CERN, 1968 (to be published), Vol. II, p. 135, find at $p_{\text{lab}}=5.1$ BeV/c,

$$\sigma(\pi^+ + n \to \omega + p) \times \frac{\Gamma(\omega \to \pi^+ + \pi^- + \pi^0)}{\Gamma(\omega \to \text{all})} = 128 \pm 3\ \mu\text{b},$$

with a t dependence e^{Bt}, $B=3.08\pm0.7$. Using the known $\omega \to \pi^+ + \pi^- + \pi^0$ branching ratio we estimate

$$\frac{d\sigma}{dt}(\pi^+ + n \to \omega + p)_{t=-0.5}^{p=5.1} = 100 \pm 50\ \frac{\mu\text{b}}{\text{GeV}^2}.$$

Assuming an $s^{2\alpha-2}$ energy dependence with $-\frac{1}{2}<\alpha<\frac{1}{2}$, we find at $p_{\text{lab}}=6$ BeV and $t=-0.5$ a value of 80 ± 40 μb/GeV^2. Barmawi (Ref. 2) quotes experiments of W. Bugg et al. and G. Benson et al. giving $d\sigma/dt \sim 0.25$ mb/GeV^2 at $t=-0.5$, $p_{\text{lab}}=3.25$–3.65 BeV/c. Assuming the same energy dependence as above we find for $p_{\text{lab}}=6$ GeV/c and $t=-0.5$, $d\sigma/dt \sim 80\pm30$ μb/GeV^2. E. Shibata and M. Wahlig, Phys. Letters 22, 354 (1966) find at $p_{\text{lab}}=10$ BeV/c,

$$\sigma(\pi^- + p \to \omega^0 + n) \times \frac{\Gamma(\omega \to \pi + \gamma)}{\Gamma(\omega \to \text{all})} = 5 \pm 2\ \mu\text{b}$$

with an e^{4t} t dependence. This gives at $t=-0.5$, $d\sigma/dt = 20\pm10$ μb/GeV^2. The same energy correction gives at $p_{\text{lab}}=6$ and $t=-0.5$, $d\sigma/dt = 60\pm45$ μb/GeV^2. The consistency among these evaluations encourages us in believing that our Eq. (4) is realistic.

[9]In addition to taking the extreme limits of Eqs. (2) and (4), we have also used the $\rho_{11}=\frac{1}{2}$ limit in the absence of concrete information. The average values of Eqs. (2) and (4) and a $\rho_{11}\sim\frac{1}{4}$ would give 0.004 μb/GeV^2 as the limit in Eq. (5).

[10]The ω' may be needed elsewhere in order to avoid the difficulty with factorization pointed out by V. Barger and L. Durand, Phys. Rev. Letters 19, 1295 (1967).

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[12]This agrees with the prediction of Dar et al., Ref. 1, who derived it using different assumptions.

[13]One could also consider the exchange of the $I=0$ component of the B-meson octet. Such a contribution would interfere with B exchange but not with ω'. Polarized-photon experiments can distinguish between such a contribution and ω' exchange. Another possibility is the introduction of a fixed pole, either in photoproduction only or in photoproduction and $\pi+N \to V+N$.

PERTURBATION METHOD FOR ATOMS IN INTENSE LIGHT BEAMS

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The problem of interaction of atoms with intense light is reformulated via a time-dependent unitary transformation. An effective electronic binding potential is obtained. The effective perturbation remains bounded as the intensity of the incident light increases.

The mechanism responsible for initiating sparking in gases irradiated by very intense laser beams (about 10^{11} W/cm^2) is usually considered to be multiphoton photoeffect. Past calculations of ionization cross sections have been low by many orders of magnitude, and calculations of threshold intensities have been high by two orders of magnitude.[1,2] The difficulty experienced in these calculations is that perturbation theory is applied to intensities far beyond the limit of validity of the usual theory, and what is equally important, the effect of the intense electromagnetic wave on the initial state has been totally neglected. Indeed, the concept of photon absorption has its roots in perturbation theory, so that even the meaning of the concept becomes unclear at the intensities that we consider.

In view of these difficulties, we propose a reformulation of the problem of the interaction of intense light with atoms. Essentially, the method consists of a transformation to an accelerated frame of reference. It is shown that, in nonrelativistic dipole approximation, an effective intensity-dependent potential that binds the electrons can be found. The remaining terms in the interaction approach a finite limit as the incident intensity is increased. The present formulation of the problem is equivalent to the usual one at low intensities.

For simplicity, we consider a hydrogen atom in the nonrelativistic dipole approximation. The method is easily generalized to the case of many electrons moving in a Coulomb potential. The Schrödinger equation is

$$\frac{1}{2m}\left[\frac{\hbar}{i}\vec{\nabla}-\frac{e}{c}\vec{A}(t)\right]^2\Psi(\vec{r},t)+V(\vec{r})\Psi(\vec{r},t) = i\hbar\frac{\partial\Psi}{\partial t}(\vec{r},t). \quad (1)$$

We introduce the new wave function

$$\psi(\vec{r},t)=\Omega\Psi(\vec{r},t), \tag{2}$$

where

$$\Omega=\exp\left\{\frac{i}{\hbar}\int_{-\infty}^{t}\left[\frac{i\hbar e}{mc}\vec{A}(\tau)\cdot\nabla+\frac{e^2}{2mc^2}\vec{A}(\tau)\right]d\tau\right\}$$

$$=\exp\left[\frac{i}{\hbar}\int_{-\infty}^{t}H_{\text{int}}{}^{t}(\tau)d\tau\right]. \tag{3}$$

Because of our dipole approximation, Ω may be written as a product:

$$\Omega=\Omega_1\Omega_2,$$

$$\Omega_1=\exp\left[-\int_{-\infty}^{t}\frac{e}{mc}\vec{A}(\tau)d\tau\cdot\nabla\right],$$

$$\Omega_2=\exp\left\{\frac{i}{\hbar}\int_{-\infty}^{t}\frac{e^2}{2mc^2}\vec{A}(\tau)d\tau\right\}. \tag{4}$$

Ω_1 is a translation operator,

$$\Omega_1 f(\vec{r})=f\left(\vec{r}-\int_{-\infty}^{t}\frac{e}{mc}\vec{A}(\tau)d\tau\right). \tag{5}$$

Thus, the transformation (2) represents a shift to an accelerated frame of reference.

For brevity, we define

$$\vec{\alpha}=-\int_{-\infty}^{t}\frac{e}{mc}\vec{A}(\tau)d\tau. \tag{6}$$

The relation

$$\ddot{\vec{\alpha}}=-\frac{e}{mc}\dot{\vec{A}}(\tau)=\frac{e}{m}\vec{E}(t) \tag{7}$$

indicates that $\vec{\alpha}$ is the classical displacement of a free electron from its center of oscillation in a radiation field $\vec{E}(t)$.

One easily shows that $\psi(\vec{r},t)$ satisfies the differential equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t)+V(\vec{r}+\vec{\alpha})\psi(\vec{r},t)$$

$$=i\hbar\frac{\partial\psi}{\partial t}(\vec{r},t) \tag{8}$$

The reader should note that no assumption concerning the validity of a Schrödinger equation in an accelerated frame of references has been made. Equation (8) follows immediately from Eqs. (1)-(3); it is somewhat similar to a result obtained by van Kampen.[3]

We consider the incident field to be a monochromatic plane wave of angular frequency ω, so that apart from a phase factor,

$$\vec{\alpha}_0=\frac{e}{m\omega c}\vec{A}_0, \tag{9}$$

where $\vec{A}_0$ is the amplitude of the incident wave.

Equations (8) and (1) must be equivalent to all orders in e, since (8) has been obtained from (1) through a unitary transformation. However, the reader may find an explicit proof for first- and second-order processes more satisfying. For small $\vec{A}_0$, the time-dependent potential term may be written

$$V(\vec{r}+\vec{\alpha})\simeq V(\vec{r})+(\vec{\alpha}\cdot\nabla)V(\vec{r})$$

$$+(1/2!)(\vec{\alpha}\cdot\nabla)^2V(\vec{r})+\cdots \tag{10}$$

with $\vec{\alpha}=\vec{\alpha}_0\sin\omega t$. For one-quantum emission or absorption processes, the matrix element is

$$\langle a|\vec{\alpha}\cdot\nabla V(\vec{r})|b\rangle=\vec{\alpha}\cdot\langle a|[\nabla,H_0]|b\rangle$$

$$=(E_b-E_a)\vec{\alpha}\cdot\langle a|\nabla|b\rangle$$

$$=\hbar\omega_{ba}\vec{\alpha}\cdot\langle a|\nabla|b\rangle. \tag{11}$$

Since for single-quantum processes $|\omega_{ba}|=\omega$, this differs from the matrix element of the lowest order interaction of (1) by a phase factor.

We next consider a second-order process in which two quanta are absorbed, so that $E_f=E_0+2\hbar\omega$. In this process, one must take the second term of (10) to second order and the third term to first order. Hence the matrix element for two-quantum absorption is

$$\langle f|H_{\text{int}}{}^{(2)}|0\rangle=\tfrac{1}{2}\langle f|(\vec{\alpha}\cdot\nabla)^2V(\vec{r})|0\rangle+\sum_m\frac{\hbar^2\omega_{fm}\omega_{m0}\vec{\alpha}\cdot\langle f|\nabla|m\rangle\vec{\alpha}\cdot\langle m|\nabla|0\rangle}{E_0-E_m+\hbar\omega}. \tag{12}$$

The first term may be written as follows:

$$\begin{aligned}\tfrac{1}{2}\langle f|(\vec{\alpha}\cdot\nabla)^2V(\vec{r})|0\rangle &= \tfrac{1}{2}\langle f|[\vec{\alpha}\cdot\nabla,[\vec{\alpha}\cdot\nabla,H_0]]|0\rangle = \tfrac{1}{2}\langle f|[\vec{\alpha}\cdot\nabla,\vec{\alpha}\cdot\nabla H_0 - H_0\vec{\alpha}\cdot\nabla]|0\rangle \\ &= \tfrac{1}{2}\langle f|(\vec{\alpha}\cdot\nabla)^2H_0 - 2(\vec{\alpha}\cdot\nabla)H_0(\vec{\alpha}\cdot\nabla) + H_0(\vec{\alpha}\cdot\nabla)^2|0\rangle \\ &= \tfrac{1}{2}(E_0+E_f)\langle f|(\vec{\alpha}\cdot\nabla)^2|0\rangle - \sum_m \langle f|\vec{\alpha}\cdot\nabla|m\rangle\langle m|\vec{\alpha}\cdot\nabla|0\rangle E_m \\ &= \sum_m \langle f|\vec{\alpha}\cdot\nabla|m\rangle\langle m|\vec{\alpha}\cdot\nabla|0\rangle(E_0+\hbar\omega-E_m). \end{aligned} \tag{13}$$

Because of the relation

$$\hbar^2\omega_{fm}\omega_{m0} + (E_0+\hbar\omega-E_m)^2 = \hbar^2\omega^2, \tag{14}$$

(12) reduces to the usual second-order matrix element for two-quantum absorption as calculated from the interaction of Eq. (1).

In the study of ionization of atoms, the potential of interest is the Coulomb potential,

$$\begin{aligned} V(\vec{r}+\vec{\alpha}) &= -\frac{e^2}{|\vec{r}+\vec{\alpha}|} \\ &= -\frac{e^2}{2\pi^2}\int \frac{e^{i\vec{q}\cdot(\vec{r}+\vec{\alpha})}}{q^2}\,d^3q. \end{aligned} \tag{15}$$

Because of the relation $\vec{\alpha}=\vec{\alpha}_0\sin\omega t$, we may write

$$e^{i\vec{q}\cdot\vec{\alpha}} = \sum_{n=-\infty}^{\infty} J_n(\vec{\alpha}_0\cdot\vec{q})e^{in\omega t}. \tag{16}$$

Equation (16) shows that the potential contains a time-independent part given by

$$V_0(\vec{r}) = -\frac{e^2}{2\pi^2}\int \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}J_0(\vec{\alpha}_0\cdot\vec{q})d^3q. \tag{17}$$

It should be noted that the expansion (16) inserted into Eq. (15) is equivalent to the expansion (10) if one approximates the Bessel functions by the leading term in their respective power-series expansions.

The effective potential $V_0(\vec{r})$ is easily understood by calculating the effective charge that would produce it:

$$\begin{aligned} -e\rho_{\text{eff}}(\vec{r}) &= -\frac{1}{4\pi}\nabla^2V_0(\vec{r}) \\ &= -\frac{e^2}{8\pi^3}\int e^{i\vec{q}\cdot\vec{r}}J_0(\vec{\alpha}_0\cdot\vec{q})d^3q. \end{aligned} \tag{18}$$

Taking $\vec{\alpha}_0$ in the positive Z direction, one obtains

$$\begin{aligned} \rho_{\text{eff}}(\vec{r}) &= \frac{e}{\pi}\delta(x)\delta(y)\frac{1}{(\alpha_0^2-Z^2)^{1/2}}, \quad Z<\alpha_0, \\ &= 0, \quad Z>\alpha_0, \end{aligned} \tag{19}$$

Equation (19) gives the time-averaged charge seen by the electron in the oscillating frame of reference. The term $(1/\pi)(\alpha_0^2-Z^2)^{-1/2}$ is the classical probability density for an oscillator oscillating in the Z direction with amplitude α_0. The effective potential can be simplified by making use of the identity

$$J_0(\vec{\alpha}_0\cdot\vec{q}) = \frac{1}{2\pi}\int_0^{2\pi} e^{i\vec{\alpha}_0\cdot\vec{q}\sin\varphi}\,d\varphi. \tag{20}$$

Insertion of (20) into (17) yields

$$V_0(\vec{r}) = -\frac{e^2}{4\pi^3}\int_0^{2\pi}\int \frac{e^{i(\vec{q}\cdot\vec{r}+\vec{\alpha}_0\cdot\vec{q}\sin\varphi)}}{q^2}d^3q\,d\varphi = -\frac{e^2}{2\pi}\int_0^{2\pi}\frac{d\varphi}{|\vec{r}+\vec{\alpha}_0\sin\varphi|}, \tag{21}$$

which is the time average of the potential in Eq. (8). This integral cannot be evaluated in closed form. One can, however, easily obtain an estimate for the order of magnitude of intensity-dependent shifts of ionization potentials by approximating $J_0(\vec{\alpha}_0\cdot\vec{q})$ by $\cos(\vec{\alpha}_0\cdot\vec{q}/\sqrt{2})$. This is reasonable in the region $\vec{\alpha}_0\cdot\vec{q}<2$. This gives a good approximation of $V_0(\vec{r})$ only for large r, but here we shall consider the result for all r as an order-of-magnitude calculation. In this approximation

$$V_0(\vec{r}) \approx -\frac{e^2}{4\pi^2}\int \frac{e^{i\vec{q}\cdot(\vec{r}+\vec{\alpha}_0/\sqrt{2})}+e^{i\vec{q}\cdot(\vec{r}-\vec{\alpha}_0/\sqrt{2})}}{q^2}d^3q = -\frac{e^2}{2|\vec{r}+\vec{\alpha}_0/\sqrt{2}|} - \frac{e^2}{2|\vec{r}-\vec{\alpha}_0/\sqrt{2}|}. \tag{22}$$

Equation (22) indicates that, in this approximation, the electron sees two charges of magnitude $\frac{1}{2}e$ separated by a distance $\sqrt{2}\,\alpha_0$. For a ruby laser beam of intensity 2×10^{11} W/cm^2, $\sqrt{2}\,\alpha_0$ is approximately $0.8a_0$. The work of Bates, Ledsham, and Stewart[4] on the wave functions of the hydrogen molecular ion indicates that in the present approximation, the ionization potential drops to 75% of its zero-intensity value. Since this estimate is based upon an approximation which is quite poor, it would not be surprising if, at 2×10^{11} W/cm^2, ionization potentials fell to half their zero-intensity values.

The present procedure is equivalent to the usual one; its merit lies in the much more rapid convergence of the perturbation series at high intensities. Thus, in the present formulation, one would first find eigenvalues and eigenstates (numerically) of the Hamiltonian

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V_0(\vec{r}), \tag{23}$$

with $V_0(\vec{r})$ given by (21), and then compute ionization probabilities due to the time-dependent perturbation

$$V_{\text{int}} = -\frac{e^2}{2\pi^2} \times \sum_{n\neq 0}\int \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2} J_n(\vec{\alpha}_0\cdot\vec{q})d^3q e^{in\omega t}. \tag{24}$$

Each term of (24) is clearly bounded as the intensity of the incident radiation increases to arbitrary values.

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ρ PRODUCTION FROM HYDROGEN BY 16-GeV BREMSSTRAHLUNG*

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A 2.2-m streamer chamber in a large electromagnet was used with a 16-GeV bremsstrahlung beam to study multibody photoproduction. This paper reports on the 851 three-prong, three-constraint kinematic fit events of the type $\gamma p \to \pi^+\pi^- p$; at energies above 6 GeV, these go almost completely to a $p\rho^0$ final state.

We wish to report on part of the results obtained in the first run of an experiment to study multibody photoproduction with the Stanford Linear Accelerator Center 2.2-m streamer chamber.[1]

A collimated bremsstrahlung beam of 16-GeV peak energy and 3-mm diameter was incident on a 1-cm-diam, 3-atm hydrogen gas target extending through the streamer chamber. The beam of low intensity (~120 equivalent quanta per 1.5-μsec pulse–180 pulses/sec) was measured by a quantameter.[2]

The chamber was mounted in a large electromagnet with a field of 8 kG and was viewed by three cameras in 18° stereo through the open upper pole of the magnet.

The streamer chamber was fired and pictures taken every time a charged particle triggered a rectangular fourfold coincidence array covering approximately ±15° around the beam direction. The large scintillation counters were split (±2°) in the horizontal plane containing the target and were perpendicular to the magnetic field in order to reduce the number of triggers from electromagnetic background events.

The interaction vertices located inside the hydrogen gas target were invisible.

Of the 87 000 pictures taken and scanned, 7055