THEORY OF CURRENTS– HOW TO BREAK THE SYMMETRY*

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A theory of currents is extended to include the symmetry-breaking effect without introducing extra currents. It is a nonspherical top in the internal space and found to be consistent with the mass-mixing model.

Recently the author proposed a model theory of currents.[1] It seems to be consistent with some basic principles of field theory, yet it does not discriminate any particle as elementary from the rest. In this sense this model or a more sophisticated version of it could be the "missing link" between the conventional field theory and the S-matrix theory.

Already some analyses have been made on the basis of this model. Bardakci, Frishman, and Halpern[2] showed that the model is a singular limit of Yang-Mills theory. Bardakci and Halpern[3] and Yoshimura and Sugawara[4] independently showed that there exists a canonical representation of the model. Gross[5] and Callan and Gross[6] showed that we can test whether this model itself can describe the real world or not. In my opinion it is quite possible that particles, even fermions, belong to a noncanonical representation of the model.

Here starting from SU(3)⊗SU(3) theory we present what we think is the most natural way to introduce symmetry breaking. As was shown in Ref. 4, the model in question is a spherical top in the internal space, the Schwinger constant being the moment of inertia. Obviously we can break the symmetry by deforming the top. The easiest way to get this deformed top theory is to follow the prescription given in Ref. 2: Write down the Yang-Mills Lagrangian and take the limit of $g_0 \to 0$, $m_0 \to 0$, and $(m_0/g_0)^2 \to C$ where g_0 is the coupling constant and m_0 is the mass of the vector meson (we call this the BFH limit). The problem is to find out what kind of symmetry-breaking term leads to the nonspherical top in the limit. The answer is

$$\mathcal{L} = -\tfrac14 F_{\mu\nu}^{\ a}F^{\mu\nu a} - \tfrac14 F_{\mu\nu}^{\ \prime a}F^{\prime\mu\nu a}$$
$$+\tfrac12 m_0^{\ 2}\{(\delta_{ab}+\epsilon d_{8ab})(\varphi_\mu^{\ a}\varphi^{\mu b}+\varphi_\mu^{\ \prime a}\varphi^{\prime\mu b}) + (\epsilon'\delta_{ab}+\epsilon'' d_{8ab})(\varphi_\mu^{\ a}\varphi^{\prime\mu b}+\varphi_\mu^{\ \prime a}\varphi^{\mu b})\},$$

where

$$F_{\mu\nu}^{\ a} = \partial_\mu\varphi_\nu^{\ a} - \partial_\nu\varphi_\mu^{\ a} - \tfrac12 g_0 f_{abc}(\varphi_\mu^{\ b}\varphi_\gamma^{\ c} + \varphi_\gamma^{\ c}\varphi_\mu^{\ b}),$$

$$F_{\mu\nu}^{\ \prime a} = \partial_\mu\varphi_\nu^{\ \prime a} - \partial_\nu\varphi_\mu^{\ \prime a} - \tfrac12 g_0 f_{abc}(\varphi_\mu^{\ \prime b}\varphi_\nu^{\ \prime c} + \varphi_\nu^{\ \prime c}\varphi_\mu^{\ \prime b}),$$

$$\varphi_\mu^{\ a} = 2^{-\frac12}(\tilde V_\mu^{\ a} + \tilde A_\mu^{\ a}),\quad \varphi_\mu^{\ \prime a} = 2^{-\frac12}(\tilde V_\mu^{\ a} - \tilde A_\mu^{\ a}).$$

This is an example of a mass mixing model in the terminology of Kroll, Lee, and Zumino.[7] The current-mixing model reduces to the original spherical top in the limit. The breaking parameters ϵ, ϵ', and ϵ'' correspond to breaking from SU(3)⊗SU(3) into SU(2)⊗SU(2), SU(3), and SU(2), respectively. Defining the currents through the renormalized fields

$$V_\mu^{\ a} = (m_0^{\ 2}/g_0)(\delta_{ab} + \epsilon d_{8ab} + \epsilon' + \epsilon'' d_{8ab})\tilde V_\mu^{\ b},$$

$$A_\mu^{\ a} = (m_0^{\ 2}/g_0)(\delta_{ab} + \epsilon d_{8ab} - \epsilon' - \epsilon'' d_{8ab})\tilde A_\mu^{\ b},$$

and taking the BFH limit we get the following expressions for the commutators, $\theta_{\mu\nu}$, and the equations of motion:

$$[V_0^a(x), V_0^b(y)] = [A_0^a(x), A_0^b(y)] = if_{abc}V_0^c(x)\delta(x-y),$$

$$[V_0^a(x), A_0^b(y)] = if_{abc}A_0^c(x)\delta(x-y),$$

$$[V_0^a(x), V_i^b(y)] = if_{abc}(\eta_b^V/\eta_c^V)V_i^c(x)\delta(x-y) - ic\delta_{ab}\eta_b^V\partial_i\delta(x-y) \quad \text{(no sum for } b\text{)},$$

$$[A_0^a(x), A_i^b(y)] = if_{abc}(\eta_b^A/\eta_c^V)V_i^c(x)\delta(x-y) - ic\delta_{ab}\eta_b^A\partial_i\delta(x-y) \quad \text{(no sum for } b\text{)},$$

$$[V_0^a(x), A_i^b(y)] = if_{abc}(\eta_b^A/\eta_c^A)A_i^c(x)\delta(x-y),$$

$$[A_0^a(x), V_i^b(y)] = if_{abc}(\eta_b^V/\eta_c^A)A_i^c(x)\delta(x-y), \tag{1}$$

where

$$\eta_a^V = 1 + \epsilon d_{8aa} + \epsilon' + \epsilon'' d_{8aa},$$

and

$$\eta_a^A = 1 + \epsilon d_{8aa} - \epsilon' - \epsilon'' d_{8aa};$$

$$\theta_{\mu\nu} = \frac{1}{2c}\sum_a \frac{1}{\eta_a^V}[V_\mu^a V_\nu^a + V_\nu^a V_\mu^a - g_{\mu\nu}(V_\rho^a V_\rho^a)] + \sum_a \frac{1}{\eta_a^A}[A_\mu^a A_\nu^a + A_\nu^a A_\mu^a - g_{\mu\nu}(A_\rho^a A_\rho^a)]; \tag{2}$$

$$\partial^\mu V_\mu^a = -\frac{1}{2c}\sum_{b,c} f_{abc}\left[\frac{1}{\eta_c^V}(V^{\mu b}V_\mu^c + V_\mu^c V^{\mu b}) + \frac{1}{\eta_c^A}(A^{\mu b}A_\mu^c + A_\mu^c A^{\mu b})\right],$$

$$\partial^\mu A_\mu^a = -\frac{1}{2c}\sum_{b,c} f_{abc}\left[\frac{1}{\eta_c^V}(A_\mu^b V^{\mu c} + V^{\mu c}A_\mu^b) + \frac{1}{\eta_c^A}(V_\mu^b A^{\mu c} + A^{\mu c}V_\mu^b)\right],$$

$$\partial_\mu V_\nu^a - \partial_\nu V_\mu^a = \frac{1}{2c}\sum_{b,c} f_{abc}\eta_a^V\left[\frac{1}{\eta_b^V\eta_c^V}(V_\mu^b V_\nu^c + V_\nu^c V_\mu^b) + \frac{1}{\eta_b^A\eta_c^A}(A_\mu^b A_\nu^c + A_\nu^c A_\mu^b)\right],$$

and

$$\partial_\mu A_\nu^a - \partial_\nu A_\mu^a = \frac{1}{2c}\sum_{b,c} f_{abc}\eta_a^A\left[\frac{1}{\eta_b^A\eta_c^V}(A_\mu^b V_\nu^c + V_\nu^c A_\mu^b) + \frac{1}{\eta_b^V\eta_c^A}(V_\mu^b A_\nu^c + A_\nu^c V_\mu^b)\right]. \tag{3}$$

It is clear from the expressions for the commutators that Gell-Mann's assumption, that the current commutation relations remain unchanged even when the symmetry is broken, is satisfied only for the time-time commutators in our model.[9] Most of the current algebra calculations depend only on the time-time commutation relations. There we have no trouble. When the weak Hamiltonian comes in we have to use the integrated space-time commutators. Here all the calculations done in the past few years should be re-examined because of our factor $\eta_a^{V,A}$. Yet the effect will be around 10% which is characteristic of symmetry breaking. We believe there is nothing so far against our commutation relations. We therefore propose here how to test these commutation relations.

We start from the expression

$$i\int e^{ikx}\langle 0|T[V_\mu{}^a(x)V_\nu{}^b(0)]|0\rangle=\int\frac{1}{k^2+m^2}\left\{\rho_{ab}{}^{(1)}(m^2)\left(g_{\mu\nu}+\frac{k_\mu k_\nu}{m^2}\right)+\rho_{ab}{}^{(0)}(m^2)k_\mu k_\nu\right\}dm^2$$
$$+\delta_{\mu0}\delta_{\nu0}\int\left[\frac{\rho_{ab}{}^{(1)}(m^2)}{m^2}+\rho_{ab}{}^{(0)}(m^2)\right]dm^2.$$

By multiplying both sides by k_μ and taking the Bjorken limit[10] we get

$$\int[\rho_{ab}{}^{(1)}(m^2)/m^2+\rho_{ab}{}^{(0)}(m^2)]dm^2=c\delta_{ab}\eta_b{}^V\quad\text{(no sum)}.$$

This is essentially the Lehmann formula for the z factor.[11] It is frequently called Weinberg's first sum rule in recent literature.[12] Our sum rule is consistent with the mass-mixing model in contrast to usual assumptions.[13] We neglect the contribution of $\rho_{ab}{}^{(0)}$ and put in vector mesons to $\rho_{ab}{}^{(1)}(m^2)$ following other people.[13] We also assume that G_ρ or f_ρ etc. defined in Ref. 13 are SU(3) symmetric.[14] Then we get, together with

$$\Gamma(\omega\to e^+e^-)/\Gamma(\varphi\to e^+e^-)=m_\omega/m_\varphi+m^2\theta_Y,$$

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to e^+e^-)=\frac{3m_\rho{}^2}{4m_{k^*}{}^2-m_\rho{}^2}[m_\varphi\Gamma(\varphi\to e^+e^-)+m_\omega\Gamma(\omega\to e^+e^-)].$$

Notice that the factor on the right-hand side is missing in the case of symmetric Schwinger constant.[13] This sum rule for the leptonic processes can be tested in the near future when the colliding-beam experiments proceed. Furthermore if we naively extend our commutation relations to include the SU(3) singlet current[15] we get the following results: For the mixing angles,

$$\theta=\theta_Y=\theta_N=\tan^{-1}[(3m_\varphi{}^2-4m_{k^*}{}^2+m_\rho{}^2)^2/(4m_{k^*}{}^2-m_\rho{}^2-3m_\omega{}^2)]^{1/2};$$

for the masses,

$$\tfrac{4}{3}m_\omega{}^2(m_{k^*}{}^2-m_\rho{}^2)^2=(m_\varphi{}^2-\tfrac{4}{3}m_{k^*}{}^2+\tfrac{1}{3}m_\rho{}^2)(3m_\rho{}^2m_{k^*}{}^2-\tfrac{3}{2}m_\rho{}^4-m_{k^*}{}^2m_\omega{}^2-\tfrac{1}{2}m_\rho{}^2m_\omega{}^2).$$

The mass formula is very well satisfied. In fact when $m_\rho=m_\omega$ this reduces to the well-known nonet formula $2m_{k^*}{}^2=m_\rho{}^2+m_\varphi{}^2$ and we also get $\tan^2\theta=\frac{1}{2}$ consistent with nonet theory.

Of course we understand that our results are for the test of the mass-mixing model and not for our theory of currents. But we will be very encouraged if these are satisfied. Colliding-beam experiments are most interesting from this point of view.

Finally it would be very interesting to see what we can get from our equations of motion assuming the existence of particles following Refs. 5 and 6. We are working along this line and hope to be able to present our results soon.

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[8]The property of d_{8ab} guarantees the conservation of the isospin and the hypercharge currents.

[9]We remark that this change of the space-time commutators is essential in our model. In fact, if the Schwinger constants are not symmetric, simple application of the Jacobi identity shows that the non-Schwinger terms should

be different from the symmetric case.

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[14]This assumption is satisfied in the mass-mixing model but we are not sure if it can be derived from our model.

[15]Which perhaps we should not do because, if we include the singlet current in our theory naively, it decouples from the system ($f_{0ab}=0$) and we get a massless scalar meson.

POSITIVE DETERMINATION OF AN INTERSTELLAR MAGNETIC FIELD BY MEASUREMENT OF THE ZEEMAN SPLITTING OF THE 21-cm HYDROGEN LINE

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Fields of the order of 2×10^{-5} G exist in the Perseus spiral arm in the direction of the radio source Cassiopeia A.

A new attempt to determine the interstellar magnetic field strength by measurement of the Zeeman splitting of 21-cm neutral hydrogen spectra has been successful. The results indicate that fields of 20×10^{-6} G (20 μG) exist in the Perseus spiral arm in the direction of Cassiopeia A, whereas no fields of this order have been found in four local spectral features.

The experiment was done at the National Radio Astronomy Observatory, using the new 416-channel digital spectrometer in conjunction with the 140-ft telescope. The dish was illuminated by a pair of crossed dipoles mounted in a shallow circular waveguide so as to produce circular beam patterns. The dipoles fed a hybrid directly, and the digital correlator alternately sampled the left- and right-hand polarized outputs by means of a reed switch operating at 1 Hz. The 416-channel autocorrelator was operated as two separate spectrometers of 192 channels, each with different overall bandwidths. Those used were 156, 312, and 625 kHz, giving an effective resolution of 0.98, 1.97, and 3.94 kHz/channel, respectively.

Four regions were examined for Zeeman splitting effects. These were the absorption spectra of Cas A ($l^{\rm II}=111.5°$, $b^{\rm II}=0.2°$) and Tau A ($l^{\rm II}=184.5°$, $b^{\rm II}=-5.8°$), and two narrow high-latitude emission spectra ($l^{\rm II}=350°$, $b^{\rm II}=+25°$ and $l^{\rm II}=11°$, $b^{\rm II}=+31°$, respectively).

The digital correlator was switched between left- and right-hand polarizations. These were recorded separately and later combined in the computer so that scans presenting both the difference between the two polarizations, as well as one of the polarizations alone, were obtained. The latter gave a comparison spectrum that was used in removing residual effects which had the absorption profile shape and were of the order of 1.1 % for Cas A.

The absorption spectra of Cas A shows two widely separated components, one due to matter in the local spiral arm (Orion arm) and the other to matter in the Perseus spiral arm. The latter in turn shows two separated features with considerable structure in them.[1] For this reason, the Perseus arm feature has not been closely studied in the Zeeman experiments in the past. It was suspected that the arm showed many separate clouds (about eight to ten), and therefore, the resultant magnetic field splittings might be incomprehensibly mixed.

The Orion arm feature has been studied previously and the limit to the field, with no regard to possible structures or component clouds, was set as 4 μG.[2,3] This limit has been reduced in the present experiment to $+0.55\pm0.87$ μG. The data used in the work reported here were from one bank of 192 channels of the spectrometer operated with a 625-kHz overall bandwidth, i.e., 3.25 kHz/channel.

The data in the lower half of Fig. 1 show 16.3 hours' integration plotted as the difference between right- and left-hand polarization incident on the feed. Also shown is the absorption profile itself, obtained simultaneously. Zeeman splitting effects will manifest themselves as the derivative of the observed absorption lines, and predicted