ature gradient will be set up along the radius of the wire with the axial region having the higher temperature and a correspondingly lower H_c. We have found that even a small relaxation of the field condition at the boundary near the center of the wire has a marked effect on the shape of the boundary near the axis, with a resultant increase in ρ. Thus, if H_c near the center is about 25% less than at the surface of the wire, the resistance ratio increases by about 8%. This effect will clearly be greater when (i) i_c is increased by lowering the temperature, and (ii) the residual resistance is increased.

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GANTMAKHER OSCILLATIONS IN THE NEIGHBORHOOD OF A HELICON WINDOW IN COPPER*

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Strong Gantmakher-Kaner size-effect oscillations have been observed for a magnetic field along the [110] direction in very pure copper. Their period is in good agreement with an extremum in $\partial A/\partial k_z$ for this direction, where A is the cross-sectional area of the Fermi surface. Their large amplitude is attributed to a damped helicon mode below the usual helicon absorption edge. Related results are given for the field along [111].

A window for the propagation of helicons below the absorption edge has been observed indirectly in copper. Although the mode is too highly damped to permit direct observation, it greatly enhances the amplitude of Gantmakher-Kaner (GK) size-effect oscillations[1] in the transmitted signal.

Antoniewicz[2] pointed out that it is possible, in a non-free-electron model, for a damped helicon mode to exist below the usual absorption edge. Such a mode can exist if there is an extremum in $\partial A/\partial k_z$, the derivative of the cross-sectional area of the Fermi surface in the direction of the static applied field $\vec{B}_0$. Since $\partial A/\partial k_z \sim m_c v_z$, where m_c is the cyclotron mass and v_z the average velocity of the electrons along $\vec{B}_0$, an extremum in this quantity implies that a large group of electrons will follow a helical path through the specimen with an effective wave vector $q_e = \omega_c/v_z$, where $\omega_c = eB_0/m_c c$. If the helicon mode is damped, it will have wave-vector components which correspond to q_e, and energy will be coupled into these electrons for an appreciable distance into the specimen. These electrons are responsible for the GK oscillations, as explained below.

Experimental technique.—Signals are coupled into the copper by means of a small, rectangular cross-section coil having about 200 turns of No. 42 wire placed as close to the sample as possible. A similar coil on the other side of the sample picks up the transmitted signal. A continuously tunable lock-in detection scheme, similar to that employed by Hansen, Grimes, and Libchaber,[3] was used to detect the component of transmitted signal in phase with a reference signal.

The copper samples were prepared from a boule having a resistivity ratio of about 35 000. The samples were spark cut in the form of thin plates (approximately 7.5 mm on a side), spark planed, and etched lightly in a dilute solution of nitric acid to remove surface damage. The crystallographic axis is normal to the face of the sample to within 0.3°. The sample fits into a probe so as to allow it to be tilted in the vertical plane by a micrometer at the top of the probe. The magnet can be rotated in the horizontal plane so that one can adjust independently the tilt angle in the vertical plane and the field rotation angle until the field is aligned along the crystallographic axis to within 0.1°. The magnetic field is aligned along the crystallographic axis by looking for symmetry in plots of signal amplitude versus tilt angle and rotation angle.

Experimental results.—When the field is precisely along the [110] direction we obtain GK oscillations, but the helicon modes which might be expected to appear at higher fields are completely damped[4] by open-orbit electrons.[5] When the magnetic field is slightly off axis, however, the open orbits become extended closed orbits and helicon propagation is observed as shown in Fig. 1(a). Since Weisbuch and Libchaber[6] did observe helicons, they apparently did not have the magnetic field exactly parallel to the [110] axis.

The GK oscillations appear from about 6 to 18 kG for $\vec{B}_0 \parallel [110]$, and change very little when $\vec{B}_0$ is turned a few degrees either way. The spacing between the peaks is not strictly periodic because of coupling to a helicon mode. The spacing ranges from about 390 G at the low-field end to about 314 G at the high-field end. The high-field value, which corresponds to the "purer" Gantmakher mode, yields $m_c v_z = 6.5 \times 10^{-20}$ g cm

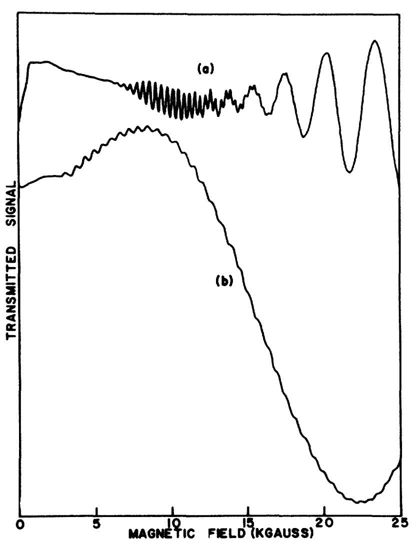

FIG. 1. (a) Transmitted signal versus B_0 for tilt angle = 0.0°, rotation angle = −2.0°, f = 246.4 kHz, sample thickness = 0.81 mm, T = 4.2°K. ($\vec{B}_0$ rotated 2° about [001] away from [110].) (b) Transmitted signal versus B_0 for $\vec{B}_0 \parallel$ [111], f = 10.0 kHz, sample thickness = 0.84 mm, T = 4.2°K.

sec^{-1}, or $\partial A/\partial k_z = 3.9 \times 10^8$ cm^{-1}, which is about 40% less than the value given by Weisbuch and Libchaber.[6] Figure 2 shows a plot of $\partial A/\partial_z$ along [110] for the Fermi surface Powell[7] has proposed to explain his magnetoresistance data. It is purportedly consistent with de Haas-van Alphen, magnetoacoustic, and cyclotron resonance data. The horizontal line which corresponds to the 314-G period reported above is seen to be consistent with the broad extremum in $\partial A/\partial k_z$ centered near $k_z/k_F = 0.6$.

Theoretical interpretation.—We first summarize the basic equations governing the propagation of electromagnetic waves in metals. The dispersion relation for a circularly polarized electromagnetic wave in a metal, in particular a helicon, may be written as

$$q^2 = (4\pi\omega/c^2)(\mathrm{Im}\sigma_\pm - i\,\mathrm{Re}\sigma_\pm), \tag{1}$$

where q is the complex wave vector, ω is the circular frequency, $\sigma_\pm = \sigma_{xx} \mp i\sigma_{xy}$ the complex conductivity of the right and left circularly polarized helicon wave, c is the speed of light, and the static magnetic field is along the z axis. It should be noted that if for some reason the real part of the conductivity is much smaller than the imaginary part, a helicon is propagated with little attenuation.

The conductivity tensor for a cylindrically symmetric surface (about the magnetic field) is

$$\sigma_\pm = \sigma_{xx} \mp i\sigma_{xy} = \frac{e^2}{4\pi^2\hbar^2}\int_{-k_z\,\max}^{k_z\,\max} \frac{m_c \tau v_\perp^{\,2} dk_z}{1 + i(\omega\tau \mp \omega_c\tau - qv_z\tau)}, \tag{2}$$

where τ is the relaxation time (assumed constant

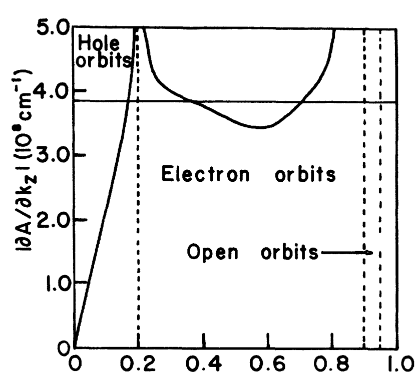

FIG. 2. $|\partial A/\partial k_z|$ vs $|k_z k_F|$ for $\vec{B}_0 \parallel$ [110]. The horizontal line indicates our experimental value for $|\partial A/\partial k_z|$.

on the surface). The ratio of the electric field in the transmitted wave to the electric field in the incident wave in the region of interest is given by[8,9]

$$\frac{E_+}{E_i} = -\frac{i4\omega}{dc}\sum_{n=1}^{\infty}\frac{(-1)^n}{(n\pi/d)^2-(\omega/c)^2\epsilon_+(n\pi/d)} + \frac{i4c}{d\omega\epsilon_+(0)}, \tag{3}$$

where $\epsilon_+(q) = K + (4\pi/i\omega)\sigma_+(q)$, K is the ionic dielectric constant, d is the sample thickness, and the assumption was made in the derivation that specular reflection took place at the boundaries of the specimen. This is a reasonable assumption if the mean free path of the electrons is of the order of or less than the specimen thickness.

A model (dumb-bell-shaped) Fermi surface proposed by Eckstein[10] was chosen to facilitate the computation. The derivative of the cross-sectional area perpendicular to the magnetic field of this Fermi surface is shown in Fig. 3(a) and has the following features: (a) There is a large number of electrons with the same velocity at the extremum $v_z = v_{z\,\mathrm{extr}}$, and (b) there are a number of electrons with velocity greater than $v_{z\,\mathrm{extr}}$. The results of the computation using this model surface are given in Fig. 3(b). The following features of Fig. 3(b) should be noted: As the magnetic field is increased [this corresponds to larger values of v_z in Fig. 3(a) to satisfy the Doppler-shifted cyclotron condition $\omega_c = qv_z$], suddenly the GK oscillations appear; then, for a higher field, a second "edge" occurs where the undamped helicons start. For low magnetic fields neither the helicon nor the GK oscillations exist. These features are interpreted as damped helicons "launching" GK oscillations for magnetic field values such that $\omega_c > qv_{z\,\mathrm{extr}}$ and a normal helicon propagating for $\omega_c > qv_{z\,\mathrm{max}}$. Here q is given by the helicon dispersion relation, i.e., the zero of the denominator of Eq. (3).

In this connection it should be remembered that GK oscillations are caused by a group of electrons with almost the same velocity, which are excited on one side of the sample and arrive on the other side in phase for certain values of the magnetic field, i.e., when $\omega_c = qv_{z\,\mathrm{extr}}$ is satisfied, where $q = \pi n/d$, and n is some integer. However, GK oscillations of sufficiently large amplitude are seen only under some special conditions. The electrons could pick up the energy from the normal skin depth, but with certain exceptions the electrons do not pick up enough energy with the right phase to give an experimentally observable result.

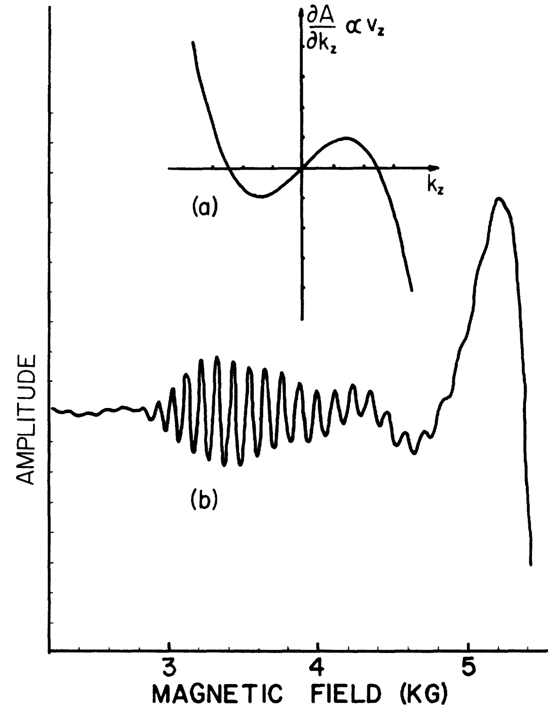

FIG. 3. (a) The derivative of the cross-sectional area with respect to k_z $[m_c v_z = -(\hbar/2\pi)(\partial A/\partial k_z)]$ as a function of k_z for a dumb-bell Fermi surface proposed by Eckstein (see Ref. 10). (b) The real part of the transmitted electric field in arbitrary units as a function of the external magnetic field. This figure is computed using the Fermi surface described in (a) for a frequency $\omega = 10^6\ \mathrm{sec}^{-1}$, relaxation time $\tau = 10^{-9}$ sec, and an electron density $n = 1.4\times 10^{22}\ \mathrm{cm}^{-3}$.

Let us examine the results for a free-electron surface. If the helicon is undamped, then only electrons with helical wavelength matching that of the helicon will pick up energy from the wave. Other electrons quickly get out of phase with the wave and do not contribute to the GK oscillations. But we cannot have the GK electrons resonate with the helicon since they are precisely the ones which would damp the helicon wave the most. Therefore for high fields where helicons are slightly damped the GK oscillations vanish. Below the absorption edge the real part of the wave vector is approximately equal to the imaginary part of the wave vector; thus the helicon wave does not exist and it therefore does not excite GK oscillations, at least not large enough to be seen experimentally. The transition region between the damped and undamped helicon is very sharp and, since the GK oscillations would have the same period (as a function of the magnetic field) as the helicons in this region, the GK oscillations are probably not observable. The same is true for a surface with an orbit edge.[11] Here the GK oscillation should be stronger but still probably would not be seen. The above conjectures were borne out by a numerical computation for a spherical Fermi surface and a surface with an orbit edge. The amplitude of the GK oscillations below the absorption edge for the latter surface is two orders of magnitude smaller than the GK oscillations occurring in the "window" region of Fig. 3(b) under the same conditions: Hence the statement that they probably would not be seen.

If, however, one has a mechanism for damping the helicon for magnetic fields such that $\omega_c = qv_z$ is satisfied by electrons with velocities v_z above the extremum in the derivative of the cross-sectional area, then GK oscillations are "launched" by the enhanced skin depth due to the damped helicon. The electrons respond to the enhanced skin depth due to damped helicons even if their helical wavelength does not match that of the predominant helicon wavelength as long as the mismatch is not too large. One can think of the Fourier component of the damped helicon exciting these electrons, and as long as this Fourier component is reasonably close to the center frequency its amplitude is reasonably large.

The above considerations may be seen in Eqs. (2) and (3). The GK oscillations arise from the large peak in the conductivity which occurs at $\omega_c = qv_{z\,\mathrm{extr}}$ but since the wave vector q is "quantized" due to the boundary conditions, a term with the large conductivity occurs only for "quantized" values of the magnetic field. There is a linear relation between the magnetic field and the the wave vector for the GK oscillations, but $q \sim B^{-1/2}$ for the helicons, so that the GK oscillations eventually die out as the wave-vector mismatch becomes too great. One should note that the width of the spectrum of wavelength in the helicon is proportional to the damping. The amplitude of this spectrum is, however, inversely proportional to the damping. If open orbits are present, they damp out the helicons[4] in the same way as pieces of Fermi surface with $v_z > v_{z\,\mathrm{extr}}$ and, if the open orbits have a small number of electrons, then the GK oscillations still exist even though the helicons are damped out. This is seen in the polar plot of Cu in the [110] direction.

Figure 1(b) shows the results for $\vec{B}_0 \parallel [111]$ and a frequency of 10 kHz. Here the GK oscillations are superimposed on the helicon signal up to at least 25 kG. Their period is about 600 G, which implies that there is an extremum for $m_c\bar{v}_z = 13 \times 10^{-20}$ g cm sec^{-1}, or $|\partial A/\partial k_z| = 7.8\times 10^8$ cm^{-1}. Powell's Fermi surface has an extremum at about 14×10^{-20} g cm sec^{-1}, or 8.4×10^8 cm^{-1}, which is in good agreement with our experimental value. The helicon edge occurs at a relatively low field for this orientation because a large fraction of the electrons execute hole-type orbits, leading to a rather small number of effective electrons. Antoniewicz[12] has obtained a similar curve for a calculation based on a Fermi surface consisting of an undulating cylinder plus a spherical hole surface.

Thus far we have not observed GK oscillations for the [100] direction, despite the fact that there is an extremum in $m_c\bar{v}_z$, and presumably a window for this direction as well. A possible explanation for their absence may lie in the fact that the electrons at the extremum have a large oscillatory component in v_z, so that they do not stay in phase with the helicon field.

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