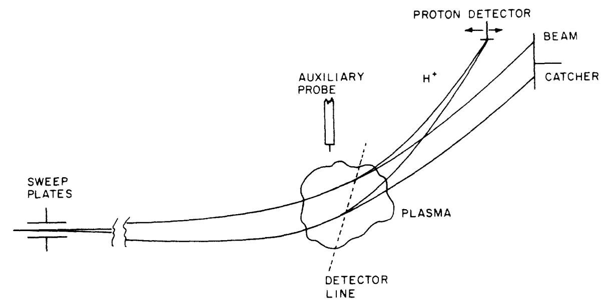

FIG. 1. Schematic of the beam probe system, showing two paths of the primary H_2^+ ion beam and also the paths of those protons which are both emitted from a point on each primary beam path and which meet at a single detector location. That detector sees all possible emission points lying on its detector line. The density measured by the beam probe is located at a point determined by the intersection of this detector line and the selected beam path.

The beam-probe density-measurement technique which we are reporting here consisted in probing the plasma with a 1.7-MeV H_2^+ beam and observing the protons produced by collisional breakup of the H_2^+ ions. The protons were separated from the main beam at the point of breakup by the magnetic field confining the plasma and therefore provide a localized measurement of the plasma density. The density of the beam was less than 10^7 cm^{-3}, and less than 1% of the beam made collisions within the plasma; hence the beam did not disturb the plasma. The plasma density was mapped throughout a plane by deflecting the beam across the plasma, measuring the breakup current from a series of points along a "detector line," and then moving the detector to measure the density along a new detector line. Figure 1 is a schematic diagram of this measurement process. For a complete mapping, the density was measured at 2000 points in a 20×100 array covering approximately 6×6 cm^2 area. Subsequently, a computer converted the data into both isometric projections and tonal maps. (In a tonal map each point has a shade of grey representative of the density at the corresponding point in the plasma.)

The plasma was a high-current magnetically confined helium arc,[3] adjusted so that the arc remained in a coherent, if somewhat noisy, oscillation near 185 kHz. The plasma density was $\approx3\times10^{13}$, $T_e\approx15$ eV, and $T_i\approx1$ eV; the arc current was ≈100 A and the confining magnetic field 3 to 5 kG. Arcs such as this are particularly awkward to measure with conventional techniques, since the density is too low to be measured with optical interferometers, yet high enough to block microwaves, and both high and hot enough to vaporize Langmuir probes. However, a number of studies of similar arcs—mostly based on peripheral Langmuir-probe measurements—have been reported recently,[4-7] and a variety of waves (ion-acoustic, drift, beam-plasma, and Alfvén) at least tentatively identified. Cylindrical symmetry and the concentricity of the total and perturbation density distributions were tacitly assumed in making these identifications; these assumptions are often made in the analysis of plasma data, yet our beam probe results show that they are not valid for our arc, and are probably not valid for other arcs or plasmas either.

Density maps that were coherent in phase with the 185-kHz oscillation in our arc were made by sampling the density signal in synchronization with a reference signal, which was obtained from the floating potential of a peripherally located Langmuir probe. In effect, these maps are stroboscopic pictures of the oscillating plasma.[8] Measurements were made with both dc and ac coupling of the density signal: With the dc coupling the recorded signal was proportional to the density at a given phase φ; with the ac coupling the recorded signal was proportional to the perturbation in density at φ. Figure 2 is an example of some of these stroboscopic measurements. The total density is shown in isometric projection in (a) and as a tonal map in (b); the nearest part of the projection corresponds to the bottom

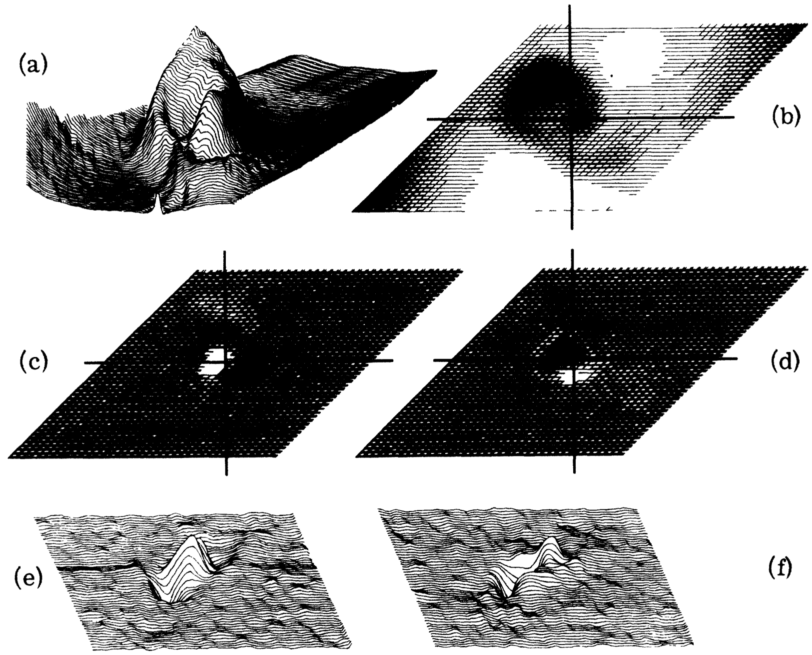

FIG. 2. Stroboscopic isometric projections and tonal maps of the density of a plasma which is sustaining an oscillation at 185 kHz. The total density at 0 μsec after a reference trigger (a) in isometric projection and (b) as a tonal map; the perturbation in density as tonal maps (c) at 0 μsec delay and (d) at 3 μsec, and (e) as isometric projections at 3 μsec, and (f) at 4 μsec. The point of view of (e) and (f) is rotated 45° counter-clockwise from the point of view of (a)-(d). The scale height of the perturbation isometric projections (e) and (f) is twice that of (a), and the contrast of the perturbation tonal maps (c) and (d) is about 10 times the contrast in (b). The high peaks at the two far corners of the total density maps (a) and (b) are the result of instrumental effects.

of the tonal map. Both (a) and (b) are at 0 μsec delay after the reference trigger; however, the time-average total density distribution and the distributions at other time delays are quite similar to these 0-μsec stroboscopic maps. The perturbation in density is shown as tonal maps at 0-μsec (c) and 3-μsec (d) delays. During this period the perturbation has rotated by approximately 190°. The axes on the tonal maps are centered on the center of this rotation. The density perturbation is also shown in isometric projection at 3 μsec (e) and 4 μsec (f), during which time it not only rotates 68° but also changes its shape quite a bit. The point of view of these two projections is rotated 45° counter-clockwise from that of the tonal maps and the total density projection (that is, the point of view is from the south-east, instead of from the south), and the scale height is twice that of the total density projection; the contrast of the perturbation tonal maps is about 10 times the contrast of the total density map.

Figure 2 clearly reveals a number of striking features of the density distribution which cannot be measured with clarity by other density-measuring techniques. These features have a strong effect on the whole nature of the plasma. (1) The total density distribution, whether time-averaged or stroboscopic, is not symmetric. (2) The density perturbation, although mostly a localized $m = 1$ azimuthal wave, does not propagate as a pure rotation of a perturbation, but rather as a sloshing of the density around the center of rotation. (3) The center of rotation is not at the center of the C-shaped total density distribution, but instead is at one end of the C. (4) The wave does not propagate along a path of nearly constant density, but instead travels over the peak of the density ridge and then out and around in the low outer plain.

The ion beam need not have the high energy of our ~2-MeV H_2^+ beam to provide adequate probing of the plasma; any ion with the same momentum, such as 20-keV Hg^+ (going to Hg^{2+}), could be used. A low-energy heavy-ion beam probe would have the great advantage that it could probably measure space potential as well as density[9] because the space potential at the point of origin of the Hg^{2+} ion modulates the energy of that ion, whereas the density modulates the number of ions created. These two properties probably could be measured simultaneously.

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INEQUALITIES FOR THE ENERGY AND FREE ENERGY OF MANY-BODY SYSTEMS*

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The Bogoliubov inequality is employed to obtain bounds on the density dependence of the free energy and energy of a wide class of interacting many-particle systems including the realistic case of multicomponent Coulomb-interacting systems.

We consider systems composed of σ species of nonrelativistic quantum or classical particles contained in a volume Ω and interacting with each other via pair potentials proportional to $1/r^n$. The masses, densities, and coupling constants can assume any values for which the system is stable.[1] The Hamiltonian is given by

$$H=\sum_{i=1}^{\sigma}T_i+\sum_{i=1}^{\sigma}\sum_{j=1}^{\sigma}V_{ij},\tag{1}$$

where T_i, the kinetic energy of the ith species, is

$$T_i=\sum_{l=1}^{N_i}\frac{(p_l^{\,i})^2}{2m_i},\tag{2}$$

and the energy of interaction between the ith and jth species is

$$V_{ij}=\frac{1}{2}\sum_{\substack{k,l=1\\k\neq l}}^{N_i}\frac{e_i^{\,2}}{|r_k^{\,i}-r_l^{\,i}|^n},\quad i=j,$$
$$=\frac{1}{2}\sum_{k=1}^{N_i}\sum_{l=1}^{N_j}\frac{e_ie_j}{|r_k^{\,i}-r_l^{\,i}|^n},\quad i\neq j.\tag{3}$$

In Eqs. (2) and (3) N_i is the number of particles of type i, m_i their mass, e_i a coupling constant (their charge, for Coulomb forces), and $r_k^{\,i}$ and $p_k^{\,i}$ the position and momentum operators for the kth particle of the ith species, respectively. We have suppressed summation over spin indices. The statistics of each species is arbitrary as long as the total system is stable.[2]

We now demonstrate that the results of some previous work[3] apply to the total energy per particle u or free energy per particle f of any system with Hamiltonian given by Eq. (1). Here the total number of particles is

$$N=\sum_i^{\sigma}N_i.\tag{4}$$

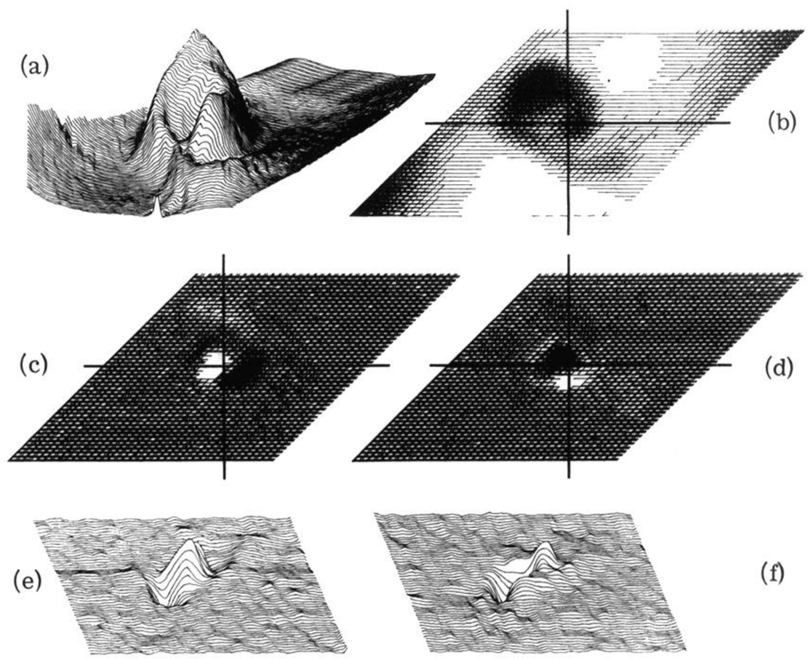
Hence u and f are given by

$$u=(1/N)\langle H\rangle,\tag{5a}$$

and

$$f=u-Ts,\tag{5b}$$

where the angular brackets denote the thermal expectation value in the ensemble characterized by H, T is the absolute temperature, and s the entropy per particle. Since (with the provisions



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