# LOCATION OF A SECONDARY $\rho$ REGGE POLE IN THE REACTION $\pi^-p\rightarrow\pi^0n$ *

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The trajectories and residues of $\rho$ and $\rho'$ Regge poles are determined in the range $0 \geq t \geq -1.0(\text{GeV}/c)^2$ from a simultaneous fit to (i) continuous-moment sum rules, using $\pi N$ phase shifts up to 2 GeV/$c$, and (ii) $\pi^-p\rightarrow\pi^0n$ scattering cross section data above 4 GeV/$c$. The $\rho'$ trajectory is found to be roughly parallel to the $\rho$ and about a half-unit lower. The $\pi^-p\rightarrow\pi^0n$ polarization is correctly predicted in magnitude and sign.

The theoretical interpretation of the nonvanishing polarization in the reaction $\pi^-p\rightarrow\pi^0n$ remains an open question.[1] Contributions from complex-$J$-plane singularities in addition to the $\rho$-meson Regge pole are required in Regge models; we denote these by $\rho'$. Possibilities for $\rho'$ include secondary Regge poles, conspiring poles, Regge cuts, or conspiring cuts. Several $\rho+\rho'$ models have been proposed,[2] but the high-energy data alone do not yield a unique solution. Additional information on the nature of the $\rho'$ amplitude can be obtained from continuous-moment sum rules (CMSR), which relate Regge parameters to integrals over low-energy phase shifts.[3] In this Letter we present results for the $\rho$ and $\rho'$ exchange amplitudes obtained from a simultaneous fit to (i) CMSR using $\pi N$ phase shifts up to 2 BeV/$c$, and (ii) $d\sigma(\pi^-p\rightarrow\pi^0n)/dt$ and $\sigma_t(\pi^-p)-\sigma_t(\pi^+p)$ data above 4 BeV/$c$. The $\pi^-p\rightarrow\pi^0n$ polarization measurements add only a weak constraint and were not included in the data set fitted; the resulting polarization predictions agree in sign and magnitude with experiment.

The contribution from a Regge cut can be parametrized to a first approximation by an effective Regge pole. If a crossing-odd $\pi N$ amplitude $a(\nu,t)$ has the asymptotic pole form

$$a(\nu,t)=-\nu\sum_i\gamma_i(t)(\nu_0^2-\nu^2)^{\frac{1}{2}\theta_i}, \tag{1}$$

the resulting CMSR are

$$\int_{\nu_0}^{\nu_1}d\nu\,\mathrm{Im}\{a(\nu,t)(\nu_0^2-\nu^2)^{\frac{1}{2}(-\epsilon-1)}\}=\sum_i\frac{\gamma_i(\nu_1^2-\nu_0^2)^{\frac{1}{2}(\theta_i-\epsilon+1)}}{(\theta_i-\epsilon+1)}\sin[\tfrac{1}{2}\pi(\theta_i-\epsilon-1)]. \tag{2}$$

Here $\nu=(s-u)/4M$; $\nu_0=\mu+t/4M$ is the normal threshold. The nucleon pole contribution at $\nu_B=(2\mu^2-t)/4M$ is understood to be added to the integral.

For the reaction $\pi^-p\rightarrow\pi^0n$ we take the two crossing-odd amplitudes[4] $A'^-$ and $\nu B^-$ with Regge asymptotic forms

$$A'^-=-\nu\sum_i\gamma_i(\nu_0^2-\nu^2)^{\frac{1}{2}(\alpha_i-1)},$$

$$\nu B^-=-\nu\sum_i\beta_i(\nu_0^2-\nu^2)^{\frac{1}{2}(\alpha_i-1)}, \tag{3}$$

where the summation is over $i=\rho,\rho'$. The $\alpha_i(t)$ are the trajectories. The coefficients $\gamma_i(t)$ and $\beta_i(t)$ contain the residue functions and also the $-1/\cos(\frac{1}{2}\pi\alpha_i)$ factors of the usual Regge parametrization. The characteristic Regge phase factors are included in the powers of $\nu_0^2-\nu^2$.

We evaluated the left-hand side of Eq. (2) for the amplitudes $A'^-$ and $\nu B^-$ using the recent CERN phase-shift results.[5] The upper limit is $\nu_1=2.075+t/4M$. We considered the following ranges of $\epsilon$ and $t$:

$1.0>\epsilon\geq-5.0$ in steps of 0.5,

$0\geq t\geq-0.5$ in steps of 0.1 $(\text{GeV}/c)^2$,

$-0.5\geq t\geq-0.95$ in steps of 0.15 $(\text{GeV}/c)^2$.

We did not ultimately use the full range of $\epsilon$ above. For $\epsilon>-1$, the integrals are very sensitive to the precise phase shifts near threshold and also to the extrapolation in the unphysical region, $\mu+t/4M\leq\nu\leq E_\pi E_N/M$. When $t<-4M\mu=-0.52$, the $s$- and $\mu$-channel cuts overlap, and the low-energy part of the integral becomes more dubious. We have therefore suppressed the low-energy contributions by taking $\epsilon \leqslant -1$, except at $t=0$. The high moments (large $|\epsilon|$) can also be misleading, since they are very sensitive to the amplitude at the upper limit $\nu_1$: Recent measurements show that some fluctuations are present above 2 GeV.[6] Sum rules will only be reliable to the extent that they average evenly over such fluctuations, without overemphasizing a small-$\nu$ region. In practice, we found that the CMSR with $|\epsilon|>3$ were not entirely consistent with the lower moments and with other data; so we dropped them.

We simultaneously analyzed the remaining CMSR and the $\pi^- p \to \pi^0 n$ cross section data[7] using three complementary approaches:

(a) Analysis at fixed-$t$ values. No assumptions about the $t$ dependence of trajectories and residues are then required. Scattering cross section data are interpolated where necessary.

(b) Analysis over a range of $t$, parametrizing the $\alpha_i(t)$, $\beta_i(t)$, and $\gamma_i(t)$ by polynomials in $t$. This approach injects a minimum of prejudices about $t$ dependence, but the polynomials tend to blow up outside the $t$ range considered.

(c) Analysis over a range of $t$, parametrizing the residues with smooth functions such as $e^{at}$, $(t-t_0)e^{bt}$, $\alpha e^{ct}$, etc. and the trajectories by $\alpha(t) = \alpha(0)+\alpha'(0)t$. The advantage here is a smoother extrapolation to larger $t$. Basically the same results were found with each of these methods.

Representative forms for the residues found from the fits to the CMSR and scattering data are shown in Fig. 1. The crosses are results of analyses at fixed $t$; the solid curves represent a parametrization with the smooth functions

$$\alpha_\rho(t) = 0.55 + t,$$

$$\alpha_{\rho'}(t) = 0.8t,$$

$$\gamma_\rho(t) = -10.8(1+t/0.23)\exp 2.75t,$$

$$\gamma_{\rho'}(t) = -58.6t(1+t/0.55)\exp 2.0t,$$

$$\beta_\rho(t) = -202\alpha_\rho \exp 1.48t,$$

$$\beta_{\rho'}(t) = -329t \exp 5.0t.$$

[Units: $t$ in $(\mathrm{GeV}/c)^2$; $\gamma_i$ in $\mathrm{GeV}^{-1}$; $\beta_i$ in $\mathrm{GeV}^{-2}$].

Our results can be summarized as follows:

(i) The $\rho$ and $\rho'$ trajectories proved to be correlated. A typical solution for the trajectories is listed above.[8] If the $\rho'$ is a bona fide Regge pole (and not an effective-cut contribution), then a straight-line extrapolation would place a $J^P = 1^-$ particle in the mass region 1000-1200 MeV. No such meson has been reported.

(ii) The coefficient $\beta_\rho$ changes sign at the point $\alpha_\rho = 0$, in accord with the usual interpretation of the dip in $d\sigma(\pi^- p \to \pi^0 n)/dt$ at $t \simeq -0.5$. The coefficient $\gamma_\rho(t)$ changes sign near $t \simeq -0.2$, as previously suggested by the crossover phenomenon in elastic scattering. Moreover, $\gamma_{\rho'}(t)$ changes sign, but near $t \approx -0.6$, so that $\mathrm{Im}A'^-$ and $\mathrm{Re}A'^-$ do not vanish at the same place.[9] The results for $\gamma_{\rho'}$ and $\beta_{\rho'}$ are not inconsistent with linear-residue zeros at $\alpha_{\rho'} = -1$.

(iii) The quantities $\alpha_{\rho'}$, $\gamma_{\rho'}$, and $\beta_{\rho'}$ are all small near $t=0$. The simplest parametrization consistent with the fixed-$t$ results is to make all three vanish at $t=0$. This parametrization is just a convenient simplification, since not all these zeros have to occur at $t=0$ or be correlated with each other. Possible origins of the behavior near $t=0$ include the following: (a) a $\rho'$

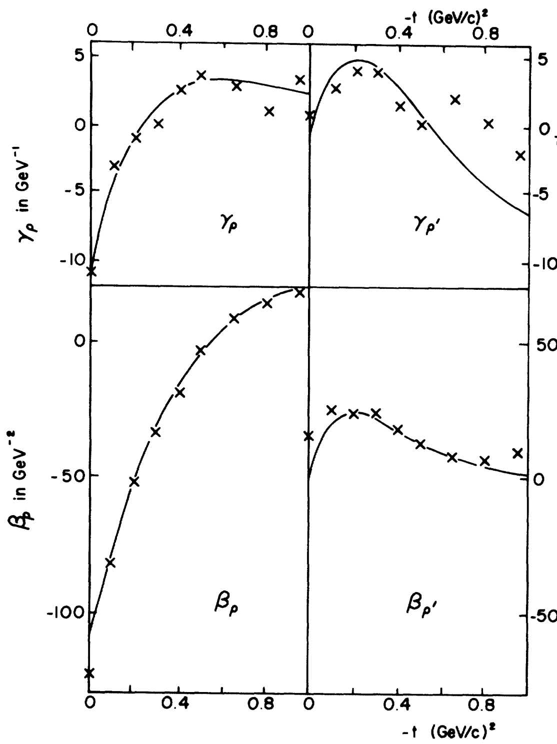


FIG. 1. Residue coefficients for $\rho$ and $\rho'$ Regge poles obtained from a simultaneous fit to CMSR and $\pi^- p \to \pi^0 n$ cross section data [cf. Eqs. (2) and (3) of text]. The crosses denote results of fixed-$t$ analyses, assuming trajectories as given. The solid curves represent the parametrization shown in the text.

Regge pole that chooses nonsense at $\alpha_{\rho'}=0$:

$$\gamma_{\rho'}\propto\alpha_{\rho'}, \quad \beta_{\rho'}\propto\alpha_{\rho'};$$

(b) a conspiring $\rho'$ Regge pole[2]:

$$\gamma_{\rho'}\propto t, \quad \beta_{\rho'}\propto\alpha_{\rho'}.$$

Both conjectures are based on the smallness of $\alpha_{\rho'}$ near $t=0$.

If the $\rho'$ represents an effective cut, then factors of $\alpha_{\rho'}$, $\alpha_{\rho'}+1$, etc. are not expected in the $\gamma_{\rho'}$ and $\beta_{\rho'}$ coefficients.

(iv) The $\rho'$ amplitudes are not negligible compared with the $\rho$ at present accelerator energies, and polarization is predicted in $\pi^-p\to\pi^0n$ of the same size and sign as experimentally observed. The predictions with the above parametrization are compared with the data[10] in Fig. 2. Although the details of the predictions are somewhat dependent on the choice of parametrization, the broad qualitative trends of the curves in Fig. 2 for $-t<0.5$ are common to all fits to the CMSR and cross sections.

The predictions for the polarization at very small $|t|$ (<0.1) are sensitive to perturbations from the given parametrization at $t=0$. The qualitative behaviors of the polarization near $t=0$ resulting from various parametrizations are

(a) $\gamma_{\rho'}\propto t, \quad \beta_{\rho'}\propto t,$

$P(\pi^-p\to\pi^0n)\propto(-t)^{3/2}$ and positive as shown in Fig. 2;

(b) $\gamma_{\rho'}\propto t, \quad \beta_{\rho'}\propto(t_0-t), \quad t_0>0,$

$P(\pi^-p\to\pi^0n)\propto(-t)^{1/2}$ and negative;

(c) $\gamma_{\rho'}\propto(t_0-t), \quad \beta_{\rho'}\propto t, \quad t_0>0,$

$P(\pi^-p\to\pi^0n)\propto(-t)^{1/2}$ and positive. The present polarization data favor possibility (c). This argues against a conspiring $\rho'$ amplitude.

(v) The $\rho+\rho'$ solution given above provides a reasonable overall representation of data[6] on $d\sigma(\pi^-p\to\pi^0n)/dt$ between 2 and 4 GeV/$c$. However, for the $t$ range $0.04<-t<0.12$ the fit does fall somewhat below the measured $d\sigma/dt$. At larger $|t|$, the fit extrapolates well through the data. Near $t=0$ the fit extrapolates through the mean of the observed energy-dependent fluctuations in $d\sigma/dt$.[6] These fluctuations are presumably direct-channel resonance effects. The CMSR of Eq. (2) are approximate to the extent that resonance contributions for $\nu>\nu_1\sim 2$ BeV have not been included.

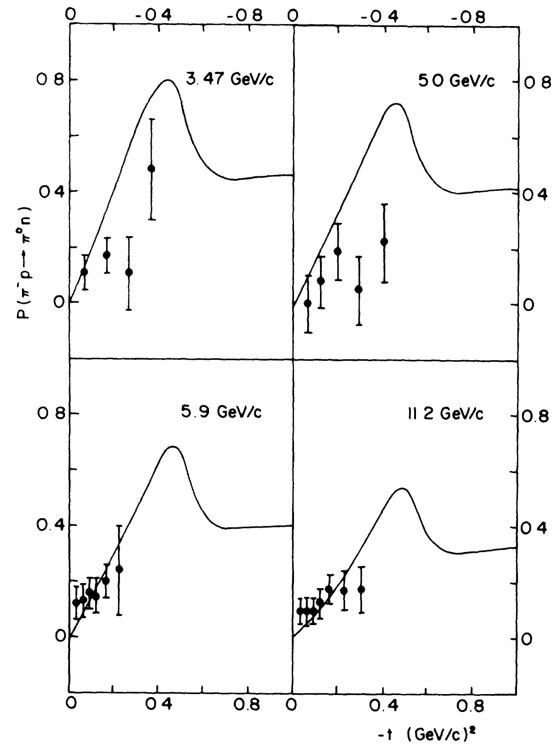


FIG. 2. Predicted $\pi^-p\to\pi^0n$ polarization compared with the high-energy data. The solid curves are calculated from the $\rho+\rho'$ parametrization in the text. The data are taken from Ref. 10.

(vi) For the most part, previous $\rho+\rho'$ models[2] do not provide adequate representations of the CMSR. The Rarita-Schwarzchild model[2] in particular disagrees dramatically with the $\nu B^-$ sum rules.

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# ASYMPTOTIC BEHAVIOR OF INFINITELY RISING TRAJECTORIES

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Infinitely rising trajectories are shown to be consistent with the usual features of dispersion theory and Regge-pole theory only if the trajectories behave, to within logarithmic factors, as $\sqrt{s}$ as $s \to +\infty$. For such trajectories the imaginary part must increase more rapidly than the real part.

Considerable interest has developed in the possibility that Regge trajectories rise indefinitely with increasing energy,[1]

$$\mathrm{Re}\alpha(s) \to +\infty \text{ as } s \to +\infty. \tag{1}$$

On the basis of the empirical forms of $\mathrm{Re}\alpha(s)$ in the resonance region, trajectories obeying (1) would be expected to be proportional to $s$ at large $s$. However, theoretical arguments[2,3] have been advanced which suggest that the real part of such trajectories should be proportional to $\sqrt{s}$ as $s \to +\infty$. Reference 2 is a "simple bootstrap model" whereas Ref. 3 is more general, the main assumption being that certain channels couple to high-spin Regge recurrences. Independently of how fast $\mathrm{Re}\alpha$ increases, there is an immediate problem with (1). In general $P_\alpha(Z)$ grows exponentially as $\alpha \to \infty$, and this brings into question the existence of the $N$-subtracted dispersion relation for the scattering amplitude. This problem has been examined by Khuri[4] on the basis of the following four assumptions: (i) The scattering amplitude $f(s,t)$ is analytic in the cut $s$ plane and bounded by $s^N$ as $s \to +\infty$ for fixed $t$; (ii) $f(s,t)$ is also bounded by $s^N$ as $s \to +\infty$ for fixed, physical values of $Z = \cos\theta$; (iii) the Sommerfeld-Watson transformation for the partial wave $\alpha_l(s)$ exists, and $\alpha_l(s)$ is bounded by $s^N$ as $s \to \infty$ for fixed $l$; (iv) the residue function $\beta(s)$[5] and the Regge trajectory $\alpha(s)$ are analytic with only a right cut and are bounded by $s^N$ as $s \to +\infty$.

Of course, if the number of trajectories obeying (1) is infinite, and a "super cancellation"[4] between the individual terms occurs, then the difficulty is no longer present. Otherwise the conclusion of Khuri's work is negative, namely, that (1) is not consistent with the above four assumptions.

In this note we wish to make two points. First, the four assumptions of Khuri are consistent with (1) if the trajectory increases as $\sqrt{s}$ to within logarithmic factors,

$$\mathrm{Re}\alpha(s) \to s^{\frac{1}{2}}(\ln s)^n(\ln\ln s)^m \cdots \text{ as } s \to +\infty. \tag{2}$$

Second, if a trajectory satisfies (2) and its phase has a limit as $s \to +\infty$, then the imaginary part of $\alpha$ must increase more rapidly than the real part,[6]

$$[\mathrm{Re}\alpha(s)/\mathrm{Im}\alpha(s)] \to 0 \text{ as } s \to +\infty. \tag{3}$$

Condition (2) is needed for consistency with assumption (i); condition (3), *per se*, is not needed by any of Khuri's four assumptions. However, it is needed to insure consistency with an additional assumption which we wish to consider, namely that (v) $f(s,t)$ is bounded by $s^N$ as $s \to +\infty$ for fixed, unphysical $Z$, excluding those values of $Z$ on the cuts of $P_\alpha(\pm Z)$. Assumption (v) is an extension of assumption (ii); it is added to the other four