(1967).
[2]E. L. Wolf, Phys. Rev. Letters 20, 204 (1968).
[3]L. C. Davis and C. B. Duke, Solid State Commun. 6, 193 (1968).
[4]F. Steinrisser, L. C. Davis, and C. B. Duke, to be published.
[5]W. L. McMillan and J. M. Rowell, to be published.
[6]R. N. Hall, J. A. Racette, and H. Ehrenreich, Phys. Rev. Letters 4, 456 (1960).
[7]J. Shewchun and R. M. Williams, Phys. Rev. Letters 15, 160 (1965).
[8]M. Hass and B. W. Henvis, J. Phys. Chem. Solids 23, 1099 (1962).
[9]G. Dolling and J. L. T. Waugh, in Lattice Dynamics, edited by R. F. Wallis (Pergamon Press, New York 1965).
[10]J. M. Rowell and L. Kopf, Phys. Rev. 137, A907 (1965).
[11]H. Ehrenreich and A. W. Overhauser, Phys. Rev. 104, 331 (1956); W. A. Harrison, Phys. Rev. 104, 1281 (1956).
[12]F. H. Pollack, C. W. Higginbotham, and M. Cardona, J. Phys. Soc. Japan Suppl. 21, 20 (1966).
[13]C. Kittel, Quantum Theory of Solids (John Wiley & Sons, Inc., New York, 1963), Chap. 7.
[14]J. M. Rowell, private communication.

DESTRUCTION OF SUPERCONDUCTIVITY BY A CURRENT

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We present a new model of the intermediate state in current-carrying superconductors. The model predicts a resistance transition which is in reasonable agreement with experimental values. An introductory treatment of secondary effects is also given.

When superconductivity in a wire is destroyed by a current, resistance returns in a manner different from that predicted by London,[1-4] and the consideration of secondary effects[5,6] does not account satisfactorily for the discrepancy. We present a treatment which gives the structure of the intermediate state, predicts a resistance transition in reasonable agreement with experimental observations, and enables the evaluation of secondary effects.

When the current in a superconducting wire reaches the value i_c at which the magnetic field at the surface has the critical value H_c, flux should start to penetrate the wire and create a series of normal regions along the wire. Throughout these regions the field must obviously be H_c or as close to H_c as possible. It follows that in the normal regions the variation of current density with radius r should ideally have the form

$$j(r)=(i_c/2\pi a)(l/r),$$

where a is the radius of the wire. However, it is not possible to find a finite structure which gives $H=H_c$ throughout the whole of the normal volumes, and the most important condition is obviously that $H=H_c$ at each normal-superconducting interface.

For a given boundary the potential distribution in one of the normal regions can be obtained by a numerical solution of Laplace's equation subject to the following boundary conditions: (i) $V=$ constant on any s-n boundary and (ii) $\partial V/\partial r=0$ at the surface of the wire. The current distribution, and hence the field distribution, can then be calculated from the potential distribution. Using trial and error methods, boundaries satisfying the field criterion mentioned above can be found for any given ratio of the wire radius a to the structure periodic length d, up to a limiting value $(a/d)_{\max}=1.4$. The axial width of the normal regions at the center of the wire falls rapidly to zero as a/d approaches 1.4, thus making it impossible to find satisfactory boundaries for a/d larger than 1.4.

As has been pointed out by Shoenberg,[7] two mutually opposing criteria will determine the optimum value of a/d: (i) That throughout the normal region H be as close as possible to H_c. Quantitatively, this amounts to minimizing

$$e=\frac{H_c^{\,2}}{8\pi}V_n-\int_{V_n}\frac{H^2}{8\pi}dv$$

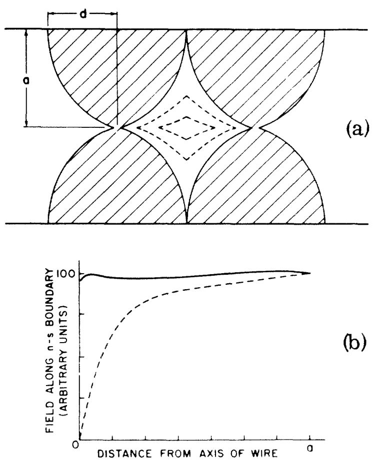

FIG. 1. (a) Intermediate-state structure for $a/d = 1.4$. (b) Variation of H along the boundary compared with that of London's model.

per unit length of wire, where V_n is the volume of the normal region. It is obvious that e decreases as a/d increases. (ii) The n-s surface energy s per unit length of wire be a minimum. This makes smaller values of a/d more favorable. Values of e and s have been calculated for indium and it has been found that for the values of a/d under consideration, s is only a few percent of e, and is therefore negligible. In type-I superconductors s becomes important only when a/d is much larger than 1.4. Hence, we have accepted $(a/d)_{\max} = 1.4$ as the optimum value, and we suggest that at $i = i_c$ the structure of the intermediate state is as shown in Fig. 1(a). Such a structure has the normalized resistance

$$\rho = (R/R_n)_{i=i_c} = 0.69,$$

and the variation of field along the n-s boundary is shown in Fig. 1(b).

When the current rises above i_c, the superconducting cores must shrink to an equilibrium shape for each value of current so as to maintain the condition $H = H_c$ along the boundaries. Boundaries to satisfy this criterion have been found for several values of $i > i_c$, and two of them are indicated by the broken lines in Fig. 1(a). The resistances of these structures have been evaluated, and the full return of resistance curve as predicted by our model is shown in Fig. 2.

Since the present model gives a definite size to the structure, it is amenable to the treatment of secondary effects such as mean free path and Joule heating.

In the above calculations we have assumed that

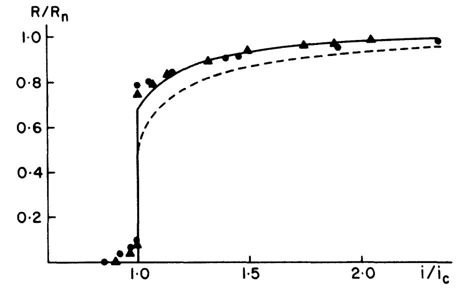

FIG. 2. Return of resistance curve: solid line, present model; dashed line, London (Ref. 1); closed circles, Scott (Ref. 2) (0.286-mm-diam In wire); closed triangles, D. C. Baird and B. K. Mukherjee (unpublished) (0.5-mm-diam In wire).

the mean free path l in the normal material is small compared with the dimensions of the normal regions. Kuper has suggested that when this is not the case, scattering of electrons at n-s interfaces will reduce the effective mean free path, thereby increasing the value of ρ. He has proposed[6] that for the London model the average effective conductivity is given approximately by

$$\sigma(r) = \sigma_0\{1-(1-p)\exp(-rp/l)\}$$

with $p = \sin\psi$, where ψ is half the angle at the apex of the normal region. Assuming that the lower part of the phase boundary in the present model may be reasonably approximated by a straight line, we have used the Kuper formula to find the values of ρ for different diameters of pure indium wire assuming that $l = 2.5\times10^{-3}$ cm, and our results are shown in Fig. 3 along with experimental results for comparison.

It is obvious that due to Joule heating a temper-

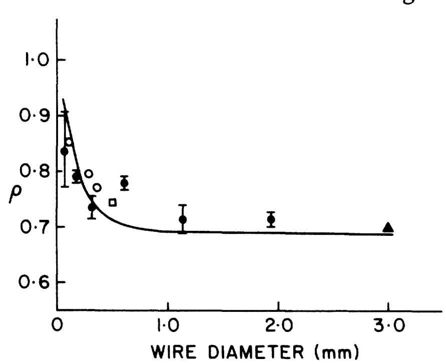

FIG. 3. Mean free path effect: The resistance discontinuity ρ at i_c plotted as a function of wire diameter. Solid line: prediction of the present model; open circles: Scott (Ref. 2), closed circles: Meissner and Zdanis (Ref. 4), open squares: D. C. Baird and B. K. Mukherjee (unpublished), and closed triangles: R. Freud, Cz. Sulkowski, and B. Makiej [Phys. Letters 27A, 187 (1968)].

ature gradient will be set up along the radius of the wire with the axial region having the higher temperature and a correspondingly lower H_c. We have found that even a small relaxation of the field condition at the boundary near the center of the wire has a marked effect on the shape of the boundary near the axis, with a resultant increase in ρ. Thus, if H_c near the center is about 25% less than at the surface of the wire, the resistance ratio increases by about 8%. This effect will clearly be greater when (i) i_c is increased by lowering the temperature, and (ii) the residual resistance is increased.

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[1]F. London, Superfluids (John Wiley & Sons, Inc., New York, 1950), Vol. 1.

[2]R. B. Scott, J. Res. Natl. Bur. Std. (U.S.) 41, 581 (1948).

[3]L. Rinderer, Helv. Phys. Acta 29, 339 (1956).

[4]H. Meissner and R. Zdanis, Phys. Rev. 109, 681 (1958).

[5]S. T. Berkovich and G. M. Lapir, Zh. Eksperim. i Teor. Fiz. 44, 1329 (1963) [translation: Soviet Phys. – JETP 17, 896 (1963)].

[6]C. G. Kuper, Phil. Mag. 43, 1264 (1952).

[7]D. Shoenberg, Superconductivity (Cambridge University Press, New York, 1952).

GANTMAKHER OSCILLATIONS IN THE NEIGHBORHOOD OF A HELICON WINDOW IN COPPER*

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Strong Gantmakher-Kaner size-effect oscillations have been observed for a magnetic field along the [110] direction in very pure copper. Their period is in good agreement with an extremum in $\partial A/\partial k_z$ for this direction, where A is the cross-sectional area of the Fermi surface. Their large amplitude is attributed to a damped helicon mode below the usual helicon absorption edge. Related results are given for the field along [111].

A window for the propagation of helicons below the absorption edge has been observed indirectly in copper. Although the mode is too highly damped to permit direct observation, it greatly enhances the amplitude of Gantmakher-Kaner (GK) size-effect oscillations[1] in the transmitted signal.

Antoniewicz[2] pointed out that it is possible, in a non-free-electron model, for a damped helicon mode to exist below the usual absorption edge. Such a mode can exist if there is an extremum in $\partial A/\partial k_z$, the derivative of the cross-sectional area of the Fermi surface in the direction of the static applied field $\vec{B}_0$. Since $\partial A/\partial k_z \sim m_c v_z$, where m_c is the cyclotron mass and v_z the average velocity of the electrons along $\vec{B}_0$, an extremum in this quantity implies that a large group of electrons will follow a helical path through the specimen with an effective wave vector $q_e = \omega_c/v_z$, where $\omega_c = eB_0/m_c c$. If the helicon mode is damped, it will have wave-vector components which correspond to q_e, and energy will be coupled into these electrons for an appreciable distance into the specimen. These electrons are responsible for the GK oscillations, as explained below.

Experimental technique.—Signals are coupled into the copper by means of a small, rectangular cross-section coil having about 200 turns of No. 42 wire placed as close to the sample as possible. A similar coil on the other side of the sample picks up the transmitted signal. A continuously tunable lock-in detection scheme, similar to that employed by Hansen, Grimes, and Libchaber,[3] was used to detect the component of transmitted signal in phase with a reference signal.

The copper samples were prepared from a boule having a resistivity ratio of about 35 000. The samples were spark cut in the form of thin plates (approximately 7.5 mm on a side), spark planed, and etched lightly in a dilute solution of nitric acid to remove surface damage. The crystallographic axis is normal to the face of the sample to within 0.3°. The sample fits into a