[1]S. Mandelstam, Phys. Rev. 168, 1884 (1968).

[2]D. Z. Freedman and J. M. Wang, Phys. Rev. 160, 1560 (1967).

[3]The O(3, 1) classification is, of course, only valid at $t=0$.

[4]An $M=1$ conspiracy has opposite parity Regge trajectories intersecting at $t=0$. For details on Lorentz pole theory, see M. Toller, Nuovo Cimento 53A, 671 (1967), and 54A, 295 (1968).

[5]By σ we mean an S-wave state of two pions, of whatever isospin and energy.

[6]H. Högaasen and P. Salin, Nucl. Phys. B2, 657 (1967).

[7]An s^{α} contribution to $f_{\frac{1}{2},-\frac{1}{2},0,0}$ would require an $M=0$ trajectory (A_1 type) at $J=\alpha+1$. Here we deal with $M=0$ or $M=1$ trajectories with leading $J=\alpha$.

[8]It must, if M is to be defined from on-shell considerations. We might mention that our special results for the $\pi\sigma$ and $\pi\rho$ couplings are easily derivable without the explicit use of conspiracy conditions from the off-shell invariant-amplitude approach given in R. F. Sawyer, Phys. Rev. 167, 1372 (1968). The argument given, leading to Eq. (4), is similar to one due to M. Le Bellac, Phys. Letters 25B, 524 (1967). Kinematic arguments leading to (2) and (3) can be found in L. Jones and H. K. Shepard (to be published).

[9]This theorem is implicit in the general residue formula for the unequal-mass case given in the paper by Cosenza, Sciarrino, and Toller (to be published),

$$\beta_{\lambda_1\lambda_3}^{\mu\sigma} \sim t^{\frac{1}{2}[-\lambda+1+|M-|\lambda_1-\lambda_3||]}.$$

[10]This is true up to certain pole terms for the pion amplitude (i.e., the Regge vertex), which correspond to bremsstrahlung graphs in the complete amplitude for Regge exchange. These graphs do not obey the conspiracy condition (1). But we can conclude that S-wave πN scattering, for example, vanishes in the $M_\pi \to 0$ limit.

[11]S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

[12]The $M=1$ pion trajectory raises an entirely distinct question: How can it possibly make a physical $J=0$ particle at an energy $E=M_\pi$ so near to the O(3, 1) symmetry point? In a model in which the symmetry breaking may be treated in a perturbation expansion in E it cannot. However, in an ingenious model due to Blankenbecler and Sugar it can, through mixing with a nearby $M=0$ trajectory. Frazer, Lipinski, and Snider have manufactured a similar model. It should be noted that neither of these models gives a dynamical understanding of the hypothesis of partially conserved axial-vector current. Furthermore, neither model is, in our opinion, even consistent with the interpretation of current-algebra results as exact results in a $M_\pi=0$ world, which are good approximate results in the $M_\pi=M_\pi$ world.

[13]We use the notation of R. F. Sawyer, Phys. Rev. 167, 1372 (1968).

[14]This type of pion coupling is frequently called evasive. But $M=0$ is the correct designation. An $M=0$ pion necessarily evades in the $\bar{N}N$ system, but it can couple to other systems at $t=0$. $M=0$ is a quality intrinsic to a trajectory; evasion is not.

[15]F. Arbab and J. Dash, Phys. Rev. 163, 1603 (1967).

[16]R. L. Omnes, Phys. Rev. 168, 1893 (1968).

FORM FACTOR RATIO ξ FROM A MEASUREMENT OF $K_{\mu3}^{+}$:K_{e3}^{+} BRANCHING RATIO

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By measuring the branching ratio of $K_{\mu3}^{+}$ relative to K_{e3}^{+} we have arrived at a value of $\xi(0)=f_-(0)/f_+(0)=-0.08\pm0.15$. This is in good agreement with the Callan-Treiman relation.

We describe a measurement of the branching ratio of $K_{\mu3}^{+}$ relative to K_{e3}^{+}. We deduce from this a value of the parameter ξ, the ratio of the form factors for vector coupling in K_{l3} decay, where

$$\xi(0)=f_-(0)/f_+(0),$$

$$f_\pm(q^2)=f_\pm(0)[1+\lambda_\pm(q^2/m_\pi^2)],$$

and

$$q^2=(p_K-p_\pi)^2=m_K^2+m_\pi^2-2m_KE_\pi.$$

There is a discrepancy between the values of ξ deduced from branching ratio and polarization methods.[1] Apart from a result given some time ago,[2] investigations of the relative muon and positron semileptonic rates indicate a comparatively high $K_{\mu3}$:K_{e3} ratio, and hence a positive value, ~0.3, for ξ.[3] On the other hand, polarization experiments have shown a systematic shift towards negative values for ξ, averaging $\xi(0)=-1.25\pm0.32$.[4]

We have carried out a K^+ experiment at Nimrod in which positron- and muon-decay rates were observed simultaneously in the same apparatus (Fig. 1). The K^+ beam entered the apparatus through a channel cut in the plates of spark chamber B3. The four brass-plate chambers B1-B4 were required for a Dalitz-plot analysis of the decays which will be reported at some later date.

Approximately 1000 K^+ per Nimrod pulse stopped in the beryllium plates of a small spark chamber. The stopping of a particle was observed electronically by the counter telescope ($123\bar{4}\bar{5}$). Decays were accepted between 4 and 34 nsec later by counters 6 and 8, and the spark chambers were triggered if a particle either was observed in the Cherenkov counter or stopped in the aluminum-plate range chamber. The trajectory was recorded by sonic spark chambers S1-S4. Events which scattered from the pole faces of the magnet were rejected by means of counter 7. The trigger criterion was ($123\bar{4}\bar{5}$) ($68\bar{7}$) ($\overline{9\bar{C}}$).

The Cherenkov counter was extensively tested and calibrated.[5] Its efficiency in the momentum band concerned here was $(98.7 \pm 0.1)\,\%$ for positrons and $(0.4 \pm 0.05)\,\%$ for muons.

For the branching-ratio determination, measured momentum limits of 110-150 MeV/c were imposed at the magnet.[6] By using the same momentum interval for both leptons, apparatus-dependent systematic effects were minimized. The choice of the limits was influenced by the size of the pion background and the ability of the 24-gap range chamber to accommodate the range straggling of both muons and pions.

The basic resolution of the spectrometer was $(1.40 \pm 0.03)\,\%$. The overall resolution was $(2.27 \pm 0.13)\,\%$ resulting from the spread in ionization losses in the beryllium chamber. The solid-angle acceptance of the spectrometer was constant at $0.42\,\%$ of 4π sr for momenta between 110 and 150 MeV/c.

Muons were selected by their range-momentum correlation. Figure 2 shows the separation of pions from muons where the measured range is plotted as a deviation from the mean empirical range of the muons. The standard deviation of the muon distribution corresponds to ± 1.2 plates or ± 1.3 g/cm^3 at 130 MeV/c. In our momentum window $(15.3 \pm 0.75)\,\%$ of non-Cherenkov events were identified as pions. The efficiency for misidentifying positrons as muons was insignificant.

In the treatment of the data, $70\,\%$ of the frames were remeasured and checked.

It was necessary to check in detail that the reconstruction of the particle trajectories through the spectrometer had no relative bias between positrons and muons. The four sonic spark chambers had two gaps each. The 16 available coordinates were fitted for minimum χ^2, to possible theoretical trajectories. Weights for the components of the χ^2 were estimated from multiple-scattering calculations and from the observed

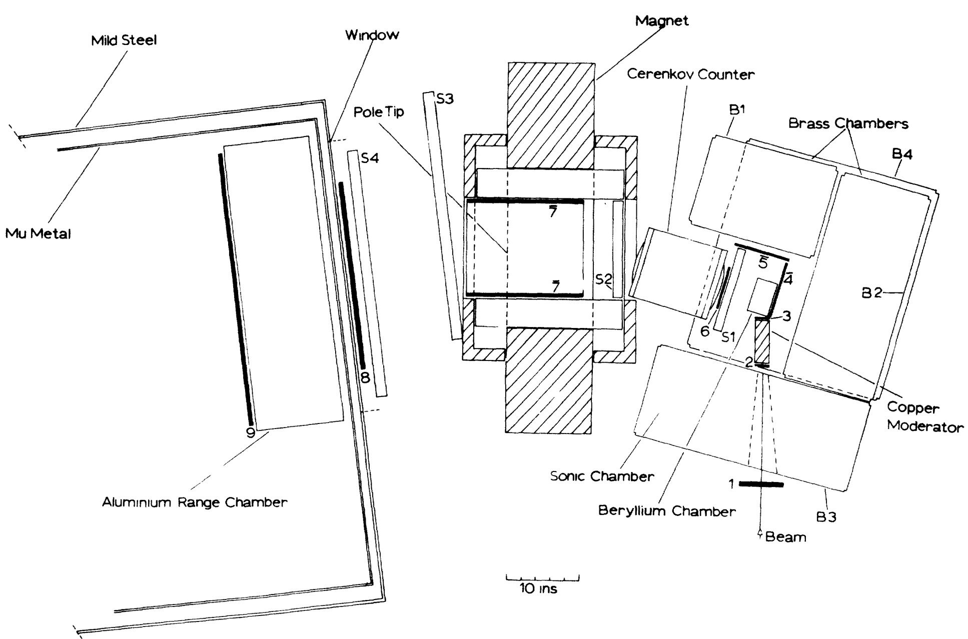

FIG. 1. Plan view of apparatus.

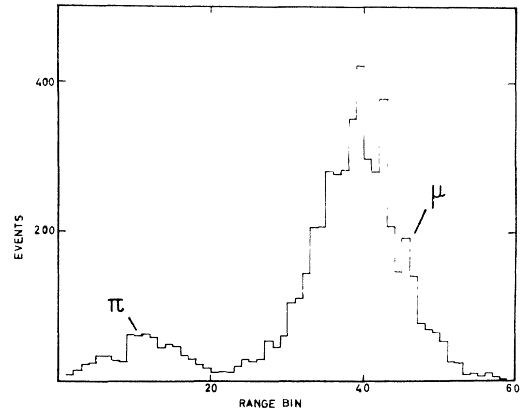

FIG. 2. Range separation of pions and muons. The separation between the peaks corresponds to 7.1 g/cm^2 at 130 MeV/c.

relative displacements of the pairs of sparks in each chamber.

A limit, which excluded 9% of the events, was imposed on the values of χ^2. The principle reason for this limit was to eliminate the large number of pions which decayed in flight within the limits of the sonic system. An examination of high-χ^2 orbits for the uncontaminated positron and $K_{\mu 2}$ events allowed us to restrict the uncertainty on the branching ratio due to this cut to ±1%.

The χ^2 weights were corrected to equalize the mean χ^2 values of positrons and muons within this limit. After a close comparison of the distributions we conclude that the treatment of the two leptons is identical within statistics.

There was a further source of background. In the region before the first sonic chamber, pions decaying in flight became indistinguishable from $K_{\mu 3}$ muons. However, within our momentum band the minimum direction change in their trajectories was 0.12 rad. Measurements of the tracks in the beryllium-plate chamber were used to calculate the effect, and the result agreed with a direct Monte Carlo computation. The contribution was (7.0 ± 0.45)% of the number of observed muons.

The process $K \to \mu\nu\gamma$ was also present. The rate was calculated from the theoretical spectrum,[7] assuming no structure amplitude. It involved a subtraction of (3.6 ± 0.4)% of the data.

The corrected numbers appearing in our momentum band, 5601 muons and 7770 positrons, are in the ratio 0.721 ± 0.017.

This number has been related to the form factors by calculating spectra from the K_{l3} matrix

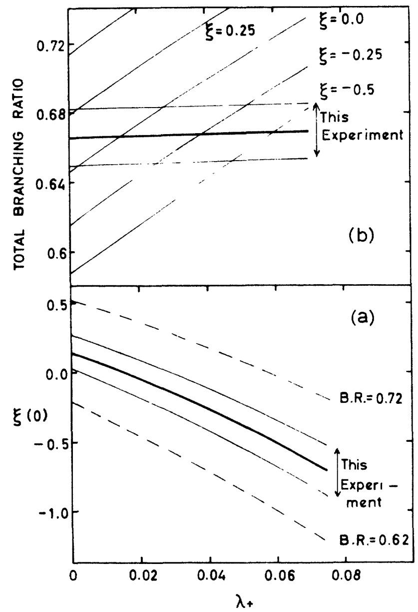

FIG. 3. Relation between experimental branching ratio and the total branching ratio.

elements for a vector interaction. Radiative corrections[8] have been taken into account for the positron spectra, but in the experimental momentum range they were barely significant. These calculated spectra were modified to allow for energy losses in all materials as far as the second sonic spark chamber, taking into account the distribution of the starting points of trajectories. For the positrons, bremsstrahlung losses were included.[9]

The contributions to our error from several experimental sources have been investigated. The effects of uncertainties in momentum resolution, momentum calibration, momentum dependence of the spectrometer solid angle, and ionization energy loss are insignificant. A possible error of ±0.5% due to uncertainty in the effects of bremsstrahlung has been allowed.

Our results are shown in Fig. 3. In Fig. 3(a) is the relation between ξ and λ_+, for $\lambda_- = 0$, given by this experiment. If we accept $\lambda_+ = 0.023$,[4] then

$$\xi(0) = -0.08 \pm 0.13.$$

Including the experimental uncertainty in λ_+ (±0.008) the error on ξ becomes ±0.15.

The possibility that λ_- is large[10] leads us to remark that the variation of $\xi(0)$ and its experimental errors is fairly linear in λ_-. At $\lambda_- = 0.1$ and $\lambda_+ = 0.02$ our experiment would give

$$\xi(0) = -0.13 \pm 0.25.$$

For the overall branching ratio, Fig. 3(b) indicates ($\lambda_+ = 0.023$, $\lambda_- = 0$) that

$$\Gamma(K_{\mu 3})/\Gamma(K_{e3}) = 0.667 \pm 0.017.$$

The $K_{\mu 3}{}^+$ branching ratio has been calculated [assuming a value of $(4.94 \pm 0.11)\,\%$ for $K_{e3}{}^+$ branching ratio[11]] to be

$$\Gamma(K_{\mu 3}{}^+)/\Gamma(\text{all}) = (3.29 \pm 0.11)\,\%.$$

In our restricted momentum window the spectrum of muons is not very sensitive to the form factors. There is, however, a second value, $\xi = -5.1$, associated with our branching ratio. The theoretical spectrum for this gives a χ^2 of 40 for 6 degrees of freedom when compared with the experimental spectrum. For $\xi = -0.08$ we obtain $\chi^2 = 7.2$.

Our value for the form-factor ratio is 1.7 standard deviations from the well-known prediction (−0.3) of a simple $K^*(890)$ intermediate state model.

A comparison can be made between the $K_{\mu 2}$ and $K_{\mu 3}$ amplitudes using the Callan-Treiman[12] relation. The momentum-transfer extrapolation has been performed between $q^2 = m_K{}^2$ and 0 assuming $\lambda_+ = \lambda_-$. We find for $\lambda_+ = 0.023$ that the theoretical value, $\xi = 0.0 \pm 0.05$, is in excellent agreement with our experimentally deduced value.

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