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CONTINUUM CONTRIBUTIONS TO VECTOR-MESON DECAYS

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A model for estimating continuum contributions to vector-meson propagators is constructed and used to predict the rate of vector-meson decay into lepton pairs, and a modification of the SU(3) prediction for the rate of φ decay into $K+\bar{K}$. We also compare the pion electromagnetic form factor obtained in our model with that determined by the Novosibirsk group using the reaction $e^+ + e^- \rightarrow \pi^+ + \pi^-$.

In calculations involving single-particle intermediate states, it has become customary to describe the particle propagator by a simple pole perhaps with an imaginary part with a simple kinematical structure added to the inverse propagator in order to take into account the finite width of the particle. This approximation is made both for numerical simplicity, and because the detailed structure of the continuum cannot be reliably calculated. Nevertheless, it is possible to construct a model which takes into account the effect of the low-mass continuum states on the propagators, and, with this model, to estimate corrections to the pole-model predictions of the leptonic decay rates of the vector mesons (ρ, ω, φ) and of the SU(3) prediction for the rate of $\varphi \rightarrow K+\bar{K}$. In passing, we examine the Novosibirsk data on the reaction $e^+ + e^- \rightarrow \pi^+ + \pi^-$ and discuss the question of fitting the width of the ρ meson.

The inverse of the ρ-meson propagator (strictly speaking, of the coefficient of $g_{\mu\nu}$ in the propagator) is written as

$$D(s) = s[1+\pi(s)] - M^2 \tag{1}$$

with

$$\pi(s) = \frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\rho(s')}{s'^2(s'-s)} ds'. \tag{2}$$

$\rho(s)$ is chosen to have the phenomenological form[1]

$$\rho(s) = \lambda \frac{(s-4m_\pi^2)^{3/2}}{s^{1/2}} \left(\frac{\alpha^2}{\nu+\alpha^2}\right)^2 \tag{3}$$

$[\nu = (s/4m_\pi^2)-1]$. α^2 is a cutoff parameter. In S-matrix language, $D(s)$ is a phenomenological D function for p-wave π-π scattering, in which the left-hand cut of the partial-wave amplitude has been approximated by a double pole, and the influence on D of inelastic cuts has been approximated by a linear function of s.

For fixed α^2, the constants λ and M^2 are adjusted to fit the mass and width of the ρ; in order to renormalize the propagator to have the form

$$(s - m_\rho^2 + i m_\rho \Gamma_\rho)^{-1} \tag{4}$$

for s near m_ρ^2, $D(s)$ must be multiplied by

$$Z_\rho = 1/\mathrm{Re}D'(m_\rho^2). \tag{5}$$

The pole-model prediction for the rate for the decay $\rho \to l^+l^-$ is

$$\Gamma(\rho \to l^+l^-) = \frac{1}{3}\frac{(e^2/4\pi)^2}{(f_\rho^2/4\pi)}\left(1+\frac{2m_l^2}{m_\rho^2}\right) \times \left(1-\frac{4m_l^2}{m_\rho^2}\right)^{1/2} m_\rho, \tag{6}$$

where $f_\rho^2/4\pi$ is related to the ρ width by

$$\Gamma_\rho = \frac{f_\rho^2}{48\pi}\frac{(m_\rho^2-4m_\pi^2)^{3/2}}{m_\rho^2} \tag{7}$$

(ignoring structure in the ρ-π-π vertex). When the continuum contributions are included, the prediction of Eq. (6) must be multiplied by a correction factor

$$C_\rho = (Z_\rho M^2/m_\rho^2)^2. \tag{8}$$

Numerically, $C_\rho = 1.09$-1.10 for a wide range of Γ_ρ and α^2.

The pion form factor

$$F_\pi(s) = -M^2/D(s) \tag{9}$$

can be compared with the experimental results obtained at Novosibirsk.[2] A least-squares fit to the data gives

$$m_\rho = 780 \pm 15 \text{ MeV}, \quad \Gamma_\rho = 160 \pm 30 \text{ MeV} \tag{10}$$

for $\alpha^2 = 100$ (the result is essentially independent of α^2). However, the quality of the fit is poor ($\chi^2 = 11.7$ with $\langle\chi^2\rangle \sim 4$-$5$), and the fit to the point at 760 MeV is especially poor. A reasonable curve can be drawn for $m_\rho = 765$ MeV and $\Gamma_\rho = 130$ MeV, but the corresponding curve for $\Gamma_\rho = 100$ MeV gives a much poorer fit (see Fig. 1).

The difference between this fit and the fit produced by the Novosibirsk group,[2]

$$m_\rho = 764 \pm 11 \text{ MeV}, \quad \Gamma_\rho = 93 \pm 15 \text{ MeV}, \tag{11}$$

can be attributed to two sources: (i) The form factor given by Eq. (9) is normalized to unity at $s = 0$ to give the correct total charge of the pion. An arbitrary normalization factor (k) has been used by Auslander et al.[2] to fit the data. (ii) The form factor used in that fit has a constant width, while the form factor used in the present analysis has an energy-dependent width appropriate to a p-wave resonance [together with a slight modification of the real part of $D(s)$ which affects mainly the scale of the form factor near $s = m_\rho^2$].

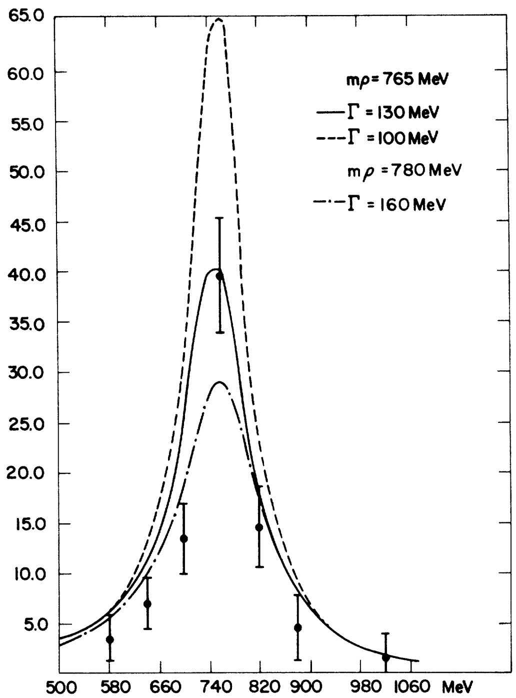

FIG. 1. Fits to the Novosibirsk data on the pion form factor using the modified ρ propagator. All curves are for $\alpha^2 = 100$. Masses and widths used are shown in the figure.

Effect (i) is the most important; for a form factor having the shape indicated by Eq. (9), but with an arbitrary normalization constant introduced as a free parameter, a fit to the data is obtained with

$$m_\rho = 769 \pm 4 \text{ MeV}, \quad \Gamma_\rho = 93 \pm 8 \text{ MeV} \tag{13}$$

for $\alpha^2 = 100$ (the result is again insensitive to α^2); here $\chi^2 = 2.2$. This shows, incidentally, that the main effect of including a p-wave energy dependence in the width[3] is to shift the peak of $|F_\pi(s)|^2$ relative to the mass defined by $\mathrm{Re}D(s) = 0$.

From a theoretical point of view, however, this fit is unsatisfactory, since the normalization of the form factor is related unambiguously in our model to the electric charge of the pion; the normalization constant required to produce the fit of Eq. (13) implies a pion charge of $0.71e$.

The curve drawn in Fig. 1 with $\Gamma_\rho = 130$ MeV can evidently be modified to produce a better fit by including a contribution to the form factor from a higher-mass vector meson (with mass ~1.5-2.0 BeV, say[4]). However, the experimental data are not yet sufficient to warrant a detailed analysis along these lines.[5] Considering

Table I. Computed values of decay rates of ω and φ, mixing angles, and K^* mass in various broken-symmetry models. Values of the ρ-meson parameters used are $m_\rho = 765$ MeV, $\Gamma_\rho = 120$ MeV, $\alpha^2 = 100$.

	Octet broken	$m(K^*)$ (MeV)	$\Gamma(\omega \to l^+ l^-)$ (keV)	$10^5 B_\omega$	$\Gamma(\varphi \to l^+ l^-)$ (keV)	$10^4 B_\varphi$	$\Gamma(\varphi \to K^+ K^-)$ (MeV)	θ_Y (deg)	θ_N (deg)
Pole dominance	SU(3)	Input	0.40	3.3	1.15	2.31	4.98	34.0	21.7
	U(3)	878	0.56	4.65	1.03	2.36	4.36	40	26.4
Continuum	U(3)	Not calculated	0.345	2.87	0.75	2.58	2.90	37.7	15.8

the rather large experimental errors and our fits (Fig. 1), the commonly accepted value for Γ_ρ (115-140 MeV) appears to be consistent with the data.

A model for the propagator describing the ω and φ, in which Eq. (1) is interpreted as a 2×2 matrix, can also be constructed. The K-$\bar{K}$ intermediate state, which couples only to the octet combination (V_8) of ω and φ, is handled like the π-π state. The absorptive part of the dispersion integral for the three-pion state is written as the product of a smooth cutoff function and the phase space for the decay of an $I=0$ vector meson into three pions via $\rho+\pi$ intermediate states.[6] A condition of octet breaking of U(3) is imposed in terms of spectral-function sum rules,[7] and SU(3) symmetry is assumed to relate the ρ-π-π and V_8-K-$\bar{K}$ coupling constants.

The resulting predictions for the rates of $\varphi \to K + \bar{K}$ and of $\omega, \varphi \to l^+ + l^-$ (together with the branching ratios B_ω, B_φ of the leptonic decays), along with the values of the ω-φ mixing angles,[8] are shown in Table I. Shown for comparison are the predictions of the pole approximation to the current-mixing model of Coleman and Schnitzer,[9] in both broken-SU(3) and broken-U(3) versions.[10] The predictions of the present model, except for B_φ, are mildly sensitive to the value of α^2; an uncertainty of ±5 % is thereby introduced, but the value $\alpha^2 = 100$ is also preferred for other reasons. These details and other applications of the model will be presented elsewhere.[11]

Note that the agreement between the total φ width and experiment is substantially improved; the prediction for the leptonic decay of φ is consistent with the recently reported experimental results[12] (which are, however, subject to large uncertainties).

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