# REGGE CUTS, ABSORPTION MODEL, AND DIFFRACTION DIPS IN INELASTIC SCATTERING*

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(Received 8 August 1968)

We propose an expression for calculating Regge-cut contributions to inelastic scattering, based on the absorption model. We present a brief justification of our expression, using the idea of an instantaneous composite structure for the scattering particles. We apply the model to $\rho$ exchange in $\pi^- p \to \pi^0 n$.

In recent years, a phenomenology of high-energy (quantum-number exchange) reactions based on Reggeon exchange has enjoyed some success.[1] We claim, however, that recent detailed experimental information makes it clear that the description of high-energy reactions based on analytic properties in the angular-momentum plane will not be simple mathematically. In particular, we show that physical arguments regarding multiple scattering as well as experimental data demand a large role for Regge cuts.[2] We present an expression for the "principal cut" associated with any pole. The expression involves only one (scale) parameter, slightly larger than unity, in addition to the parameters needed to describe the pole.

The experimental evidence indicating a strong role for cuts is shown through (1) forward peaking in $\pi$-exchange processes,[3] and (2) dips and secondary maxima. There is also a variety of relatively detailed experimental indicators such as (3) polarization in $\pi$ charge-exchange scattering and (4) the crossover in the $\pi^{\pm} p$ differential cross sections. All these phenomena can probably be accurately understood in terms of the principal cut and its interference with the pole contribution. Detailed applications to (2)-(4) are presented. Good agreement with experiment is found. For $\pi$ exchange at very small momentum transfer our formalism is qualitatively the same as the absorption model of Gottfried and Jackson.[4] We will discuss $\pi$-exchange processes in a future publication.

The model we use is an absorption model with Regge-pole exchange input. We start with the Sopkovich formula[4,5]

$$T_l^{\mathrm{Abs}} = (S_l^{\mathrm{el}})^{\frac{1}{2}} T_l^{\mathrm{ex}} (S_l^{\mathrm{el}})^{\frac{1}{2}}. \tag{1}$$

In this formula $T_l^{\mathrm{ex}}$ is an amplitude for the exchange of some quantum number in a single scattering and $S_l^{\mathrm{el}}$ is the elastic $S$-matrix element in either the initial or final state. $T_l^{\mathrm{Abs}}$ is the amplitude to be compared with experiment. We expand the root of $S_l^{\mathrm{el}} = 1 + 2ik T_l^{\mathrm{el}}$ in a binomial series:

$$\delta T_l = T_l^{\mathrm{Abs}} - T_l^{\mathrm{ex}}$$
$$= ik(T_l^{\mathrm{el}} + T_l^{\mathrm{ex}} + T_l^{\mathrm{ex}} T_l^{\mathrm{el}}) + \cdots.$$

We drop higher terms, which go to zero if the elastic scattering is the same in the initial and final states, and which are different in different absorption models. The terms we retain in $\delta T_l$ are exactly the double-scattering terms (and, in particular, shadowing terms) in a multiple-scattering theory.[6] They represent an elastic and an exchange scattering occurring in either order. Thus, neglecting spin for ease of exposition, if we partial-wave analyze the single-scattering (or pole-exchange) amplitude

$$T^{\mathrm{ex}} = \sum (2l+1) T_l^{\mathrm{ex}} P_l$$

and the usual elastic amplitude

$$T^{\mathrm{el}} \equiv f = \sum (2l+1) T_l^{\mathrm{el}} P_l,$$

the Sopkovich formula for the full amplitude (equal final and initial scattering) is

$$T^{\mathrm{Abs}} = \sum (2l+1) S_l^{\mathrm{el}} T_l^{\mathrm{ex}} P_l = T^{\mathrm{ex}} + \delta T$$

with

$$\delta T = 2ik \sum (2l+1) T_l^{\mathrm{el}} T_l^{\mathrm{ex}} P_l.$$

One finds immediately

$$\delta T = \frac{2ik}{4\pi} \int d\Omega\, T^{\mathrm{ex}}(\Omega_1) T^{\mathrm{el}}(\Omega_2). \tag{2}$$

For $T^{\mathrm{el}}$ we use observed elastic cross sections, setting the final elastic scattering equal to the initial by averaging. For $T^{\mathrm{ex}}$ we use Regge-pole exchange, with the usual Regge-pole parameters varied in order to give a good fit to experi-

mental data. The Regge-pole amplitudes have the most simple structure; they are evasive and have no nonsense dips.

Our model must be justified. Both Reggeization and absorption involve composite structure, and it is not a priori clear that the same effect has not been included twice. A detailed argument that two different effects are involved will be presented in a later paper; we only outline the argument here. The original absorption model[4,5] used elementary particle exchange for the single-scattering amplitude $T^{ex}$. It clearly lacks the composite structure of the exchanged particle. This showed up in the incorrect energy dependence in the original absorption model.

The effect absorption takes into account is the geometric (or composite) structure of the scattering particles. For example, in the eikonal approximation derivation of the absorption model, the scattering is pictured as the passage of a particle through an extended optical potential, with the scattering which effects the quantum-number exchange occurring at some intermediate point on the trajectory of the particle. Regge-pole exchange involves a form factor, which is also a manifestation of the structure of the scattering particles. However, the form factor only involves the time-averaged structure, whereas the absorption at high energies involves the instantaneous structure.

In summary, the original absorption model lacks the structure of the exchanged particle, and the Regge-pole model lacks the instantaneous structure of the scattering objects. Our model includes both.

In the $t$-channel angular-momentum plane, our correction $\delta T$ is a Regge cut, with the usual branch-point trajectory (if elastic scattering is given by a Pomeranchuk-Regge pole). The fact that its singularity structure is completely different from the input Regge pole is further evidence that a new effect is included.[7]

The justification of our model requires that the exchanged Reggeon effecting the quantum-number exchange be a "particle." It is not clear that such a description applies to elastic scattering (or "diffraction dissociation" scattering). There is no known particle definitely associated with the Pomeranchuk Regge trajectory, so that this may not be a simple Regge pole representing single elastic scattering. Related models have assumed a related description for elastic scattering, involving single and multiple Pomeranchuk Regge-pole exchange.[8] Although this assumption is plausible, our justification of our model does not extend to these models. Therefore we apply our model only to inelastic (quantum-number exchange) scattering.

We consider the reaction $\pi^- p \to \pi^0 n$ via $\rho$ exchange in order to illustrate the formulation and especially the phenomenon of dip plus secondary maximum. The amplitude is the sum of $\rho$ Regge-pole exchange and the associated absorption term:

$$M_{\lambda'\lambda} = M_{\lambda'\lambda}^{\rho} + M_{\lambda'\lambda}^{\rho P}, \tag{3}$$

$$M_{\lambda'\lambda}^{\rho P} = \frac{i}{32\pi^2}\sum\int d\Omega\, M_{\lambda'\lambda''}^{\rho} M_{\lambda''\lambda}^{P}, \tag{4}$$

where, for unpolarized nucleons,

$$d\sigma/dt = (64\pi k^2 s)^{-1}(|M_{++}|^2 + |M_{+-}|^2).$$

For the purposes of computation the Regge-trajectory exchange factor is approximated so that at high energy and small angles,

$$M_{+-}^{\rho} = e^{i\varphi}\gamma(-t)^{\frac{1}{2}}\left[\frac{2}{3} - \frac{1}{\alpha(t)-1}\right] \times ie^{-\frac{1}{2}i\pi\alpha(t)}\left(\frac{E}{E_0}\right)^{\alpha(t)}, \tag{5}$$

$$M_{++}^{\rho} = \gamma^{+}\left[\frac{2}{3} - \frac{1}{\alpha(t)-1}\right] \times ie^{-\frac{1}{2}i\pi\alpha(t)}\left(\frac{E}{E_0^{+}}\right)^{\alpha(t)}, \tag{6}$$

where $\gamma$'s and $E_0$'s are real constants to be chosen. In other words $E_0$ is chosen to optimize the approximation $\gamma = \text{const}$. Here $E$ is the lab energy. Two more constants enter in the assumed linear trajectory: $\alpha = \alpha_0 + \alpha_1 t$.

The elastic-scattering amplitude will be taken to be

$$M_{\lambda'\lambda}^{P} = \delta_{\lambda\lambda'}(i+\rho)s\sigma e^{Gt}. \tag{7}$$

We will take $\rho$, $\sigma$, and $G$ from experiment, including their gradual energy dependence.

The double-scattering integration can be carried out analytically to yield a simple form when some high-energy small-angle approximations and an expansion[9] in the parameter $\alpha_1/[(1-\alpha_0)(F$

$+G)]$ are made:

$$M_{+\mp}^{\rho P} = -\lambda A_{\mp}\frac{(i+\rho)\sigma}{8\pi(F+G)}e^{\alpha_0 F/\alpha_1}e^{FGt/(F+G)}\left[\frac{2}{3}+\left(1-\alpha_0+\frac{\alpha_1}{F+G+\alpha_1/2(1-\alpha_0)}-\frac{\alpha_1 G^2 t}{(F+G)^2}\right)^{-1}\right], \quad (8)$$

where $F=\alpha_1[\ln(E/E_0)-\frac{1}{2}i\pi]$, $A_-=\gamma G(-t)^{1/2}/(F+G)$, and $A_+=\gamma^+$, with the value of $E_0$ appropriate to the (+−) or (++) case and where the factor $\lambda$, a scale factor for the double-scattering term expected to be somewhat larger than unity, has been introduced.

A fit has been made to the differential cross-section data,[10] using helicity flip only (omitting consideration of data very near 0° where helicity nonflip must dominate). The results are shown in Fig. 1 using the parameters given in the caption.[11] The value for trajectory slope $\alpha_1$ is 1.2 $(\mathrm{BeV}/c)^{-2}$, similar to values currently accepted. The value of $\alpha_0$ is significantly less than the effective value $\alpha(0)\approx 0.6$, which, in the absence of consideration of cuts, has been considered the proper value for the $\rho$. Using these new parameters, $\alpha=1$ yields $m_\rho=0.725$ BeV, in good agreement. The value $\lambda=1.9$ seems reasonable to us in terms of the contributions of other intermediate states such as $A_1$ instead of $\pi$ and other $N_{I=\frac{1}{2}}$ trajectories instead of the nucleon.

There is no difficulty in describing the dip at $t=-0.6$ in this theory.[12] The dip moves slowly as a function of energy so that experiments at 70 or 200 BeV/$c$ will probably be needed to observe its motion. It is the logarithmic shrinkage of the single-scattering term embodied in the $e^{\alpha_1 \ln(E/E_0)t}$ factor that determines the motion of the dip. This competes with the decrease in total cross section which reduces the magnitude of the double-scattering or cut term. At very high energy, assuming $\sigma_T \to$ const, the shrinkage dominates the motion.

The diffraction dip, as observed in $\pi$ charge-exchange scattering, should occur in most other processes. The position should vary with the range of interaction. In most cases rather careful examination may be required to see dips because different exchanges and especially different helicity amplitudes each contribute a dip at a different position tending to obscure the phenomenon.

The charge-exchange helicity-nonflip amplitude has also been calculated using the same value for $\lambda$ and the trajectory, choosing a new value for $E_0$ $(E_0^+)$ to fit the difference in total cross sections[13]:

$$\mathrm{Im}M_{++}=k(s)^{1/2}[\sigma_T(\pi^-p)-\sigma_+(\pi^+p)]. \quad (9)$$

A fair fit is found for $E_0^+\approx 0.1$.[11] With these parameters the crossover of $d\sigma(\pi^-p)/dt$ and $d\sigma(\pi^+p)/dt$,[14] which is assumed to satisfy

$$\mathrm{Re}[(-i-0.2)M_{++}]=0, \quad (10)$$

occurs near $t=-0.15$ at 8 GeV/$c$ in excellent agreement with experiment. It is associated with

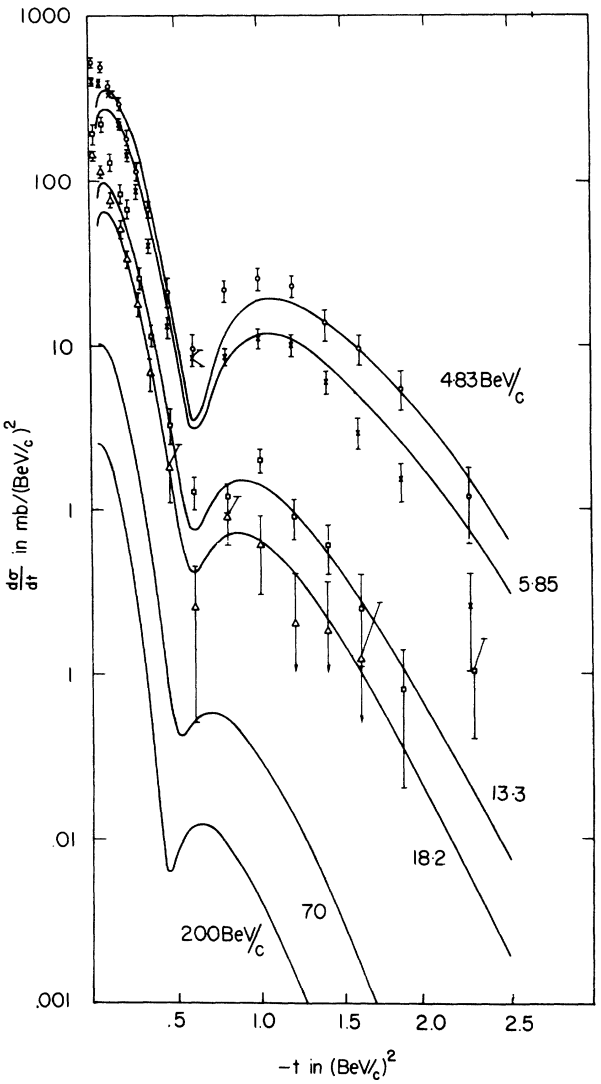


FIG. 1. Fit to the differential cross section for $\pi^-p \to \pi^0 n$ (see text). The parameters used at 4.8 GeV/$c$ are $\rho=0.2$, $G=3.8$ $(\mathrm{BeV}/c)^{-2}$, and $\sigma=29$ mb. These change slightly in the appropraite directions at higher energy. The other parameters are $\alpha_1=1.2$ $(\mathrm{BeV}/c)^{-2}$, $E_0=0.54$ BeV, $\alpha_0=0.37$, and $\lambda=1.9$.

a minimum in $|M_{++}|$. The polarization in charge-exchange scattering at small momentum transfer has a maximum of about 10% near $t = -0.15$, which is in reasonably good agreement with experiment.[15]

Disagreements between the predictions of simple Regge-pole models and experimental data have often been resolved by adding extra trajectories. In particular, the trajectory of the $B$ meson has been used to resolve such disagreements as polarization and details of differential cross sections. These are just the sort of disagreements that the cuts can resolve. As an example consider $\pi^0$ photoproduction. Here the $B$ has been used[16] to explain the partial filling in of the dip at $t \approx -0.5$ BeV$^2$. In our model, the dip will be partially filled in because the single scattering and double scattering are not exactly 180° out of phase. The difference between our model involving absorbed $\omega$ exchange, and a model having an $\omega$ pole plus $B$ exchange could be determined by a polarized-photon experiment.[17]

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*Work supported in part by the U. S. Atomic Energy Commission.

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[1]See, e.g., the review article by L. Bertocchi in the Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967 (North-Holland Publishing Company, Amsterdam, The Netherlands, 1968), p. 197.

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[8]See Arnold, Arnold and Blackmon, Frautschi and Margolis, and Cohen-Tannoudji et al. (Ref. 2).

[9]The error in all approximations made is 1-2%.

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[11]At this writing, we are making a best fit to the differential cross section with both flip and nonflip contributions and to the total cross section difference [Eq. (9)]. The parameters presented in this Letter are not final.

[12]The dip is due to destructive interference between $T^{ex}$ and the double scattering (or shadow correction) which is, for general reasons, broader in $t$. The situation is the same as for the dip in $p$–light-nucleus elastic scattering.

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