Since the regions studied occur at galactic longitudes 350, 11, 111, and 184 deg, we cannot easily explain the absence of fields of 10 μG in the local arm as a selection effect. If spiral-arm magnetic fields are all similar and longitudinal, we might have expected a smaller field component in the Perseus arm than in the Orion arm at the longitude of Cas A. Since the reverse is true, the above two points suggest one of two conclusions: Either the sun is in a region of the galaxy with a low field strength, or the region in the direction of Cas A is unique. Rickard[6] has suggested that the latter might be so. He claims that a supernova occurred somewhere at the outer edge of the Perseus arm, which has distorted that arm to produce the double nature of the 21-cm emission spectra as well as of the optical lines. Shock waves, such as he envisaged, may have compressed and amplified the magnetic field so that we now see the amplified fields in this experiment.

An interesting aspect of this measurement is that either model may be easily checked. Since we are now dealing with fields of 10 μG and not 1 μG, the search for Zeeman effects in other absorption or emission spectra throughout the galactic plane will be made much easier.

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WILL THE RESONANCES ON THE LEADING TRAJECTORIES BECOME STABLE TO STRONG DECAY AT HIGH SPIN?

Hyman Goldberg
Department of Physics, Northeastern University, Boston, Massachusetts
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A heuristic discussion is given of the decay modes of high-spin particles lying on a linearly rising trajectory. Among other things, it is concluded that for particles with masses $\gtrsim$3500 MeV ($J \gtrsim 12$), the total decay widths decrease with J and go to zero approximately like $(J\ln J)^{-0.28J^{1/2}}$ as $J\to\infty$ $(0.28 = 2m_\pi b^{1/2}$, where $b \simeq 1\text{ GeV}^{-1/2}$ is asymptotic slope of the rising trajectory).

A simple relationship between the spin J and mass M (in GeV),

$$J = J_0 + M^2 \tag{1}$$

with $|J_0| < 1$, seems to characterize a number of families of baryonic[1] and bosonic[2] resonances. In the present communication we address ourselves to the consequences of Eq. (1) to the decay modes and widths of the higher resonances governed by it. The major conclusions, based on admittedly heuristic arguments, are the following: (1) After $J\sim 12$, the total widths of all resonances decrease fairly rapidly (but slower than $e^{-\alpha J}$). By $J = 50$ ($M^* \simeq 7$ GeV), the total widths of all "resonances" should be less than 1 MeV. (2) The dominant decay mode for $J \gtrsim 10$ is to a pion and a resonance with the highest kinematically allowed mass. Decays into the lowest mass states (e.g., $\pi\pi$) are predicted to vanish very rapidly for $J \geqslant 6$.

We shall consider only the two-body decay modes (these seem to be dominant if not exclusive) of a heavy particle of mass M and spin $J \simeq M^2$ into two particles which also lie on trajectories described by (1). With negligible error entailed in the ensuing arguments, J_0 will be taken equal to 0 in all cases, except when a pion may be involved.

The first part of the discussion is based on a result of wave equation theory; namely, centrifugal barrier effects will significantly inhibit a two-particle decay if

$$k^2 \ll l(l+1)/R^2, \tag{2}$$

where k is the c.m. momentum, R is the longest range of the hadronic force bewteen the decay products, and l is the orbital angular momentum of the final state. Since R is of the order of a few GeV^{-1}, the condition (2) becomes (for $l \gtrsim 2$)

$$k^2/l^2 \ll 1 \tag{2a}$$

when k is expressed in GeV. It is now a simple algebraic exercise to show that the mass formula (1) forces (2a) to be satisfied in all two-body decays of a particle of spin $J \gg 1$. It is conveniently done by dividing the possibilities into three cases.

Case 1.—Decay into two heavy particles of spins j_1, j_2, with j_1 and $j_2 \sim O(J)$. In units of GeV, Eq. (1) (with $J_0 = 0$) allows us to write

$$k^2 = \tfrac{1}{4}J[1-(\xi_1^{1/2}+\xi_2^{1/2})^2][1-(\xi_1^{1/2}-\xi_2^{1/2})], \tag{3}$$

where $j_1 = \xi_1 J$, $j_2 = \xi_2 J$, and ξ_1, ξ_2 are finite as $J \to \infty$. l, on the other hand, satisfies

$$l \geq J - j_1 - j_2 = J[1-(\xi_1+\xi_2)]. \tag{4}$$

Since $1-(\xi_1+\xi_2) \geq 2(\xi_1\xi_2)^{1/2}$ [Eq. (3)], we must have $l \geq 2(\xi_1\xi_2)^{1/2}J$. Combined with $k^2 \leq \frac{1}{4}J$, this gives

$$k^2/l^2 \leq (\xi_1\xi_2 J)^{-1} \sim J^{-1}. \tag{5}$$

Case 2.—Decay into two light particles $j_1, j_2 \ll J$. Then we allow ξ_1, $\xi_2 \to 0$ in Eqs. (3) and (4), with the result that $k^2 \simeq \frac{1}{4}J$, $l \simeq J$, and

$$k^2/l^2 \simeq \tfrac{1}{4}J^{-1}. \tag{6}$$

Case 3.—Decay into a light particle (mass $\mu \ll \sqrt{J}$) and a heavy one, spin J', mass $\sqrt{J'}$, $J' \sim O(J)$.

In this case, we have without approximation

$$k^2 = \tfrac{1}{4}[J - 2(J'+\mu^2) + (J'-\mu^2)^2/J] = (J - J' + \mu^2)^2/4J - \mu^2. \tag{7}$$

Again,

$$l \geq J - J' - \mu^2 \text{ if } \mu \text{ has spin,} \tag{8a}$$

$$\geq J - J' \text{ if } \mu \text{ has zero spin.} \tag{8b}$$

However, Eq. (7) implies

$$J - J' + \mu^2 \geq 2\mu J^{1/2} \tag{9}$$

which, for $\mu \ll J^{1/2}$ (definition of light particle), becomes $J - J' \geq 2\mu J^{1/2}$ with negligible error. As a result, we may safely substitute for (8a) or (8b)

$$l \geq J - J' + \mu^2 \geq 2\mu J^{1/2}. \tag{10}$$

Thus, for $\mu \ll \sqrt{J}$,

$$k^2 \leq (l^2/4J) - \mu^2$$

and

$$k^2/l^2 \leq \tfrac{1}{4}J^{-1} - \mu^2/l^2 \leq \tfrac{1}{4}J^{-1}. \tag{11}$$

This completes the demonstration that Eq. (1) forces $k^2/l^2 \ll 1$ for all two-body decays of a particle of $J \gg 1$. The conclusion is that barrier effects will be of major inportance in these decays.

To proceed in a more quantitative mannor, we make use of another result of wave equations; namely, the partial decay width of a resonance of spin J into a channel c with $k_c R_c \ll l_c$ behaves like[3]

$$\Gamma_c \sim \left[\frac{(k_c R_c)^{l_c}}{(2l_c - 1)!!}\right]^2 \sim \left(\frac{e k_c R_c}{2l_c}\right)^{2l_c} \tag{12}$$

for large l_c.[4] The last result may be most easily obtained from the eikonal approximation for the transmission coefficient:

$$T_l = \exp\{-\int_R^{l/k}(l^2/r^2 - k^2)^{\frac{1}{2}}dr\} \simeq (ekR/2l)^l \tag{13}$$

when $l \gg kR$ and $\Gamma_c \propto |T_l|^2$.

In both Cases 1 and 2 above, $k^2/l^2 \sim J^{-1}$, $l = aJ$, and hence for such channels,

$$\Gamma \sim (\text{const}/J)^{aJ}, \quad a > 0 \text{ and finite as } J \to \infty. \tag{14}$$

In particular, in Case 2 involving two light particles, $l \simeq J$, $k^2 \simeq \frac{1}{4}J$, and

$$\Gamma \sim (c/J)^J \tag{15}$$

with

$$c = (eR/4)^2 \tag{16}$$

expressed in GeV^{-2}. Equation (15) is a statement that the decay widths of heavy particles into a light pair will drop faster than a exponential with J. There has been some experimental confirmation of this in the decays $N_\gamma \to N\pi$ and $\Delta_\delta \to N\pi$,[5] and a form similar to (15) has been conjectured for a rising trajectory by Jones and Teplitz[6] on the basis of the Mandelstam symmetry.[7]

Of ultimate interest, however, will be Case 3. We proceed to show that large J, this mode (heavy → heavy + light) completely dominates the others.

Since $l \geq 2\mu\sqrt{J}$ [Eq. (10)], we set

$$l = 2\mu J^{1/2}(1+\alpha), \quad \alpha > 0. \tag{17}$$

Equations (11), (12), and (17) result in

$$\Gamma \leq \left[\frac{c}{J}\frac{\alpha(2+\alpha)}{(1+\alpha)^2}\right]^{2\mu J^{1/2}(1+\alpha)} \tag{18}$$

with c given in Eq. (18). For $\ln(J/c) \gtrsim 1$, Γ has a maximum for $\alpha^{-1} \simeq \ln(J/c)$. Consequently, for Case 3 the dominant width for $\ln(J/c) \gtrsim 1$ is[8]

$$\Gamma \sim \left[\frac{2}{(J/c)\ln(J/c)}\right]^{2\mu J^{1/2}}. \qquad (19)$$

Comments and conclusions.—(1) The partial width given in Eq. (19) for a process J(heavy) $\to J'$(heavy) + μ(light) with $l \simeq 2\mu J^{1/2}$ manifestly dominates the partial widths for the other decay modes [Eqs. (14) and (15)] at large J.[9]

(2) The largest partial width of the dominant type (Case 3) will occur for smallest μ, which is $m_\pi = 0.14$. Hence the dominant decay mode will be $J \to J' + \pi$, with $J' \simeq J - 2m_\pi J^{1/2}$ = largest kinematically allowed spin, $k^2 = 2m_\pi^2/\ln(J/c)$ [from Eqs. (7), (17), and footnote 8].

(3) Since the total number of decay channels for fixed J varies only as J^2, the multiplicity of channels is insufficient to allow the sum of all other partial widths to compete with the width for the dominant mode $J \to J' + \pi$. Hence the dominant behavior of the total width for all resonances at large J is of the form

$$\Gamma_{\rm tot} \sim \left[\frac{2}{(J/c)\ln(J/c)}\right]^{0.28\sqrt{J}} \qquad (20)$$

with $c = \frac{1}{16}e^2R^2$ expressed in units of GeV^{-2}.

(4) The total width will begin to suffer marked reduction when both $(J/c)\ln(J/c) \geq 2$ and $0.28J^{1/2} > 1$. With $R \simeq 0.7$ F,[10] $c \simeq 6$, and subsequently these conditions are fulfilled for $J > 12$. Total widths should fall below 1 MeV at $J \sim 36$ ($M = 6$ GeV) for bosons and $J \sim 48$ ($M = 7$ GeV) for baryons. (The branching ratio for other channels is down to $\sim 10^{-12}$ at these masses.) We await with great interest future experimental checks on this crucial conclusion in the energy region corresponding to $J \gtrsim 12$ ($M \gtrsim 3500$ MeV).

(5) As $J \to \infty$, $\Gamma_{\rm tot} \to 0$ and all the "particles" become stable to hadronic decay. For masses >20 GeV, the dominant decay mode will be electromagnetic.

(6) The total widths at large J decrease much faster than the interparticle mass spacing ($\sim J^{-1/2}$), removing in principle any overlap question.

(7) Finally, these results depend crucially on the trajectory $\alpha(s)$ growing more quickly than $\sqrt{s}$ for large s.

The arguments leading to these somewhat potent conclusions are based on the centrifugal barrier properties of wave equations. Can we make use of any of the current data to test these concepts, albeit qualitatively? We illustrate with an example.

The $l = 5$ decay into two pions of the $I = 1$ T meson[2] (2200 MeV, $J^P = 5^-$ hypothetical) may be described by the effective Lagrangian

$$(4\pi\gamma_{T\pi\pi})^{\frac{1}{2}}(m^{-4})\vec{T}^{\mu_1\cdots\mu_5}\cdot\vec{\varphi}\times\partial_{\mu_1}\cdots\partial_{\mu_5}\vec{\varphi},$$

giving a decay width[11]

$$\Gamma(T \to 2\pi) = \gamma_{T\pi\pi}(16/693)(k/m)^{10}(m/m_T)^2 k$$

(m has been introduced to make $\gamma_{T\pi\pi}$ dimensionless). With m in GeV, we deduce that $\gamma_{T\pi\pi} \simeq 86\,\Gamma(T \to 2\pi)m^8$. The data of Ref. 2 seem to indicate a total width of the order of 10-20 MeV for the T meson, with at most 15% branching into 2π. We are probably safe in assuming that $\Gamma(T \to 2\pi) < 1$ MeV, with the result that

$$\gamma_{T\pi\pi} < m^8, \qquad (21)$$

with m in GeV.

A large reduction in the value of $\gamma_{T\pi\pi}$ compared with $\gamma_{\rho\pi\pi} \simeq 2.4$ would be an indication of important barrier effects.[12] To assess this, we perforce need a value of m. Consistency with the barrier approach requires us to choose $m^{-1} \simeq$ (longest range of hadronic force) $= R$, which then supplies the canonical $(kR)^{2l}$ factor in the decay rate. If we take $m^{-1} = 0.7$ F as before,[10] then $\gamma_{T\pi\pi} < 0.3 \times 10^{-4}$, and $\gamma_{T\pi\pi}/\gamma_{\rho\pi\pi} < 0.12 \times 10^{-4}$. This is in qualitative agreement with the suppression factor $(e/2l)^{2l} = 0.02 \times 10^{-4}$ for $l = 5$, predicted from barrier pentration theory [Eq. (12)]. The somewhat precarious dependence of the quantitative aspect of this argument on the size of m is apparent. In addition, kR is not $\ll l$, but $\simeq \frac{1}{2}l$ in this case. However, as we wish only to establish a plausible illustration, it does not behoove us to dwell on these points at present.

Finally, we may view our results in the framework of the analytic properties of the trajectory α. If $\alpha(s)$ has only a right-hand cut and $\mathrm{Im}\,\alpha > 0$ for $s > 0$, then α has a Herglotz representation[13]

$$\mathrm{Re}\,\alpha(s) = A + Bs + \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{Im}\,\alpha(s')(1+ss')ds'}{(1+s'^2)(s'-s)}, \quad B > 0. \qquad (22)$$

If $B \neq 0$, then the linear term in s cannot be cancelled as $s \to \infty$ by the integral if

$$\lim_{s\to\infty} s^{\epsilon}\,\mathrm{Im}\,\alpha(s) = 0 \text{ for some } \epsilon > 0.$$

Proof. – The limit of the integral as $s\to\infty$ in that case equals $\int[\,\mathrm{Im}\alpha(s')ds'/s']$, which exists and is a constant. Now, for a linearly rising trajectory, $\mathrm{Im}\alpha(s)=s^{1/2}\Gamma\,\mathrm{Re}\alpha'(s)=s^{1/2}\Gamma B$, where Γ is the total width. Consistency requires that

$$s^{1/2}\Gamma \underset{s\to\infty}{\longrightarrow} 0$$

like some power of s. This is certainly more than fulfilled by our widths.

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[8]With $\alpha^{-1}\sim\ln J$, l is not substantially different from $J-J'+\mu^2$ (see Eq. 10). Hence we drop the $\leq$ sign.

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[10]Corresponding to 2π exchange, the edge of the double spectral function at high energies in systems with baryon number ≤ 1.

[11]In general, the decay width for an $I=1$ spin-J meson into two pions is

$$2\gamma\frac{J!}{(2J+1)!!}\left(\frac{k}{m}\right)^{2J-2}\frac{k^3}{m_J^{\,2}},$$

with the Lagrangian defined analogously to the one in the text. This formula and others relating to meson decay will be derived in a subsequent publication.

[12]The reasoning presumes no other dynamical suppression of 2π decay, such as is shown by the $f'(1500)$. Support for this assumption lies in the fact that lower members of the trajectory [$\rho(750)$ and $g(1650)$] have substantial 2π decays.

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η ASYMMETRY WITHOUT C NONCONSERVATION*

H. Yuta and S. Okubo
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 26 July 1968)

We calculate the charge asymmetry for the reaction $\pi^-p\to n\pi^+\pi^-\pi^0$ assuming C invariance for the η decay. We find that interference terms with a 3π background amplitude give a nonzero charge asymmetry. Assuming that the background in the η mass region (10 MeV) is about 10 % of the η signal, we obtain a maximum asymmetry of about 2 %, implying that the experimental asymmetry does not necessarily imply C nonconservation.

The discovery of apparent CP nonconservation[1] in the K_L decay has led to the suggestion[2] that C invariance might not hold for the electromagnetic interaction. Since then, a number of experiments[3-15] have been performed to find evidence for a C nonconservation in the electromagnetic decay of the eta meson. The summary of these experiments shows no strong evidence for C nonconservation in the eta decay and if C invariance is violated, the charge asymmetry is at most order of 10^{-2} or 10^{-3}, which is rather near to the value suggested by several theoretical estimates.[2]

However, it is the purpose of this note to show that such a small value of the charge asymmetry in η decay can be produced without C nonconservation by interference effects so that a presence of an η asymmetry does not necessarily imply C nonconservation. In this note, we would like to consider the reactions

$$\pi^-p\to n\eta,\quad \eta\to\pi^+\pi^-\pi^0 \tag{1}$$

$$\pi^-p\to n\pi^+\pi^-\pi^0, \tag{2}$$

and to show that an interference effect between the η and 3π-background amplitudes can create a charge asymmetry in the η-mass region without C nonconservation in the η decay. In the following discussion, we assume that C invariance holds for the η decay and that the charge asym-