Therefore, the effective ionic charge of the Ti ion is $e^* = 0.3P_S/(\Delta x)N_0$. With $N_0 = 2.7\times10^{22}/\mathrm{cm}^3$, e^* is ~2.1 electronic charges. If the Ti ion is assumed to have about the same effective charge in $SrTiO_3$, then the displacement of the Ti ion must be about 0.012 Å. This estimate also contains important uncertainties, but it is quite consistent with the value deduced from the induced optical transition.

This is the first observation of an electric-field–induced intensity change of a sharp line in a solid. Such effects for vibrational spectra of symmetric molecules were predicted by Condon[9] and observed by Crawford and Dagg.[10] For ions in a solid, observation is complicated by the magnetic dipole transitions which can have appreciable intensity even when an electric dipole transition is completely forbidden by symmetry. But now that the effect has been observed, the induced electric dipole transition can be measured as a function of the induced polarization which is also a measurable quantity. Further studies will be needed to confirm that the induced dipole transition is the only factor contributing to the intensity change. It is conceivable that part of the change might be an increased magnetic dipole intensity arising from changes in the crystal-field parameters when the electric field is applied. This question could be decided by polarization measurements, preferably in a magnetic field, but such studies are greatly complicated by the domain structure of the crystals.

The field-induced intensity change observed here may permit observed oscillator strengths to be used for estimates of the relationship between fields and displacements of ions. It also provides a new way to modulate the fluorescence intensity.

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PHONON-LIMITED DRIFT OF THE ELECTRON BUBBLE IN SUPERFLUID HELIUM*

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We present measurements of the negative ion mobility in He II for $T \leq 0.5$°K. We show that the usual kinetic-theory approach to ion drift in He II is inadequate, and we describe a more realistic model. It is found that inelastic phonon scattering is the dominant process determining the mobility of the electron bubble at these temperatures.

The limiting drift velocities of ions[1] in He II, under the influence of an electric field, have been investigated in some detail by Meyer and Reif.[2-4] Their results showed that at low fields the ion drift velocity v_D is linearly proportional to the applied electric field E, where the proportionality constant μ (the mobility) is temperature dependent. At higher temperatures, where roton scattering dominates, μ was seen to vary roughly as the inverse of the roton number density, while at their lowest temperatures ($T \sim 0.5$°K) phonon scattering became important. According to the data of Meyer and Reif, the temperature dependence of the phonon-limited mobility μ_p is $\mu_p \propto T^{-l}$ where $l = 3.3 \pm 0.3$ for positive ions and $l = 2.4 \pm 0.4$ for negative ions. This qualitatively implies that μ_p varies as the inverse of the phonon number density.

The interpretation of this behavior has generally proceeded along the lines of elementary kinetic theory. The ion moving through the field of thermal excitations is treated as analogous to a particle moving through a gas. In the limit where v_D is much less than the mean thermal velocity of the ion, such a model yields

$$v_D = (eE/M)\tau = \mathrm{const}\,En^{-1}, \tag{1}$$

where e is the ion charge, M the ion effective mass, E the electric field, τ the relaxation time, and n the number density of scatterers. Equation

(1) apparently explains the observed field and temperature dependences. The purpose of this paper, however, is to present experimental data which show that the model leading to Eq. (1) is unrealistic when applied to the motion of the electron bubble in the temperature region below 0.5 °K, where phonons completely dominate the scattering.

We have accurately determined the mobilities of negative ions by means of direct time-of-flight measurements. The ions were created by an Am^{241} α source and injected into the drift region by a weak electric field between the source and a grid located 1 cm in front of it. They then drifted through a region 28.2 cm in length. By means of 26 guard rings, the electric field in the drift space was kept constant to $\frac{1}{2}$%. The ions were collected at the end of the drift space, and the resulting currents were detected by an operational amplifier[5] the output of which was fed into a 400-channel analyzer. Well-defined pulses of ion current were produced by applying voltage pulses across the source region, and times of flight were read directly off the analyzer.

A typical plot of measured drift velocity versus field is given in Fig. 1. It was found that the strict linear dependence seen in Fig. 1 was maintained up to quite high velocities. E.g., at 0.5°K deviations from straight-line behavior were observed only above about 16 m sec^{-1}. Our measurements stayed well within the linear region at all temperatures. The mobilities obtained in this way proved to be very closely proportional to T^{-3} for $T \leqslant 0.5$°K. This is illustrated in Fig. 2. Fitting $\mu = AT^{-n}$ to the data gives $n = 3.00 \pm 0.01$. The rms deviation of the points below 0.5 °K from the equation $\mu = 32.47T^{-3}$ cm^2 V^{-1} sec^{-1} is ~0.3%. However, the coefficient is estimated to be accurate to only 1% in absolute terms, due to the possible existence of small systematic errors. Corrections for roton scattering are already negligible at 0.5°K.

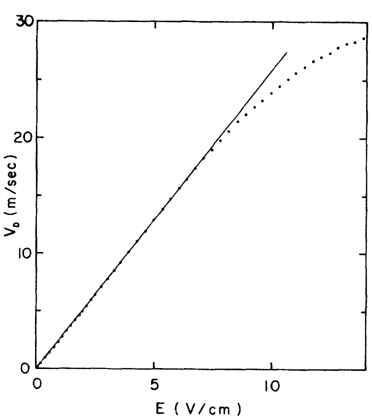

FIG. 1. Drift velocity versus electric field at 0.5°K.

Two points should be made here. First, at 0.5 °K the mean thermal velocity of the electron bubble is about 5 m sec^{-1}, assuming an effective mass of order 100 He^4 masses.[6] The linear variation of v_D with E at velocities as large as 16 m sec^{-1} shows that the usual kinetic-theory model is not applicable. Second, at thermal velocities the negative ion carries about 100 times the momentum of the average phonon, so that small-angle scattering must dominate.

A more realistic picture can be obtained as follows. From the reference frame of the electron bubble moving with velocity $\vec{v}$ through the phonon field, the momentum flux tensor due to phonons occupying states between $\vec{k}$ and $\vec{k}+d\vec{k}$ is given by

$$p_{\alpha\beta} = n(k)\hbar k_\alpha (ck_\beta/k - v_\beta) d^3k/8\pi^3, \tag{2}$$

where $n(k)$ is the Bose-Einstein distribution function, $\hbar\vec{k}$ is the phonon momentum, and c is the velocity of sound. If the momentum-transfer cross section is $\sigma(k)$, the rate of momentum loss of the bubble is $\sum v_\alpha P_{\alpha\beta}$, where

$$P_{\alpha\beta} = \frac{\hbar}{8\pi^3}\int \frac{\sigma(k)k^{-1}}{\exp(c\hbar k/k_B T)-1} k_\alpha k_\beta d^3k. \tag{3}$$

Only diagonal terms are nonzero, and these are all equal. Setting $eE_\beta = \sum v_\alpha P_{\alpha\beta}$ and $\vec{v} = \vec{v}_D$, one obtains

$$e\vec{E} = \frac{\hbar}{6\pi^2}\vec{v}_D \int_0^\infty \frac{\sigma(k)k^3 dk}{\exp(c\hbar k/k_B T)-1} \tag{4}$$

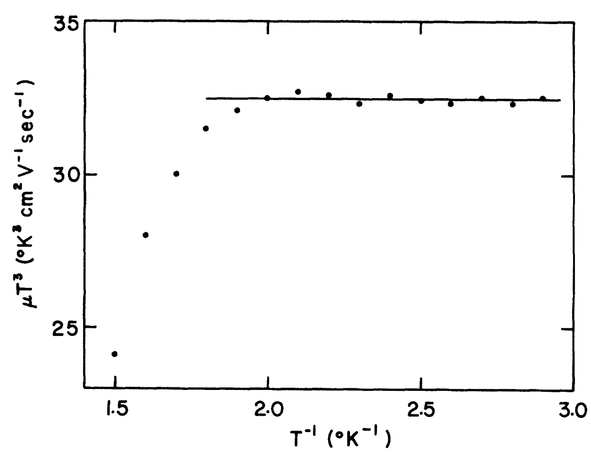

FIG. 2. Electron bubble mobility times T^3 as a function of inverse temperature. The line corresponds to $\mu = 32.47T^{-3}$ cm^2 V^{-1} sec^{-1}.

provided that $v_D \ll c$. If one now assumes $\sigma(k) = Ak^n$, one obtains $\vec{v}_D = \epsilon \vec{E} T^{-(n+4)}$, with

$$\epsilon = \frac{6\pi^2 e}{A\hbar}\left(\frac{\hbar c}{k_B}\right)^{n+4} \frac{1}{\Gamma(n+4)\zeta(n+4)}, \tag{5}$$

where ζ is the Riemann zeta function and Γ is the gamma function. The linear dependence of v_D on E up to quite high velocities thus arises naturally. The strict T^{-3} dependence we have observed strongly implies $\sigma = Ak^{-1}$.

Over the temperature range of our experiment the average phonon wavelength varies from about 300 to 500 Å, as compared with commonly accepted values of the electron bubble radius[7,8] which range from 16 to 21 Å. Hence, the classical scattering of sound from a deformable sphere in the long-wavelength limit should constitute a good approximation to the elastic scattering of phonons by the bubble. Such a model yields a cross section[9]

$$\sigma(k) = 4\pi a^2(\gamma - \tfrac{1}{3})^2 \frac{(ka)^4}{(ka)^2 + [\gamma(ka)^2 - 1]^2}, \tag{6}$$

where a is the bubble radius and γ is one-third the ratio of the volume compressibility of the bubble to the volume compressibility of the bulk liquid. For a rigid bubble this yields the Rayleigh limit $\sigma(k) \to (4\pi/9)a^2(ka)^4$, while for a very soft bubble $\sigma(k) \to 4\pi a^2$. In no case does Eq. (6) exhibit a k^{-1} dependence, thus leading directly to the conclusion that elastic scattering cannot explain our observed temperature dependence.

We note, however, that a k^{-1} dependence is precisely the characteristic feature of inelastic scattering cross sections in the long-wavelength limit.[10] In the light of the above discussion, our data therefore indicate that inelastic processes provide the dominant mechanism for energy and momentum transfer between the phonon field and the electron bubble, leading very naturally to the exact T^{-3} dependence of the measured mobility.

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