Proof. – The limit of the integral as $s\to\infty$ in that case equals $\int[\mathrm{Im}\alpha(s')ds'/s']$, which exists and is a constant. Now, for a linearly rising trajectory, $\mathrm{Im}\alpha(s)=s^{1/2}\Gamma\,\mathrm{Re}\alpha'(s)=s^{1/2}\Gamma B$, where Γ is the total width. Consistency requires that

$$s^{1/2}\Gamma \underset{s\to\infty}{\longrightarrow} 0$$

like some power of s. This is certainly more than fulfilled by our widths.

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$$2\gamma\frac{J!}{(2J+1)!!}\left(\frac{k}{m}\right)^{2J-2}\frac{k^3}{m_J^{\,2}},$$

with the Lagrangian defined analogously to the one in the text. This formula and others relating to meson decay will be derived in a subsequent publication.

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η ASYMMETRY WITHOUT C NONCONSERVATION*

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We calculate the charge asymmetry for the reaction $\pi^-p\to n\pi^+\pi^-\pi^0$ assuming C invariance for the η decay. We find that interference terms with a 3π background amplitude give a nonzero charge asymmetry. Assuming that the background in the η mass region (10 MeV) is about 10 % of the η signal, we obtain a maximum asymmetry of about 2 %, implying that the experimental asymmetry does not necessarily imply C nonconservation.

The discovery of apparent CP nonconservation[1] in the K_L decay has led to the suggestion[2] that C invariance might not hold for the electromagnetic interaction. Since then, a number of experiments[3-15] have been performed to find evidence for a C nonconservation in the electromagnetic decay of the eta meson. The summary of these experiments shows no strong evidence for C nonconservation in the eta decay and if C invariance is violated, the charge asymmetry is at most order of 10^{-2} or 10^{-3}, which is rather near to the value suggested by several theoretical estimates.[2]

However, it is the purpose of this note to show that such a small value of the charge asymmetry in η decay can be produced without C nonconservation by interference effects so that a presence of an η asymmetry does not necessarily imply C nonconservation. In this note, we would like to consider the reactions

$$\pi^-p\to n\eta,\quad \eta\to\pi^+\pi^-\pi^0 \tag{1}$$

$$\pi^-p\to n\pi^+\pi^-\pi^0, \tag{2}$$

and to show that an interference effect between the η and 3π-background amplitudes can create a charge asymmetry in the η-mass region without C nonconservation in the η decay. In the following discussion, we assume that C invariance holds for the η decay and that the charge asymmetry for the 3π background is negligible.

The matrix element M for Reactions (1) and (2) can be written

$$M = \frac{M_S M_D}{S-(m-\frac{1}{2}i\Gamma)^2} + \frac{B}{\mu^2}, \tag{3}$$

where M_S and M_D are the Lorentz-invariant matrix elements for production, $\pi^- p \to n\eta$, and the decay, $\eta \to \pi^+\pi^-\pi^0$, respectively. μ, m, and Γ are the pion mass, the η mass, and the total decay width of the η, respectively. $\sqrt{S}$ is the invariant mass of the 3π system. B is the 3π background amplitude and may be decomposed into the charge-symmetric part B_+ and the charge-asymmetric part B_- by

$$B = B_+ + B_-.$$

The factor $1/\mu^2$ in the second term in Eq. (3) is introduced to make the same dimensionality for $M_S M_D$ and B. Setting $S^{1/2} + m - \frac{1}{2}i\Gamma \simeq 2m$, the cross section for 3π production near η mass region is now given by

$$\sigma = \int |M|^2 d\Omega = \int \left[\frac{1}{4m^2} \frac{|M_S M_D|^2}{(S^{1/2}-m)^2 + (\Gamma/2)^2} + \frac{|B|^2}{\mu^4} + \frac{S^{1/2}-m}{m\mu^2} \frac{\mathrm{Re}(M_S M_D B^*)}{(S^{1/2}-m)^2+(\Gamma/2)^2} + \frac{\Gamma}{2m\mu^2} \frac{\mathrm{Im}(M_S M_D B^*)}{(S^{1/2}-m)^2+(\Gamma/2)^2} \right] d\Omega; \tag{4}$$

$d\Omega$ is the phase volume and is given by

$$d\Omega = \left(\frac{1}{2\pi}\right) \frac{M^2}{WP} \delta^4(Q-K_1-K_2-K_3-P')\delta(K_1^2-\mu^2)\delta(K_2^2-\mu^2)\delta(K_3^2-\mu^2)\delta(P'^2-M^2) d^4K_1 d^4K_2 d^4K_3 d^4P',$$

where Q, K_1, K_2, K_3, and P' are the four-component momenta for the initial system, the outgoing three pions, and neutron with mass M, respectively; P and W are the momentum magnitude of incoming proton with mass M and the total energy of the system in the c.m. system, respectively. $d\Omega$ can also be written in terms of $d\sqrt{S}$ and $d\overline{\Omega}$, which includes all the other integral elements except $d\sqrt{S}$:

$$d\Omega = d\overline{\Omega} d\sqrt{S}. \tag{5}$$

We should note that the fourth term has exactly the same Breit-Wigner form as the first term. This is the fact on which our subsequent argument is based.

The limits of the integral in Eq. (4) depend on the 3π mass cut and the geometry of counter settings used in each experiment. Since we are treating Eq. (4) in the η-mass region, the matrix elements M_S, M_D, and B can be replaced by $\overline{M}_S$, $\overline{M}_D$, and $\overline{B}$, the integrals over the η-mass region. Furthermore, the observed 3π-mass distribution should be smeared by an experimental mass resolution. Assuming the resolution function to be the familiar Gaussian form, with a resolution σ,

$$g(\sqrt{S}, \sqrt{S'}) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left[-\frac{(S^{1/2}-S'^{1/2})^2}{2\sigma^2}\right],$$

the 3π-mass distribution then is written from Eqs. (4) and (5):

$$\frac{d\sigma}{d\sqrt{S'}} = \int |\overline{M}(\sqrt{S})|^2 g(\sqrt{S}, \sqrt{S'}) d\sqrt{S} = \frac{1}{4m^2} \frac{2\pi}{\Gamma} |\overline{M}_S \overline{M}_D|^2 g(m, \sqrt{S'}) + \frac{1}{\mu^4} |\overline{B}|^2 + \frac{S^{1/2}-m}{m\mu^2} \frac{2\pi}{\Gamma} \mathrm{Re}(\overline{M}_S \overline{M}_D \overline{B}^*) g(m, \sqrt{S'}) + \frac{\Gamma}{2m\mu^2} \frac{2\pi}{\Gamma} \mathrm{Im}(\overline{M}_S \overline{M}_D \overline{B}^*) g(m, \sqrt{S'}). \tag{6}$$

In this integral, we made the approximations[16]

$$\int \frac{1}{(S^{1/2}-m)^2+(\Gamma/2)^2} g(\sqrt{S}, \sqrt{S'}) d\sqrt{S} \simeq \frac{2\pi}{\Gamma} g(m, \sqrt{S'}), \quad \int \frac{(S^{1/2}-m)}{(S^{1/2}-m)^2+(\Gamma/2)^2} g(\sqrt{S}, \sqrt{S'}) d\sqrt{S} \simeq \frac{\pi}{\Gamma}(S'^{1/2}-m) g(m, \sqrt{S'}).$$

Since we are assuming C invariance for the η decay and a negligible charge asymmetry in the background $|B|^2$, which appears to be experimentally correct, the asymmetry in the η-mass region will come from the third and fourth terms in Eq. (6). However, if we integrate Eq. (6) over $\sqrt{S'}$ from $m - \frac{1}{2}\Delta m$ to $m + \frac{1}{2}\Delta m$ (Δm being the order of σ), the third term vanishes. Therefore, for these integral limits, the experimentally observable charge asymmetry is given by the ratio of the first and fourth terms, i.e.,

$$\alpha=\frac{2m\Gamma}{\mu^2}\frac{\mathrm{Im}(\bar{M}_S\bar{M}_D\bar{B}_-{}^*)}{|\bar{M}_S\bar{M}_D|^2}=\frac{2m\Gamma}{\mu^2}\frac{|\bar{B}_-|}{|\bar{M}_S\bar{M}_D|}\sin\delta, \tag{7}$$

where δ is the phase angle for $\bar{M}_S\bar{M}_D\bar{B}_-{}^*$. The maximum charge asymmetry will be obtained if we set $\sin\delta=1$ and $B_-=B$:

$$\alpha_{\max}=\frac{2m\Gamma}{\mu^2}\frac{|\bar{B}|}{|\bar{M}_S\bar{M}_D|}=\left(\frac{2\pi\Gamma}{\Delta m}\frac{\sigma_B}{\sigma_\eta}\right)^{1/2}=0.16\left(\frac{\sigma_B}{\Delta m\sigma_\eta}\right)^{1/2} \tag{8}$$

where Δm is measured in MeV, $\Gamma\simeq4$ keV,[17] and σ_η and σ_B are the cross sections for Reactions (1) and (2) in the 3π-mass region from $m-\frac{1}{2}\Delta m$ to $m+\frac{1}{2}\Delta m$; i.e.,

$$\sigma_B=(|\bar{B}|^2/\mu^4)2m\Delta m,\quad \sigma_\eta=\pi|\bar{M}_S\bar{M}_D|^2/m\Gamma.$$

When Δm is 10 MeV and the ratio of the background to signal is $\sigma_B/\sigma_\eta=0.1$ as in the most experimental circumstances, then we obtain the maximum charge asymmetry 1.6%. This $\alpha_{\max}$ is, in fact, comparable with the order of the charge asymmetry for the η decay observed in the recent experiment.[15] Furthermore, it is also possible to have $B\simeq B_-$ in the case of small charge asymmetry in the 3π background. We will discuss this next.

The charge asymmetry in the 3π background α_B is given by

$$\alpha_B=\frac{2\,\mathrm{Re}(\bar{B}_+\bar{B}_-{}^*)}{|\bar{B}_+|^2+|\bar{B}_-|^2}=\frac{2|\bar{B}_+||\bar{B}_-|}{|\bar{B}_+|^2+|\bar{B}_-|^2}\cos\delta_B,$$

where δ_B is the phase angle for $\bar{B}_+\bar{B}_-{}^*$. There are three cases in which α_B can have a small value: (a) $|\bar{B}_+|\gg|\bar{B}_-|$, (b) $|\bar{B}_-|\gg|\bar{B}_+|$, and (c) $|\cos\delta_B|\ll1$. Apparently, case (b) can give us $B\simeq B_-$, which makes α in Eq. (7) to be $\alpha_{\max}$. Although we do not have enough information on the background, it is not inconceivable to have $B\simeq B_-$, since the background term may come from reactions such as $\pi^-p\to\pi^-\pi^0N^{*+}(N^{*+}\to n\pi^+)$ or $\pi^-p\to\rho^0N^{*0}[\rho^0\to\pi^+\pi^-(\rho^0$ being virtual$), N^*\to n\pi^0]$.

The charge asymmetry due to the third term in Eq. (6) is also interesting to note. If we evaluate the charge asymmetries, α_L and α_R, due to the third term in two mass regions (α_L from $m-\frac{1}{2}\Delta m$ to m and α_R from m to $m+\frac{1}{2}\Delta m$), then α_L and α_R should have the same magnitude, but opposite sign. Therefore, if we observe different asymmetries in the two mass regions, then this indicates existence of the interference effect due to the third term in Eq. (6).

In conclusion, we make the following remarks: (a) A charge asymmetry of the order of 10^{-2} is possibly produced by the interference effect between the η and 3π background. (b) If our mechanism is right, then the experimentally observed charge asymmetry must vary with the incident pion energy. Also, it must depend upon the production mechanism of the experiment concerned. Hence, in order to establish experimentally a C nonconservation in η decay of as small as 1% or less, we have to change the incident energy as well as the production mechanism. Also, we should make $\alpha_{\max}$ as small as possible by a proper experimental setup. If the asymmetry still persists in all these different experiments, then we are safe to infer C nonconservation. It is interesting to note in this connection that this kind of interference effect is absent for the reaction $p\bar{p}\to$ many pions.[18] This is obvious for slow-$\bar{p}$ reactions since the initial channel is symmetric under charge conjugation. However, the same holds valid also for moving-$\bar{p}$ annihilation reactions. This results from the fact that in the experimental analysis, we are only interested in analyzing the $\pi^+\pi^-\pi^0$ mass spectrum by averaging over all other kinematical variables, or it can be understood by using the operation CP in the c.m. system. Hence, if we find a charge asymmetry in $p\bar{p}$ annihilation, then we must have C nonconservation. (c) A similar argument also holds for the charge asymmetry for $\eta\to\pi^+\pi^-\gamma$. (d) Finally, a large C nonconservation in the electromagnetic interaction may conflict[19] with the experimentally found small electric dipole moment of neutron.[20] This would probably indicate that the η asymmetry, if it exists at all, would be much smaller than 0.1%. In that case, even if we have the opposite case of say $|B_-| \simeq \frac{1}{10}|B_+|$ instead of $|B_-| \gg |B_+|$, we could easily reproduce the η asymmetry as small as 0.1%, indicating that it would be quite difficult experimentally to verify the C nonconservation in η decay.

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