The solid error bars are statistical; the dashed bars are estimations of our absolute normalizations. Both the $D_{15}(1688)$ and $F_{37}(1920)$ resonances are evident from our data. However, the location of the 1920-MeV resonance seems to be shifted downwards about 75 MeV. Around 800 MeV our data seem to agree with the polynomial fit of π^0 angular distribution data by DeStaebler et al.[3] who used four powers of cosine and two crossed exchange poles with masses 938 and 1238 MeV and a pole at a mass of 700 MeV. At higher incident photon energies our data are in agreement with results of Buschhorn et al.[4] Since the angular distribution of π^0 photoproduction is not well known above 1.3 GeV, we compare our π^0 data with π^+ photoproduction data of Alvarez et al.[5] in the same angular region. The ratios $(d\sigma/d\Omega)_{\pi^+}:(d\sigma/d\Omega)_{\pi^0}$ at D_{15} and F_{37} resonances are 2.2:1 and 1:2.4, respectively. If it is assumed that the cross section at the peak of each resonance is due solely to a single resonance amplitude, then by isospin arguments one gets ratios of 2:1 and 1:2. The fact that the experimental ratios are very close to the ratios obtained by simple isospin arguments indicates that up to our energy region the S channel contribution in photoproduction of pions still plays a dominant role.

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PRODUCTION AND DECAY OF A_1 AND A_2 RESONANCES IN 16-GeV/c π^-p INTERACTIONS*

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The total cross sections for production of A_1 and A_2 mesons produced in 16-GeV/c π^-p interactions are presented together with angular distributions for their decay. An attempt has been made to treat the A_2 as a superposition of two resonant states.

We present here data from an exposure of 60 000 pictures in the Brookhaven National Laboratory 80-in. hydrogen bubble chamber, using the rf separated beam[1] in an unseparated mode to obtain a beam of 16-GeV/c π^- with $\Delta p/p = \pm 0.25\%$. The film was scanned for four-prong events, and the analysis was performed using the TVGP-SQUAW series of programs. The resultant sample contains 1995 events which constrain to the hypothesis

$$\pi^-p \to p\pi^+\pi^-\pi^-, \quad (1)$$

and which are consistent with a visual check of ionization. The total cross section for this reaction is 1.08 ± 0.12 mb.

In Fig. 1 the unshaded histogram shows the mass spectrum for the $\pi^+\pi^-\pi^-$ system with no cuts imposed on the data; the shaded histogram shows those events having at least one $\pi^+\pi^-$ combination in the mass interval 0.66-0.90 GeV (ρ^0 region) and with no associated $\Delta^{++}(1236)$. There is a very strong enhancement in the mass region 1.0-1.4 GeV which we associate with the A_1 and A_2 mesons.[2] The separation into two peaks is consistent with the production of two resonant states with parameters shown in Table I. In order to understand the background beneath these peaks, we use the one-pion-exchange (OPE) calculation of Wolf.[3] There are three OPE diagrams which contribute to Reaction (1).

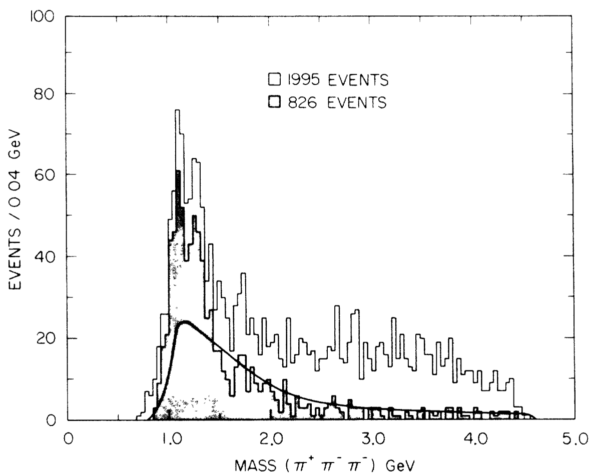

FIG. 1. Mass spectra for the $\pi^+\pi^-\pi^-$ system. The shaded histogram shown events associated with ρ^0 and excluding Δ^{++}. The solid curve shows the background estimate for ρ^0, no Δ^{++} events, based on the OPE calculation of Wolf (see text).

Their contribution has been calculated neglecting possible interference terms between them. The absolute cross sections for Δ^{++}, ρ^0 (not associated with the A mesons), and f^0 predicted by this calculation are in good agreement with the data of this experiment, and in fact, there is quite credible agreement with data from experiments over a very wide range of energies. Furthermore, the angular dependences predicted for events selected to have three-pion effective mass near the A region (1.4-1.8 GeV) agree fairly well with the data of this experiment. We therefore use this calculation to estimate the amount of background and its angular dependences in the ensuing analysis of the properties of the A_1 and A_2 resonances. The resonance parameters given in Table I were obtained by fitting the shaded histogram to two Breit-Wigner resonances and the form of the Wolf background. The solid curve in Fig. 1 shows the background normalized downward by 15% from the absolute prediction in accordance with the best fit.

Figure 2(a) shows the helicity angular distribution for A_1 events with $\pi^+\pi^-$ mass combinations in the ρ^0 region [and excluding events with $m(p\pi^+)$ in the Δ^{++} region], where β is the angle in the ρ^0 rest frame between the two π^-'s. Events where both $\pi^+\pi^-$ mass combinations lie in the ρ^0 region have been plotted twice. Fits have been made to the data for various spin and parity assignments assuming the background has an angular dependence given by Wolf's calculation. We use the functional form

$$F_s(\cos\beta)$$

$$= |A_l^s e^{ia_s} f_l^s(\cos\beta) + \text{Wolf background}|^2.$$

The spin of the A_1 is denoted by s, the orbital angular momentum state of the decay by l, the amplitude for the decay (assumed real) by A_l; a_s is the relative phase between the decay amplitude and the background to account for possible interference effects, and $f_l^s(\cos\beta)$ is the Bose-symmetrized angular function appropriate to the decay.[4] The pronounced peaking of the distribution at $\cos\beta = \pm 1$ excludes assignments in the natural spin and parity series 1^-, 2^+, etc., and we list fits only for the assignments 1^+ and 2^-. These are quite insensitive to background (confidence levels change by $\lesssim 5\%$ over the full range of background levels) and we give the confidence for the background estimate shown in Table I: 12% for 1^+ S wave, 35% for 1^+ D wave, 17% for 2^- P wave, and 8% for 2^- F wave. The assignment 1^+ D wave is clearly favored by these data although one cannot exclude the other possibilities. The preference for D wave is indeed surprising and appears to indicate that if the 1^+ assignment is correct, an appreciable amount of D wave must be included to explain these data.

For the assignment 1^+, a number of theoretical predictions have been made concerning the relative amounts of S and D waves in A_1 decay.[5-8] Those of Ref. 5 are "hard-pion" or effective-Lagrangian calculations and predict predominant-

Table I. Resonance parameters an background levels for A_1 and A_2 production. Cross sections have been corrected for the unmeasured decay mode $\rho^-\pi^0$.

Resonance	Mass (GeV)	Width Γ (GeV)	σ (μb)	Resonance cuts (GeV)	% background inside cuts
A_1	1.089 ± 0.012	0.140 ± 0.031	250 ± 50	1.00 to 1.16	49 ± 5
A_2[a]	1.282 ± 0.015	0.125 ± 0.040	180 ± 60	1.20 to 1.36	54 ± 8

[a]This assumes the presence of a single resonant state.

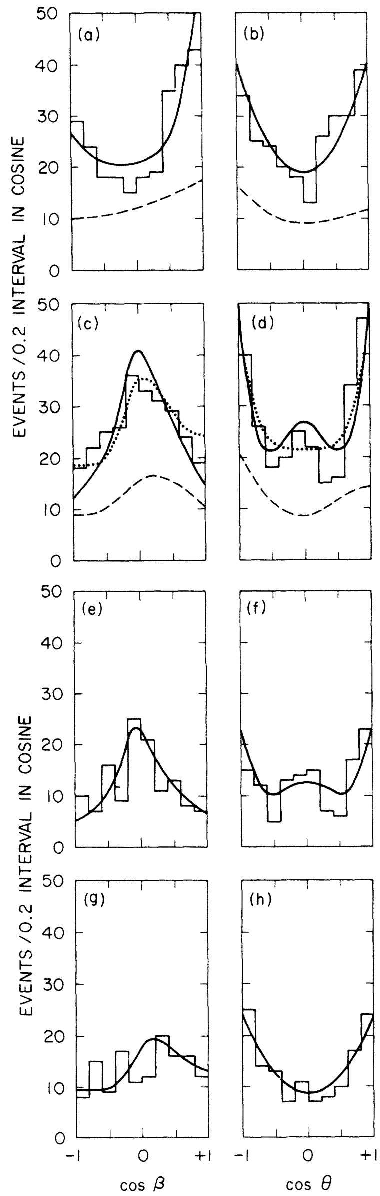

FIG. 2. Helicity (cosβ) and Jackson (cosθ) angular distributions. The dashed curves indicate the contribution from the OPE calculation of Wolf. (a) Helicity distribution for A_1 events. The solid curve shows the fits for $J^Pl=1^+$ D wave. (b) Jackson distribution for A_1 events. The solid curve represents the fit for $|g_T/g_L|^2=0.16\pm0.08$. (c) Helicity distribution for A_2 events. The solid curve is the fit for $J^P=2^+$; the dotted curve for $J^P=2^-$. (d) Jackson distribution for A_2 events where the solid curve shows the fit for 2^+ and the dotted curve for equal amounts of 2^+ and 2^- (see text). (e), (f) Helicity and Jackson distributions for $1.28 \le m(\rho^0\pi^-)\le 1.36$ GeV; solid curves indicate fits for $J^P=2^+$. (g), (h) Helicity and Jackson distributions for $1.20\le m(\rho^0\pi^-)\le 1.28$ GeV; solid curves represent fits for $J^P=2^-$.

ly S-wave decay. Those of Ref. 6 and "quark" calculations and predict predominantly transverse ρ^0's in the decay. Finally, Refs. 7 and 8 summarizes those papers using current algebra and superconvergence ideas. One of these, Ademollo et al.,[8] predicts dominantly S-wave decay. The other three[7] (Gilman and Harari, Bishari and Schwimmer, and Frampton and Taylor) find predominantly longitudinal ρ^0's. In terms of S- and D-wave amplitudes, production of transverse ρ^0's implies equal amplitudes but of opposite sign (if the amplitudes are relatively real), while the appearance of longitudinal ρ^0's requires $A_D/A_S=2$ under the same reality assumption.

Further information concerning the longitudinal or transverse nature of ρ^0's in A_1 decay can be inferred from the distribution in $\cos\theta$, where θ (the so-called Jackson angle) is defined as the angle in the A_1 rest frame between the incident beam particle and the bachelor π^-. If the A_1 resonance is diffraction produced (as seems indicated by the relative energy independence of the production cross section), the state should have zero helicity. This alignment leads to a distribution in $\cos\theta$ given by

$$W(\cos\theta) \propto \left(\frac{m_A^{\,2}}{m_\rho^{\,2}}|g_L|^2\cos^2\theta+|g_T|^2\sin^2\theta\right), \qquad (2)$$

where we have used the notation of Gilman and Harari.[7] The quantities g_L and g_T are the coupling constants for longitudinal and transverse ρ^0 helicity states. This formula neglects the effects of Bose symmetrization in the decay. Figure 2(b) shows this distirbution for A_1 events where the solid curve indicates a fit to the data using Eq. 2 superimposed on the Wolf background. The agreement is quite good and yields a value for the ratio of the coupling constants

$$|g_T/g_L|^2=0.16\pm0.08.$$

This is in good agreement with the prediction of Gilman and Harari who find a value of approximately 0.1. Although the relative phase of the couplings must be known in order to calculate the amounts of S and D wave, a determination of the magnitude of g_T/g_L allows us to set limits on the ratio of D wave to S wave. For the value of $(g_T/g_L)^2$ indicated, we find

$$0.4\le|A_D/A_S|\le 22.$$

The preference of D wave in the helicity distributions appears to indicate that the ratio is larger than the lower limit, but the presence of two orbital states makes a more precise measurement from the helicity distributions quite difficult.

Similar analyses have been performed for the A_2 resonance. The distribution in $\cos\beta$ is shown in Fig. 2(c). The absence of peaks at $\cos\beta=\pm1$ excludes the assignments 1^+ D wave and 2^- F wave, and the data do not show a dip at $\cos\beta=0.2$, which is characteristic of 1^-. Therefore, fits have been made only for the assignments 1^+ S wave, 2^+ D wave, and 2^- P wave. Confidence levels for these fits at the level of background indicated in Table I are 73 %, 67 %, and 87 %, respectively. Only the 2^+ assignment is sensitive to the background level giving poor fits (3 % confidence or less) for background levels less than 30 %. Clearly all these fits to the data are acceptable.

In order to distinguish between these assignments we must examine the distribution in $\cos\theta$ (as defined above) which is presented in Fig. 2(d). The theoretical distributions for the possible spin and parity assignments are

$$W_{2^+D}(\cos\theta) = N[\rho_{22}(1-\cos^4\theta)+\rho_{11}(1-3\cos^2+4\cos^4\theta) + 3\rho_{00}\cos^2\theta(1-\cos^2\theta)], \qquad (3a)$$

$$W_{2^-P}(\cos\theta) = N[2\rho_{22}(1-\cos^2\theta)+\rho_{11}(1+\cos^2\theta) + \rho_{00}(\tfrac{1}{3}+\cos^2\theta)], \qquad (3b)$$

$$W_{1^+S}(\cos\theta)=N[2\rho_{11}+\rho_{00}], \qquad (3c)$$

where ρ_{ii} represent spin-density matrix elements and N is a normalizing constant. The data clearly deviate from isotropy, eliminating 1^+ S wave. For the remaining two possibilities we have made fits to the data for each, once again assuming the background of Wolf and also assuming Pomeranchuk and ρ exchanges for the 2^- P-wave hypothesis [i.e., $\rho_{11}\neq0$, $\rho_{00}\neq0$, and $\rho_{22}=0$ in Eq. (3b)] and ρ exchange for the hypothesis 2^+ [i.e., $\rho_{11}\neq0$ and $\rho_{00}=\rho_{22}=0$ in (3a)]. Confidence levels are 5 % and 2 %, respectively for 2^+ D wave and 2^- P wave. The preference for 2^+ results from the peaking in the data about $\cos\theta=0$ and appears to indicate that at least part of the enhancement proceeds through a state with these quantum numbers. A fit to the data assuming the presence of both 2^+ and 2^- also gives a confidence level of 5 % and yields a ratio of 1.0 ± 0.25 for the relative amounts of these two states; the 2^- state occurs entirely through Pomeranchukon exchange.

Recent experiments have indicated a splitting in the A_2 mass spectrum.[9] Although the resolution of this experiment is insufficient to detect such splitting, we have separately examined the angular distributions for the upper and lower halves of the A_2 peak. Figures 2(e)-2(h) shows that the data are dramatically different under this selection. In the mass interval 1.28-1.36 GeV $J^P=2^+$ is favored over 2^- by confidence levels of 28 % to 15 % in the fit to the $\cos\beta$ distribution and by 14 % vs 2 % for $\cos\theta$; in the mass interval 1.20-1.28 GeV $J^P=2^-$ is preferred by 33 % vs 1 % for $\cos\beta$ and 59 % vs 4 % for $\cos\theta$. The data are indeed consistent with production of two unresolved resonances each with a cross section of about one-half the value shown in Table I.

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CONTINUUM CONTRIBUTIONS TO VECTOR-MESON DECAYS

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A model for estimating continuum contributions to vector-meson propagators is constructed and used to predict the rate of vector-meson decay into lepton pairs, and a modification of the SU(3) prediction for the rate of φ decay into $K+\bar{K}$. We also compare the pion electromagnetic form factor obtained in our model with that determined by the Novosibirsk group using the reaction $e^++e^-\rightarrow\pi^++\pi^-$.

In calculations involving single-particle intermediate states, it has become customary to describe the particle propagator by a simple pole perhaps with an imaginary part with a simple kinematical structure added to the inverse propagator in order to take into account the finite width of the particle. This approximation is made both for numerical simplicity, and because the detailed structure of the continuum cannot be reliably calculated. Nevertheless, it is possible to construct a model which takes into account the effect of the low-mass continuum states on the propagators, and, with this model, to estimate corrections to the pole-model predictions of the leptonic decay rates of the vector mesons (ρ, ω, φ) and of the SU(3) prediction for the rate of $\varphi\rightarrow K+\bar{K}$. In passing, we examine the Novosibirsk data on the reaction $e^++e^-\rightarrow\pi^++\pi^-$ and discuss the question of fitting the width of the ρ meson.

The inverse of the ρ-meson propagator (strictly speaking, of the coefficient of $g_{\mu\nu}$ in the propagator) is written as

$$D(s)=s[1+\pi(s)]-M^2 \tag{1}$$

with

$$\pi(s)=\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\rho(s')}{s'^2(s'-s)}ds'. \tag{2}$$

$\rho(s)$ is chosen to have the phenomenological form[1]

$$\rho(s)=\lambda\frac{(s-4m_\pi^2)^{3/2}}{s^{1/2}}\left(\frac{\alpha^2}{\nu+\alpha^2}\right)^2 \tag{3}$$

$[\nu=(s/4m_\pi^2)-1]$. α^2 is a cutoff parameter. In S-matrix language, $D(s)$ is a phenomenological D function for p-wave π-π scattering, in which the left-hand cut of the partial-wave amplitude has been approximated by a double pole, and the influence on D of inelastic cuts has been approximated by a linear function of s.

For fixed α^2, the constants λ and M^2 are adjusted to fit the mass and width of the ρ; in order to renormalize the propagator to have the form

$$(s-m_\rho^2+im_\rho\Gamma_\rho)^{-1} \tag{4}$$

for s near m_ρ^2, $D(s)$ must be multiplied by

$$Z_\rho=1/\mathrm{Re}D'(m_\rho^2). \tag{5}$$

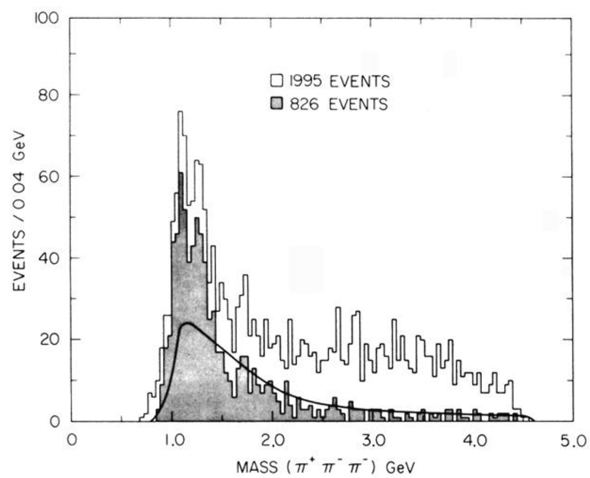

FIG. 1. Mass spectra for the $\pi^+\pi^-\pi^-$ system. The shaded histogram shown events associated with ρ^0 and excluding Δ^{++}. The solid curve shows the background estimate for ρ^0, no Δ^{++} events, based on the OPE calculation of Wolf (see text).