GEDANKEN EXPERIMENTS INVOLVING HADRONS

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A class of possible experiments which are not of the asymptotic scattering type are described. Use is made of the Einstein time dilatation to slow down the internal processes of an hadronic system so that it may be probed at controllable times.

Experimental particle physics, and in particular strong-interaction physics, has been restricted to the domain of asymptotic scattering experiments in which an asymptotically free set of initial particles are prepared by macroscopic, adjustable sources and a similar set of asymptotically free final particles are detected by macroscopic detectors. This circumstance has led some authors[1] to speculate that only experiments of such a type are possible. Thus the S-matrix theory of particle phenomena has become the orthodox theory while the traditional Schrödinger picture with its instantaneous state vector $|\psi\rangle$ satisfying

$$-i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle \tag{1}$$

has come into disfavor.

Our purpose here is to describe a wide class of possible experiments which can directly probe the evolution of the instantaneous state of a system of interacting hadrons.

Let us first define exactly what is meant by a nonscattering experiment. We wish to introduce an external probe into a system during the course of a scattering event which will interact with the system for an arbitrarily short period of time Δt centered about some adjustable time t_0 such that we can read off from the effects on the probe and system a set of parameters characterizing the evolution of the system between asymptotic states.

It is not obvious that this can be accomplished because of the finite size of hadronic systems. In general the size characterizing hadrons is $\sim 10^{-13}$ cm. Relativistic considerations may prevent any probe from getting into and out of the system in less than $10^{-13}/c \sim 10^{-23}$ sec. Since this is the time scale on which entire strong-interaction collisions and reactions take place, it may seem difficult if not impossible to probe a process in the manner described.

The experiments we shall propose probe systems whose rest frames are moving very close to the velocity of light in the experimenter's frame. The Einstein time dilatation is relied upon to slow down the internal mechanics of the system to a rate such that the internal motions of the system take long in comparison with 10^{-23} sec, thereby allowing them to be probed. For a complete description of the quantum mechanics of systems moving near the speed of light we refer the reader to Susskind[2] and Susskind and Frye.[3]

Consider a system with a total z component of momentum αL, transverse (x,y) momentum k_x and k_y, and other internal degrees of freedom labeled $\{\xi\}$. The center-of-mass energy invariant is an operator function of the set $\{\xi\}$ and the Schrödinger equation is

$$-i\frac{\partial}{\partial t}\psi(\alpha,k,\xi) = [(\alpha L)^2 + k^2 + m(\xi)^2]^{1/2}\psi. \tag{2}$$

We now let $L\to\infty$ and expand the square root in Eq. (2):

$$-i\frac{\partial}{\partial t}\psi(\alpha,k,\xi) = [\alpha L + \frac{k^2+m^2}{2\alpha L} + \cdots]\psi(\alpha,k,\xi). \tag{3}$$

Let ψ be an eigenvector of p_z with eigenvalue αL. Then according to Eq. (3),

$$\psi = e^{i\alpha Lt}\varphi(k,\xi,t). \tag{4}$$

The wave function φ carries all the information about the relative and transverse motions of the system. According to Eqs. (3) and (4) it satisfies

$$-i\frac{\partial}{\partial\tau}\varphi(k,\xi) = \frac{k^2+m(\xi)^2}{\alpha}\varphi(k,\xi), \tag{5}$$

where $\tau = t|2L$. Thus we see the Einstein time dilatation manifesting itself in the new time scale τ which is the time scale for internal and transverse motions.

We consider in addition to the infinite momentum system (called the incident system) a second target system at rest in the laboratory frame. The incident system has a well defined value of α which allows an uncertainty in p_z small in comparison with L. Let it equal Δp_z.

The incident system strikes the target at time $0\pm\Delta t$ with $\Delta t \sim \hbar/\Delta p_z c$. The purpose of this collision is to set up an interesting nonasymptotic configuration for study. After interacting with the target for a time comparable with the transit time of 10^{-23} sec, the incident system emerges with a configuration $\varphi(\xi)$. We need not be too precise in specifying the time at which the internal configuration is φ since because of time dilatation φ will change very slowly. Calling that time t_0, we need only demand $10^{-23} < t_0 + \Delta t \ll L \times 10^{-23}$. In general φ will be neither a single particle nor a set of well separated particles because the time required for the "internal Hamiltonian" $[k^2 + m(\xi)^2]/\alpha$ to bring the system to asymptotically free particles is of order $L \times 10^{-23}$ sec.

We now station a third system called the apparatus downstream from the target by an adjustable distance $yL \times 10^{-23}$ cm with y being an adjustable parameter. The entire experiment may be repeated over and over for each value of the adjustable parameter y. In this manner, we sample the state $\varphi(\xi\tau)$ with $\tau = y$ during the entire course of its evolution from $\varphi(\xi)$ to asymptotic freedom. For example, the apparatus could be the charge distribution of an atom and the experiment could study the evolution of the charge distribution in the transverse plane associated with the evolving state.

The total probability for the atom to make a transition from a state A to a state B is given by

$$Q_{AB} = \langle \varphi(\tau) | \int \rho(r_\perp)\rho(r_\perp{}')S(r_\perp)S(r_\perp{}') \times d^2r d^2r' | \varphi(\tau)\rangle, \quad (6)$$

where $\rho(r_\perp)$ is the transverse charge density $\int j_0(x,y,z)dz$ of the hadronic system, $S(r_\perp)$ is the matrix element

$$\langle A | \int \frac{\sigma(r')}{|r-r'|} d^3r' dr_3 | B\rangle,$$

σ is the charge density of the atom, and τ is the time of the measurement. Thus such an experiment can be used to obtain direct experimental information about the instantaneous correlation function $\langle \rho(r_\perp)\rho(r_\perp{}')\rangle$.

In order that our experiment may make use of macroscopic distances (let us say atomic dimensions), the Einstein dilatation factor must be $\sim 10^5$ so that energies like 10^5 BeV are necessary for the incoming system.

We have made use of some hidden assumptions which are worth pointing out.

In order that the infinite momentum limit be useful for the products of collision the following must be satisfied:

(1) The products of collision must be grouped into two distinguishable subsets, one associated with the incident system with all submomenta small in comparison with L in the frame of the initial particle, and a similar subset for the target. For example, the production of particles with transverse momentum comparable with L would prevent the use of the infinite-momentum limit which assumes $k \ll 1$.

(2) The invariant m associated with final infinite-momentum set should remain small in comparison with L for similar reasons.

It is interesting that conditions 1 and 2 are very prominent features of the actual experimental facts concerning hadronic collisions.

[1]F. G. Chew, S-Matrix Theory of Strong Interactions (W. A. Benjamin, Inc., New York, 1961).

[2]L. Susskind, Phys. Rev. 165, 1535 (1968).

[3]L. Susskind and G. Frye, Phys. Rev. 165, 1533 (1968).